Unusually strong attraction in the presence of continuum bound state

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(Received 3 June 1992)

The result of few-particle ground-state calculation employing a two-particle nonlocal potential supporting a continuum bound state in addition to a negative-energy bound state has occasionally revealed unusually strong attraction in producing a very strongly bound ground state. In the presence of the continuum bound state the difference of phase shift between zero and infinite energies has an extra jump of π as in the presence of an additional bound state. The wave function of the continuum bound state is identical with that of a strongly bound negative-energy state, which leads us to postulate a pseudo bound state in the two-particle system in order to explain the unexpected attraction. The role of the Pauli forbidden states is expected to be similar to these pseudo states.

PACS number(s): 21.45.+v, 03.65.Nk

I. INTRODUCTION

Usually, in quantum mechanical problems with short-range local potentials the binding energy of the two-particle system reflects the strength of the underlying potential. Also Levinson has shown that for a short-range local potential the phase-shift at zero energy $\delta(0)$ and the phase-shift at infinite energy $\delta(\infty)$ are related by the condition [1]

$$\delta(0) - \delta(\infty) = N\pi, \tag{1}$$

where the quantity $N \equiv [\delta(0) - \delta(\infty)]/\pi$ is the number of bound states and should be directly related to the strength of the potential.

The above simple argumentation does not necessarily work in the case of a general nonlocal potential. Such nonlocal potentials appear in various areas of physics, for example, in the resonating group method [2], or in the Feshbach unified theory of nuclear reactions [3]. The nonlocal intercluster interaction in the resonating group method seems to be, in general, stronger than an equivalent local counterpart. Also, the expression $\delta(0) - \delta(\infty)$ may have an extra jump of π (or several π's) not dictated by Levinson's theorem (1). This is true for the S-wave α-α scattering in the resonating group method [2] and for the S-wave spin quartet neutron-deuteron scattering [4] using the Faddeev equations [5]. These facts suggest the existence of excited state(s) not present in the system. They are the so-called Pauli forbidden states [1, 4] which should contribute to N on the right-hand side of Eq. (1).

All the above-mentioned features of a forbidden state can be simulated in a simple nonlocal rank-one potential possessing a continuum bound state (CBS) [1, 6] in addition to the usual negative-energy bound state(s). The difference $\delta(0) - \delta(\infty)$ for such a potential has the required extra jump of π [1, 4]. Somehow, the behavior of the phase shift or the wave function suggest the presence of an excited state of the system. In the presence of a CBS the binding energy of the usual bound state does not necessarily reflect the strength of the potential. Whenever this potential is used in a multiparticle system it produces stronger binding compared to that produced by an "equivalent" local potential which produces the same negative-energy bound state [7].

A CBS is an S-matrix pole on the real positive-energy axis [1]. The association with the bound state lies in the fact that the corresponding wave function, ϕ, has no outgoing propagation. It is in fact a localized function of momentum $\phi(p)$ and its Fourier transform $\phi(r)$ decreases exponentially like a bound-state wave function. The number N in the Levinson's theorem (1) should include all exponentially decaying states – bound state(s) and CBS. A virtual state is exponentially growing and does not contribute to the Levinson's theorem, although it implies attraction.

Tabakin [8] constructed a rank-one separable NN potential with a two-term form factor, capable of reproducing S-wave NN phase shifts reaching moderately high energies where these phase shifts become negative in agreement with experiment. This potential supports a CBS in addition to a bound deuteron. A class of similar potentials has been identified later. The use of these potentials in (three-nucleon) $3N$ and four-nucleon bound-state calculations [7] leads to ground states of unusually large binding of several hundred MeV's in addition to a weakly bound excited state. [7, 9, 10]

One may ask if the Tabakin potential reproduces the experimental NN phase shifts reasonably well then why does it fail drastically in the $3N$ bound-state problem? Strictly speaking, the experimental NN phase shifts become negative at higher energies and eventually goes to zero at infinite energy in accordance with the Levinson's theorem reflecting the presence of a single bound deuteron. On the other hand, the Tabakin phase shift

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becomes negative and goes to $-\pi$ at infinite energy reflecting the presence of a CBS in addition to the bound deuteron in violation of the usual Levinson's theorem (1); for a Tabakin-type potential $\delta(0) - \delta(\infty) = 2\pi$.

We shall see that because of an unexpected pole cancellation the momentum-space CBS wave function is found to be independent of the energy of the CBS and be identical with that of a negative-energy bound state of a one-term separable potential. Both the binding-energy and the potential parameters of this negative-energy bound state, which we shall call a pseudo bound state, are determined by the parameters of the original potential. There are no free parameters in defining the pseudo bound state. We shall see that whenever the energy of this pseudo bound state is large the three- and four-particle systems experience bound state collapse (BSC). If we would like to use a single energy to reflect upon the strength of these nonlocal potentials in situations of collapse, it is not the binding energy of the bound state but the binding energy of the pseudo state which should reflect the real strength of the potential.

First, we illustrate the idea of the pseudo state in the case of the S wave spin triplet NN channel. We use a nonlocal separable potential, possessing a CBS but no real bound state, to simulate the bound state and scattering properties in this channel. This nonlocal potential simulates a pseudodeuteron at the correct energy. Consequently, the t matrix for the model containing the CBS simulates the actual state of affairs reasonably well except near the deuteron pole.

In Sec. II we present the separable potential model that we employ. We also provide arguments for replacing the CBS by the pseudo bound state. In Sec. III we illustrate our idea in the case of the NN problem in the S-wave spin-triplet channel. In Sec. IV we present the alternative model incorporating the pseudo bound state. We also compare the results for the original nonlocal separable potential model and the alternative model for the three-particle system and find that the alternative model provides a realistic description of the actual state of affairs in situations of collapse. Finally, in Sec. V we present some concluding remarks.

II. THE MODEL

We consider the following Tabakin-type S-wave two-particle potential in the momentum space [6]

$$V(p,q) = \lambda g(p) g(q), \tag{2}$$

with

$$g(p) = \frac{\alpha_1}{(p^2+\beta_1^2)} - \frac{\alpha_2}{(p^2+\beta_2^2)} \tag{3}$$

$$= \frac{\kappa(p^2 - p_c^2)}{(p^2+\beta_1^2)(p^2+\beta_2^2)}, \tag{4}$$

where $\kappa \equiv \alpha_1 - \alpha_2$ and $p_c^2 \equiv (\alpha_2\beta_1^2 - \alpha_1\beta_2^2)/\kappa$ is the position of the CBS. All the equations of this study are written in units $\hbar = 2\mu = 1$, where μ is the reduced mass. For the NN system the calculation was performed with $\hbar^2/2\mu = 41.47$ MeV fm^2.

The two-particle t matrix for this potential, at energy $E = k^2$, is given by

$$t(p,q,k^2) = g(p)\frac{\lambda}{\tau(k^2)} g(q), \tag{5}$$

where

$$\tau(k^2) = 1 - \frac{2\lambda}{\pi}\int_0^\infty \frac{g^2(p)p^2dp}{k^2 - p^2 + i0}. \tag{6}$$

For potential (2) the momentum space Schrödinger equation for a bound state or CBS at energy E_b, described by the wave function ϕ, is given by

$$\phi(p) = \frac{2\lambda}{\pi}\frac{g(p)}{E_b - p^2 + i0}\int_0^\infty q^2 dq\, g(q)\phi(q) \tag{7}$$

$$= C\frac{g(p)}{E_b - p^2 + i0}. \tag{8}$$

The normalization constant C is determined by the condition

$$\frac{2}{\pi}\int_0^\infty \phi^2(p)p^2dp = 1. \tag{9}$$

If the system has a bound state at $E_b = -\gamma^2$, its wave function is given by

$$\phi(p) = -C\frac{g(p)}{\gamma^2 + p^2}. \tag{10}$$

In the present study the binding energy γ^2 of the NN system is always fixed at the value (2.225/41.47) fm^{-2}, when we vary the parameters of potential (2). This corresponds to a deuteron binding of 2.225 MeV.

If the potential (2) possesses a CBS at energy p_c^2, in addition to $g(p_c) = 0$ we also have $\tau(p_c^2) = 0$, and its wave function $\phi_c(p)$ is given by

$$\phi_c(p) = -C\frac{g(p)}{p^2 - p_c^2} \tag{11}$$

$$= -C\frac{\kappa}{(p^2+\beta_1^2)(p^2+\beta_2^2)}, \tag{12}$$

where we have substituted for $g(p)$ from Eq. (4). Unlike the bound-state wave function (10) which depends on binding energy γ^2, the CBS wave function (12) is independent of energy p_c^2. We note that at large distances the CBS wave function decays exponentially as $\exp(-\beta_i r)$ and not as $\exp(-p_c r)$. A comparison between Eqs. (10) and (12) reveals that the CBS wave function (12) is identical with the bound-state wave function (10) provided that we identify the binding energy γ^2 with β_i^2 and the form factor $g(p)$ with $(p^2+\beta_j^2)^{-1}$, where β_i (β_j) is the smaller (greater) of β_1 and β_2. This association is made because in the configuration space the bound-state wave function of binding γ^2 should behave as $\exp(-\gamma r)$ asymptotically.

The CBS wave function (12) has the behavior of a normal negative-energy bound-state wave function with a purely attractive separable potential. It is this negative-energy bound state which we call a pseudo bound state which will be seen to play a crucial role in predicting the collapse.

The phase-shift difference $\delta(0) - \delta(\infty)$ of potential (2-4) has an additional jump of π not accounted for by the normal bound state, or its wave function has a node [1] at short distances reflecting the presence of a second bound state. This is the pseudo bound state predicted by the CBS.

III. ILLUSTRATION IN A SIMPLE MODEL

We consider the NN problem in the S-wave spin-triplet channel employing a purely attractive Yamaguchi potential. This is the special case of potential (2) with $\alpha_2 = 0$. The parameters of this potential are determined by fitting to a deuteron binding of 2.225 MeV and a NN spin triplet scattering length of 5.4 fm. This results in $\lambda\alpha_1^2 = -7.45$ fm^{-3} and $\beta_1 = 1.4$ fm^{-1}.

We would like the full nonlocal potential (2-4), possessing only a CBS and no true bound state, to simulate the results of the above Yamaguchi potential via a pseudo deuteron of 2.225 MeV binding. The discussion related to Eq. (12) suggests that we choose $\beta_1 = 0.23$ fm^{-1}, and $\beta_2 = 1.4$ fm^{-1}; β_1^2 is the deuteron binding. The parameter α_2 is taken to be 1. We would like this potential to have a CBS. The position of this CBS, p_c^2, is still arbitrary. Once this position is given the other parameters of the potential, λ and α_1, are determined using the conditions $g(p_c) = 0$ and $\tau(p_c^2) = 0$. We varied the position of the CBS a little in order to have a good agreement between the calculations of the two potentials. Finally, we used $p_c = 0.1$ fm^{-1}, $\alpha_1 = 0.032$, and $\lambda = -9.57$ fm^{-3}. The result is very weakly sensitive to the value of p_c provided that for p_c^2 we choose an energy smaller than deuteron binding.

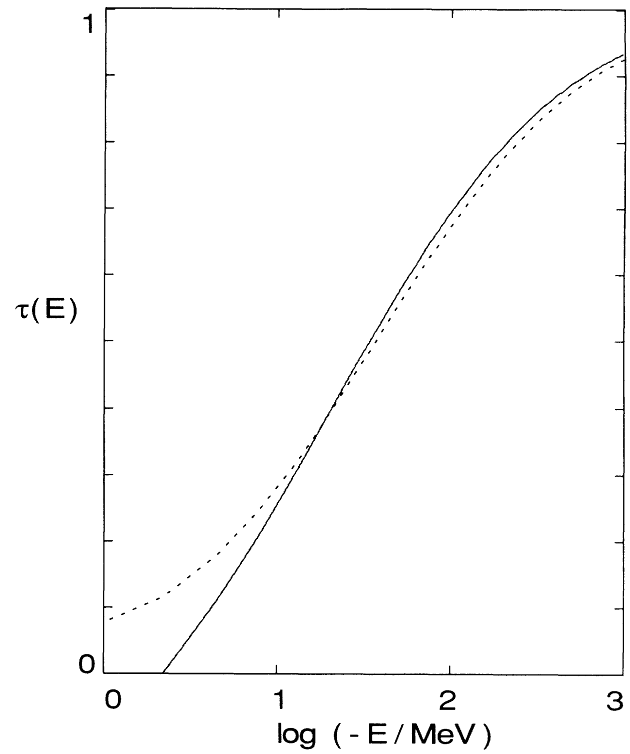

FIG. 2. The Fredholm determinant $\tau(k^2)$ for the above Yamaguchi (dashed line) and Tabakin-type (full line) potentials at negative energies.

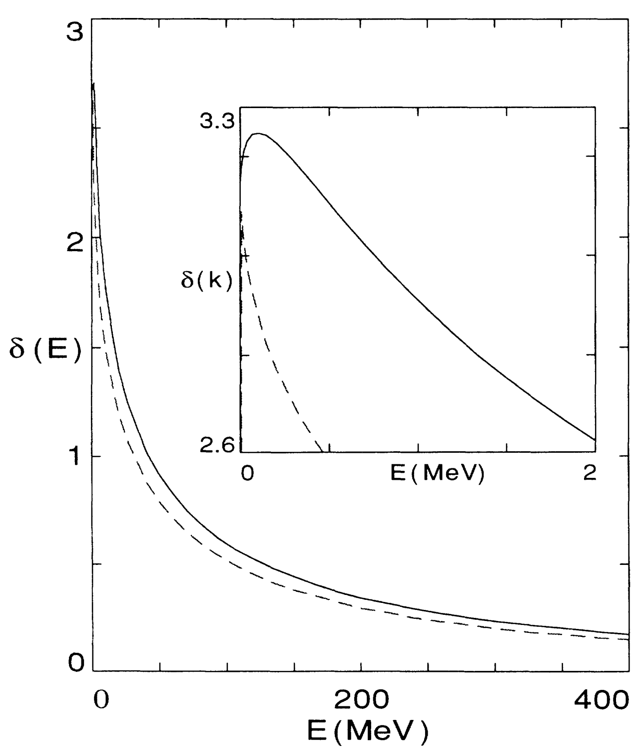

FIG. 1. Phase shifts of the Yamaguchi potential (dashed line) possessing a deuteron of 2.225 MeV binding, and the Tabakin-type potential (full line) possessing a CBS, which simulates this deuteron via a CBS.

In Fig. 1 we plot the phase shifts of the original Yamaguchi potential possessing a deuteron and those simulated via the Tabakin-type potential possessing a CBS. In Fig. 2 we plot the function $\tau(k^2)$ of Eq. (6) for the two potentials. The agreement between the two calculations is good except near zero energy. In both cases $\delta(0)-\delta(\infty) = \pi$. For the Yamaguchi potential, the bound state contributes to N and for the Tabakin-type potential the CBS contributes to N in the Levinson's theorem (1). For the Yamaguchi potential, $\tau(k^2)$ has a zero at the deuteron energy; for the Tabakin-type potential, $\tau(k^2)$ does not have a zero at negative energies. If we exclude a small domain, -10 to 10 MeV, the results for the two calculations are surprisingly similar. For a multiparticle ground-state calculation the two potentials should produce similar results. We performed a three-boson calculation with this Tabakin potential and find a three-boson binding of 38.8 MeV to be compared with 24.7 MeV for the Yamaguchi potential. These two numbers are in qualitative agreement with each other.

IV. RESULTS AND DISCUSSIONS

Next, we performed numerical calculations in a simple separable potential model including this pseudo state both for the two- and three-particle systems and compared them with the original potential model supporting the CBS. On the one hand, we have potential (2)–(4) supporting a regular bound state and a CBS, on the other hand, we consider the following form factor to be used in potential (2):

$$g(p) = \frac{1}{p^2 + \beta_j^2}, \quad (13)$$

where β_j is the greater of β_1 and β_2. By adjusting the strength parameter λ in Eq. (2) the alternative potential given by Eqs. (2) and (13) is made to reproduce a (pseudo) bound state at energy β_i^2, where β_i is the smaller of β_1 and β_2. The pseudostate and the potential (13) follow uniquely from the original nonlocal potential; there are no free parameters.

As the energy of the pseudo state is much larger than the energy of the normal state in situations of collapse, in order to illustrate our idea of the dominance of the pseudostate we have neglected the normal state and considered only the pseudostate in the alternate potential model.

We consider the full potential (4) with $\alpha_2 = 1$ and $\beta_1 = 1.4$ fm^{-1}, which allows for the possibility of a CBS to appear. The constants α_1 and β_2 were varied arbitrarily, and then λ is adjusted to yield a fixed two-particle binding of 2.225 MeV and a CBS. In the calculation we have maintained the parameter β_1 constant (=1.4 fm^{-1}) and the parameter β_2 is always larger than β_1. The pseudostate always has a binding of β_1^2, or 81.28 MeV produced by the alternative potential (13). The three-boson binding in this model should be typically about several times the two-particle binding. This puts a threshold for the three-boson ground state binding of about couple of hundred MeV's. Qualitatively, this explains a large binding for the three-particle ground state via a pseudostate.

In Table I we exhibit the potential parameters α_1, β_2, the position of the CBS, p_c, and the corresponding three-particle binding B_3 for the potential (2)–(4) and the alternative potential (13). We also exhibit in this table the lower bound (LB) calculation on B_3, from Ref. [11], given by

$$(B_3)_{LB} = 2B_2(\tfrac{3}{2}\lambda) \quad (14)$$

for the two potentials. Here B_2 stands for the two-particle binding energy, calculated when the potential strength is $3\lambda/2$. The LB calculations are shown in the parenthesis in the last two columns. The exact results for the three-particle ground state for both these potentials are qualitatively similar; the LB calculations provide good approximations to the exact calculations.

In Table I, as pointed out in a previous study [10], the increase in the three-particle binding in the case of the original nonlocal potential is due to the Thomas effect [12]. The Thomas effect also manifests in the case of the alternative potential (13), where there is only one range parameter β_2 and this parameter should reflect the range of the potential. An increase of β_2 denotes a passage to a smaller range while maintaining the two-particle (pseudo) bound-state energy fixed at 81.28 MeV.

V. SUMMARY

Here we have investigated the origin of the unusual strong attraction, known as BSC, in the problem of multiparticle ground state while using two-particle nonlocal potential that supports a CBS. We analyze the wave function of the CBS in a simple separable nonlocal potential model (2)–(4) and find that it is identical to the wave function of a negative-energy bound state which is called a pseudo bound state. The inclusion of this pseudo bound state in a simple potential model explains the unusual attraction in the presence of the CBS.

The pseudo bound state provides excess binding for the multiparticle bound state but it cannot really explain the whole multiparticle spectrum. For example, the presence of the strongly bound pseudo state eliminates in the simple model the appearance of the weakly bound excited states of the multiparticle system present in the original potential. The purpose of the present study is not to simulate all the subtle intricacies of a nonlocal potential with attractive and repulsive parts via a simple potential model but just to understand how to visualize some of the consequences of a CBS.

We studied the simple separable potential model containing a CBS because such a potential model simulates the essential features of a nonlocal intercluster potential which excludes the Pauli forbidden states. Though we cannot provide a general proof, it seems quite plausible that many of our conclusions should carry over to the case of Pauli forbidden states. Such a proof, or even a rigorous demonstration, will be a welcome addition to the literature.

TABLE I. The three-boson binding B of the original Tabakin-type potential (2)–(4) and that calculated in a simple Yamaguchi model (13) incorporating the pseudo state for $\alpha_2 = 1$ and $\beta_1 = 1.4$ fm^{-1}. The simple Yamaguchi model has the range parameter given by β_2 and is fitted to a bound state at energy 1.4 fm^{-2}. The numbers in parenthesis in the last two columns refer to the LB calculation employing Eq. (14).

α_1	β_2 (fm^{-1})	p_c (fm^{-1})	B (MeV)(Tabakin)	B (MeV)(Yamaguchi)
0.05	8.47	1.307	930.5 (1173)	902.3 (1086)
0.1	5.73	1.213	545.3 (698)	644.8 (748)
0.13	4.92	1.167	446.1 (576)	578.5 (660)
0.15	4.53	1.149	396.3 (518)	539.4 (619)
0.2	3.82	1.097	317.3 (420)	486.5 (549)

ACKNOWLEDGMENTS

L.T. thanks I.R. Afnan for the kind hospitality at the Flinders University of South Australia and A.D. thanks E.F. Redish for the kind hospitality at the University of Maryland. This work was supported in part by the Conselho Nacional de Desenvolvimento–Científico e Tecnológico (CNPq) of Brasil.

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