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Nonuniqueness of the Fierz-Pauli mass term for a nonsymmetic tensor

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Starting with a description of massive spin-2 particles in $D = 4$ in terms of a mixed symmetry tensor $T_{[\mu\nu]\rho}$ without a totally antisymmetric part ($T_{[[\mu\nu]\rho]} = 0$), we obtain a dual model in terms of a nonsymmetric tensor $e_{\mu\nu}$. The model is of second order in derivatives, and its mass term ($e_{\mu\nu}e^{\nu\mu} + ce^2$) contains an arbitrary real parameter c. Remarkably, it is free of ghosts for any real value of c and describes a massive spin-2 particle as expected from duality. The antisymmetric part $e_{[\mu\nu]}$ plays the role of auxiliary fields, vanishing on shell. In the massless case the model describes a massless spin-2 particle without ghosts.

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I. INTRODUCTION

Motivated mainly by applications in the large scale gravitational physics, there has been intense work on infrared modifications of gravity in the literature [1–5]; for review works see Refs. [6,7]. Some ingenious solutions to two basic problems of massive gravity, i.e., the appearance of ghosts [8] and the van Dam-Veltman-Zakharov mass discontinuity [9,10], have been suggested based on Ref. [11].

It is fair to say that the above problems are born in the free massive Fierz-Pauli (FP) theory [12]. In particular, the absence of ghosts in the free theory requires a fine tuning ($c = -1$) of the mass term ($h_{\mu\nu}^2 + ch^2$), which amounts to setting the mass of the ghost to infinity. It is therefore desirable to look for alternative descriptions of massive spin-2 particles. This is the subject of the present work.

In terms of a symmetric tensor $h_{\mu\nu} = h_{\nu\mu}$, which is the minimal tensor structure required for a spin-2 particle, the massive FP theory is unique[1] as a second-order theory. This point has been addressed in Ref. [13] and more recently in Ref. [14]. In Ref. [14], starting with the massless case, one notices that there is a whole continuous family of theories which contains a massless spin-2 particle and is free of ghosts. These theories have been named transverse diffeomorphisms (TDIFF) Lagrangians. In general, they also contain an extra scalar particle. There are only two points in the parameter space where we can get rid of the scalar field. One is the popular massless FP theory [linearized Einstein-Hilbert (LEH)], which might also be called DIFF theory, and the other possibility is the Weyl transverse diffeomorphisms (WTDIFF) model.[2] It turns out that only LEH can be consistently (ghost-free) deformed in order to accommodate a massive spin-2 particle, and that leads to the uniqueness of the massive FP theory in terms of a symmetric tensor.

If we want to generalize the massive FP theory we may, for instance, increase the number of symmetric tensors, allow for nonsymmetric tensors, or increase the rank of the tensor. In the next section we start with the third possibility and end up with the second one via a master action approach [15]. Remarkably, we derive a consistent model for a massive spin-2 particle in terms of a nonsymmetric tensor which differs from the FP theory in both the kinetic and massive terms. There is an arbitrariness in the mass term which is not necessarily the usual Fierz-Pauli one. The model is proved to be ghost-free via an analysis of the analytic structure of the propagator. In Sec. III we study the massless case and show that the gauge symmetries correspond to WDIFF (Weyl and linearized reparametrizations) plus a reducible vector symmetry in the antisymmetric sector; see (28). In Sec. IV we draw our conclusions. In both Secs. II and III we analyze the particle content via equations of motion and the analytic structure of the propagator.

II. THE MASSIVE CASE

It is possible to formulate the Einstein-Hilbert gravity in a first-order framelike formalism in terms of the spin connection $\omega_{\mu ab} = -\omega_{\mu ba}$ and the vierbein e_μ^a treated as independent variables. An analogous formulation for a massless spin-2 particle exists in flat space. If we keep using the curved space notation with only Greek indices and add a Fierz-Pauli mass term for $e_{\mu\nu}$, the first-order Lagrangian density in the flat space can be written symbolically (dropping some indices) as $\mathcal{L}[e,\omega] = -\omega^2 + \omega\partial e - m^2(e_{\mu\nu}e^{\nu\mu} - e^2)$. If we Gaussian integrate over the rank-3 tensor $\omega_{\mu[\alpha\beta]}$, we derive the massive Fierz-Pauli theory which describes a massive spin-2 particle in terms of the nonsymmetric tensor $e_{\mu\nu}$. The antisymmetric part $e_{[\mu\nu]}$ appears only in the mass term and, as such,

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[1]Up to trivial field redefinitions $h_{\mu\nu} \to \tilde{h}_{\mu\nu} + \frac{a}{2}\tilde{h}\eta_{\mu\nu}$ with a an arbitrary real number, except $a = -1/2$ which is not invertible.

[2]In flat space TDIFF stands for transverse ($\partial_\mu \xi^\mu = 0$) linearized reparametrizations $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$, while the WTDIFF model is invariant also under linearized Weyl transformations $\delta_W h_{\mu\nu} = \phi\eta_{\mu\nu}$. The term DIFF stands for unconstrained linearized reparametrizations which explains why there is one less unity in the number of degrees of freedom in the WTDIFF and DIFF theories as compared to TDIFF.

decouples trivially without any important contribution. Instead, if we Gaussian integrate over $e_{\mu\nu}$, we end up with a dual theory for the rank-3 tensor. It has been shown in Ref. [16], which makes use of Ref. [17], that such a higher-rank description of a massive spin-2 particle in $D = 4$ is the theory suggested by Curtright [18,19] in terms of a mixed symmetry tensor $T_{[\mu\nu]\rho}$, which is a kind of dual spin connection. See also a more recent derivation in Ref. [20]. As pointed out in Ref. [16], the key point in order to obtain the theory in Refs. [18,19] from the Fierz-Pauli theory is to keep the antisymmetric part $e_{[\mu\nu]} \neq 0$. If $e_{[\mu\nu]}=0$ the dual higher-rank model differs (see Ref. [21]) from Refs. [18,19]. The theory of Refs. [18,19], after a scaling by a mass factor, can be conveniently written [20] as[3]

$$\mathcal{L}_C[T] = E^{\mu\nu}{}_\beta T_{[\mu\nu]\rho} E^{\lambda\sigma\rho} T_{[\lambda\sigma]}{}^\beta - 2m^2[T^{[\mu\nu]\rho}T_{[\mu\nu]\rho} - 2T^\mu T_\mu], \tag{1}$$

where

$$T_{[\mu\nu]\rho} = -T_{[\nu\mu]\rho}; \qquad T_\mu = \eta^{\nu\rho}T_{[\mu\nu]\rho} \tag{2}$$

$$T_{[[\mu\nu]\rho]} = \frac{1}{3}(T_{[\mu\nu]\rho} + T_{[\nu\rho]\mu} + T_{[\rho\mu]\nu}) = 0 \tag{3}$$

$$E^{\mu\nu\alpha} = \epsilon^{\mu\nu\alpha\rho}\partial_\rho. \tag{4}$$

Introducing[4] a nonsymmetric tensor $e_{\mu\nu}$ we define the first-order Lagrangian density,

$$\mathcal{L}[e,T] = -e_{\beta\rho}e^{\rho\beta} - ce^2 + 2V_{\beta\rho}(T)e^{\rho\beta} - 2m^2[T^{[\mu\nu]\rho}T_{[\mu\nu]\rho} - 2T^\mu T_\mu], \tag{5}$$

where c is an arbitrary real constant, $e = e_\mu{}^\mu$, and

$$V_{\beta\rho}(T) = E^{\mu\nu}{}_\beta T_{[\mu\nu]\rho}, \qquad \partial^\beta V_{\beta\rho} = 0, \qquad V = V_\mu{}^\mu = 0. \tag{6}$$

The above properties of the nonsymmetric tensor $V_{\beta\rho}(T)$ are due to the transverse nature of the operator $E^{\mu\nu\rho}$ and the property (3), respectively. We can rewrite (5) as follows:

$$\mathcal{L}[e,T] = -[e_{\beta\rho} - V_{\beta\rho}(T)][e^{\rho\beta} - V^{\rho\beta}(T)] - ce^2 + \mathcal{L}_C[T]. \tag{7}$$

Since $V_{\beta\rho}(T)$ is traceless, after the shift $e_{\beta\rho} \to \tilde{e}_{\beta\rho} + V_{\beta\rho}$ we have two nondynamic terms for $\tilde{e}_{\beta\rho}$ decoupled from $\mathcal{L}_C[T]$. The fields $\tilde{e}_{\beta\rho}$ can thus be trivially integrated out in the path integral. We conclude that the particle content of $\mathcal{L}[e,T]$ is the same as that of Curtright's theory (1), i.e., one massive spin-2 particle. Notice that $e_{\beta\rho}$ does not need to be traceless to be shifted. Although the equations of motion of (5) lead (if $c \neq -1/4$) to $e = 0$, we have $e \neq 0$ off shell. This can be checked by adding a source term for the trace in (5) and defining $\mathcal{L}_J = \mathcal{L}[e,T] + eJ$. After the shift $e_{\beta\rho} \to \tilde{e}_{\beta\rho} + V_{\beta\rho}(T) + \eta_{\beta\rho}J/[2(1+4c)]$ the linear term in the scalar source J in $\mathcal{L}_J$ drops out and we have $\mathcal{L}_J = -\tilde{e}_{\beta\rho}\tilde{e}^{\rho\beta} - c\tilde{e}^2 + \mathcal{L}_C[T] + J^2/(1+4c)$, which leads to a nonvanishing two-point function $\langle e(x)e(y)\rangle = [2/(1+4c)]\delta(x-y)$. Those contact terms do not allow us to set $e(x) = 0$ strongly in the action where we have coinciding points.

On the other hand, instead of integrating over $e_{\beta\rho}$ we can Gaussian integrate over $T_{[\mu\nu]\rho}$. We end up with a dual massive Lagrangian $\mathcal{L}^*_m(e) = \mathcal{L}[e, T(e)]$ where

$$T_{[\mu\nu]\rho}(e) = T_\mu(e)\eta_{\rho\nu} - T_\nu(e)\eta_{\rho\mu} - \frac{1}{6m^2}[2E_{\mu\nu\beta}e_\rho{}^\beta + E_{\rho\nu\beta}e_\mu{}^\beta + E_{\mu\rho\beta}e_\nu{}^\beta], \tag{8}$$

$$T_\mu(e) = \frac{1}{4m^2}E_{\mu\alpha\beta}e^{\alpha\beta}. \tag{9}$$

The tensor $T_{[\mu\nu]\rho}(e)$ is obtained from the equations of motions of (5) and satisfies (3). The Lagrangian density $\mathcal{L}^*_m(e)$ must describe a massive spin-2 particle with 5 propagating degrees of freedom. Other interesting features of $\mathcal{L}^*_m(e)$ can be anticipated from (5). Since the last three terms of (5) can only generate, after Gaussian integration, kinetic terms of second order in derivatives of $e_{\mu\nu}$, it is already clear that we can have a massive spin-2 particle without necessarily a Fierz-Pauli mass term which corresponds to $c = -1$. Given that $T_{[\mu\nu]\rho}$ is coupled to $e^{\rho\beta}$ via $V_{\beta\rho}(T)e^{\rho\beta}$, due to the properties (6), the kinetic terms (mass-independent terms) will be invariant under the linearized reparametrizations and Weyl transformations:

$$\delta e_{\alpha\beta} = \partial_\beta\xi_\alpha + \eta_{\alpha\beta}\phi. \tag{10}$$

In the special case $c = -1/4$ the whole massive theory is invariant under the Weyl transformations $\delta_W e_{\alpha\beta} = \eta_{\alpha\beta}\phi$. Explicitly, after a redefinition $e_{\alpha\beta} \to me_{\alpha\beta}/\sqrt{2}$, the Gaussian integrals over the mixed symmetry tensors furnish a massive dual model for arbitrary values of c, which is our main result, i.e.,

$$\mathcal{L}^*_m = -\frac{1}{2}\partial^\mu e^{(\alpha\beta)}\partial_\mu e_{(\alpha\beta)} + [\partial^\alpha e_{(\alpha\beta)}]^2 - \frac{1}{3}(\partial^\alpha e_{\alpha\beta})^2 - \frac{m^2}{2}(e_{\alpha\beta}e^{\beta\alpha} + ce^2) + \frac{1}{6}\partial^\mu e\partial_\mu e - \frac{1}{3}\partial^\alpha e_{\alpha\beta}\partial^\beta e, \tag{11}$$

where $e_{(\alpha\beta)} = (e_{\alpha\beta} + e_{\beta\alpha})/2$ and $e_{[\alpha\beta]} = (e_{\alpha\beta} - e_{\beta\alpha})/2$. The reader can check that the mass-independent terms of (11) are indeed invariant under (10). At this point one might try to bring the arbitrary mass term in (11) to the

[3] Throughout this work we use $\eta_{\mu\nu} = \text{diag}(-,+,+,+)$.
[4] Although we use the same notation, the new field should not be confused with $e_{\mu\nu}$ appearing in $\mathcal{L}[e,\omega]$.

Fierz-Pauli form with $c = -1$ by means of a local change of variables $e_{\mu\nu} = \tilde{e}_{\mu\nu} + \frac{a}{2}\tilde{e}\eta_{\mu\nu}$ by tuning the real constant a conveniently without affecting the kinetic terms which are Weyl invariant. However, this is not always possible. Explicitly, we have

$$e_{\mu\nu}e^{\nu\mu} + ce^2 \to \tilde{e}_{\mu\nu}\tilde{e}^{\nu\mu} + \tilde{c}\tilde{e}^2, \tag{12}$$

where $\tilde{c} = c + (1 + 4c)(a + a^2)$. We have three classes of mass terms according to $c < -1/4$, $c > -1/4$, and the fixed point $c = -1/4$. Except for the fixed point, any representative of a class is continuously connected to any other mass term of the same class by varying the parameter a. No interclass jump is allowed. Thus, without loss of generality we can simply pick up, for instance, $c = -1$, $c = 0$, and $c = -1/4$, where the first case corresponds to the usual Fierz-Pauli mass term while the other two cases cannot be brought into the FP form.

Defining the massless action $S^*_{m=0} = \int d^4x \mathcal{L}^*_{m=0}$, the equations of motion of the massive dual model (11) can be written as

$$m^2(e_{\beta\alpha} + c\eta_{\beta\alpha}e) = K_{\alpha\beta} \tag{13}$$

with the massless Killing tensor given by

$$K_{\alpha\beta} = \frac{\delta S^*_{m=0}}{\delta e^{\alpha\beta}} = \frac{\Box}{2}(e_{\alpha\beta} + e_{\beta\alpha}) - \frac{1}{2}\partial_\delta(\partial_\alpha e_\beta{}^\delta + \partial_\beta e_\alpha{}^\delta) + \frac{\eta_{\alpha\beta}}{3}(\partial_\mu\partial_\nu e^{\mu\nu} - \Box e) - \frac{1}{2}\partial_\beta\partial_\delta e^\delta{}_\alpha + \frac{1}{6}\partial_\alpha\partial_\mu e^\mu{}_\beta + \frac{1}{3}\partial_\alpha\partial_\beta e. \tag{14}$$

Due to the symmetries (10) we have $\eta^{\alpha\beta}K_{\alpha\beta} = 0 = \partial^\beta K_{\alpha\beta}$. Thus, from $\eta^{\alpha\beta}$ and ∂^β in (13) we have

$$e = 0, \tag{15}$$

$$\partial^\beta e_{\beta\alpha} = 0. \tag{16}$$

Although (15) only holds for $c \neq -1/4$, it can be implemented as a gauge condition of the Weyl symmetry if $c = -1/4$. So we assume (15) and (16) henceforth for all values of c. Moreover, the antisymmetric part of (13) now leads to

$$e_{\alpha\beta} - e_{\beta\alpha} = 0. \tag{17}$$

Therefore, although $e_{[\alpha\beta]}$ appear under derivatives in $\mathcal{L}^*_m$, they play the role of auxiliary fields. Finally, (13) becomes the Klein-Gordon equation:

$$(\Box - m^2)e_{\alpha\beta} = 0. \tag{18}$$

Equations (15)–(18) are the Fierz-Pauli conditions. They guarantee that we have 5 propagating degrees of freedom corresponding to a massive spin-2 particle for any value of the constant c.

We have also checked unitarity by calculating the two-point amplitude $A_2(k)$. Introducing arbitrary sources $T_{\mu\nu}$, we have

$$e^{\int d^4k A_2(k)} = \int \mathcal{D}e_{\alpha\beta}e^{iS^*_m[e]+i\int d^4x e_{\alpha\beta}T^{\alpha\beta}}. \tag{19}$$

From (19) it can be shown that $A_2(k)$ is given in terms of the saturated propagator in momentum space as follows:

$$A_2(k) = -i(T^{\mu\nu}(k))^*[G^{-1}_{\mu\nu\alpha\beta}(k)]T^{\alpha\beta}(k), \tag{20}$$

where $T^*_{\mu\nu}(k)$ is the complex conjugate of the Fourier transform of the sources. In general, in the massive theory there are no constraints on the source except at $c = -1/4$ where the source must be traceless due to the Weyl symmetry. The propagator in momentum space can be obtained $[G^{-1}(k) = G^{-1}(\partial \to ik)]$ from the differential operator below in coordinate space; we have suppressed the four indices for simplicity,

$$G^{-1} = \frac{P^{(2)}_{SS}}{\Box - m^2} - \frac{2}{m^4}\left(\frac{\Box}{3} + m^2\right)P^{(1)}_{SS} - \frac{2}{m^4}\left(\frac{\Box}{3} - m^2\right)P^{(1)}_{AA} - \frac{\Box}{3m^4}[P^{(1)}_{AS} + P^{(1)}_{SA}] + \frac{P^{(0)}_{AA}}{m^2} - \frac{1+c}{m^2(4c+1)}P^{(0)}_{SS} - \frac{1+3c}{m^2(4c+1)}P^{(0)}_{WW} + \frac{\sqrt{3}}{m^2(4c+1)}[P^{(0)}_{WS} + P^{(0)}_{SW}]. \tag{21}$$

The spin-s projection operators $P^{(s)}_{JJ}$ and the transition operators $P^{(s)}_{IJ}$, $I \neq J$, are given in the Appendix. They satisfy the simple algebra

$$P^{(s)}_{IJ}P^{(r)}_{KL} = \delta^{sr}\delta_{JK}P^{(s)}_{IL}. \tag{22}$$

We have used (22) in order to obtain (21) by inverting[5] $G_{\alpha\beta\mu\nu}$, which in turn is defined by $S^*_m = \int d^4x \mathcal{L}^*_m = \int d^4x e^{\alpha\beta}G_{\alpha\beta\mu\nu}e^{\mu\nu}$. The only pole in (21) occurs in the spin-2 sector at $\Box = m^2$, just like in the usual Fierz-Pauli theory where $c = -1$. The calculation of the imaginary part of the residue (R_m) of $A_2(k)$ at $k^2 = -m^2$ proceeds in the same way as in the massive FP theory. The fact that $T_{\mu\nu}$ is not symmetric in our case does not make any difference since $P^{(2)}_{SS}$ projects out in the symmetric, transverse, and traceless sector anyway. Namely,

$$R_m = \lim_{k^2 \to -m^2}(k^2 + m^2)\Im m[A_2(k)] = (T^{\mu\nu})^*[P^{(2)}_{SS}]_{\mu\nu\alpha\beta}T^{\alpha\beta} = (T^{\mu\nu}_{TT})^*(T_{TT})_{\mu\nu} = \sum_{i,j}|T^{ij}_{TT}|^2 > 0, \tag{23}$$

[5] There is no inverse at $c = -1/4$ due to the Weyl symmetry; however, in this case we can add a gauge fixing term $-\lambda e^2$ which amounts to substituting $c \to -1/4 + \lambda$ in (21).

where the symmetric, transverse, and traceless tensor is given by $T_{TT}^{\mu\nu} = (P_{SS}^{(2)})^{\mu\nu\alpha\beta}T_{\alpha\beta}$ and we have used $T_{TT}^{0\mu} = 0$, which follows from $k_\alpha T_{TT}^{\alpha\beta} = 0$ in the frame $k_\alpha = (m, 0, 0, 0)$. Thus, our massive dual model $\mathcal{L}_m^*$ is free of ghosts and describes one spin-2 massive particle for arbitrary real values of c.

For a closer comparison with the usual massive FP model, it is instructive to decompose $e_{\mu\nu}$ into symmetric and antisymmetric parts. From (11) we have

$$\mathcal{L}_m^* = -\frac{1}{2}\partial^\mu h^{(\alpha\beta)}\partial_\mu h_{(\alpha\beta)} + \frac{1}{8}\partial^\mu h \partial_\mu h - \frac{1}{2}(\partial^\alpha B_{\alpha\mu})^2 + \frac{m^2}{2}B_{\alpha\beta}^2 + \frac{2}{3}\left(\partial^\alpha h_{\alpha\mu} - \frac{1}{2}\partial^\alpha B_{\alpha\mu} - \frac{1}{4}\partial_\mu h\right)^2 - \frac{m^2}{2}(h_{\alpha\beta}^2 + ch^2), \quad (24)$$

where

$$e_{\mu\nu} = h_{\mu\nu} + B_{\mu\nu}, \quad h_{\mu\nu} = h_{\nu\mu}, \quad B_{\mu\nu} = -B_{\nu\mu}. \quad (25)$$

For convenience we write down the usual Fierz-Pauli theory:

$$\mathcal{L}_m^{\rm FP} = -\frac{1}{2}\partial^\mu h^{\alpha\beta}\partial_\mu h_{\alpha\beta} + \frac{1}{4}\partial^\mu h\partial_\mu h + \left(\partial^\alpha h_{\alpha\beta} - \frac{1}{2}\partial_\beta h\right)^2 - \frac{m^2}{2}(h_{\alpha\beta}^2 - h^2). \quad (26)$$

The vectors inside the large parentheses in Eqs. (24) and (26) are related to harmonic gauges (de Donder gauges) for the massless theory [to be discussed in the next section—see (30)].

We see in (24) that the coupling between $B_{\mu\nu}$ and $h_{\mu\nu}$ is nontrivial and cannot be undone by means of any local field redefinition. In fact, the functional integral over $B_{\mu\nu}$ leads to a nonlocal effective action for the symmetric field $h_{\mu\nu}$. Moreover, if we simply set $B_{\mu\nu} = 0$ the remaining theory is no longer ghost free.

The absence of ghosts in (24) is surprisingly not only because of the coupling between $B_{\mu\nu}$ and $h_{\mu\nu}$ (see Ref. [13]), but also because of the non-Fierz-Pauli mass term (for $c \geq -1/4$). Another example of ghost-free symmetric-antisymmetric coupling has been found recently in Ref. [22], where the mass term must be of the usual Fierz-Pauli type ($c = -1$). Its kinetic terms do not contain the trace h and cannot be brought to the form appearing in (11) or in (26) by local transformations.

At this point we comment on another work in the literature. In Ref. [17] one also finds a first-order master action depending on a mixed symmetry tensor $\omega_{\mu[\nu\alpha]}$ and $e_{\mu\nu} = h_{\mu\nu} + B_{\mu\nu}$, with an arbitrary real constant a in the mass term similar to (5). Symbolically, the Lagrangian of Ref. [17] can be written as

$$\mathcal{L}_I = \omega\cdot\omega + \omega(\partial h + \partial B) - \frac{m^2}{2}(h_{\mu\nu}^2 - h^2) + aB_{\mu\nu}^2. \quad (27)$$

Integrating over $\omega_{\mu\nu\alpha}$ one gets the usual massive Fierz-Pauli action [12] displayed in (26) plus $aB_{\mu\nu}^2$. Therefore, the antisymmetric field $B_{\mu\nu}$ does not play any physical role and vanishes on shell, which is similar to the trace e [see (15)] in our model (11). Thus, (27) describes one massive spin-2 particle for any value of a, similar to (11). The case $a = 0$ is special since we have a gauge symmetry $\delta B_{\mu\nu} = \Lambda_{[\mu\nu]}$, $\delta\omega_{\mu[\nu\alpha]} = \partial_\mu\Lambda_{[\nu\alpha]}$ (see Ref. [23]). This is the analogue of the $c = -1/4$ case in our model (11). In general, after the change of variables $\omega_{\mu[\nu\alpha]} \to \omega_{\mu[\nu\alpha]} + \partial_\mu B_{\nu\alpha}$ the field $B_{\mu\nu}$ disappears from (27) except for the last term. This shows that there is no physical coupling between $B_{\mu\nu}$ and $\omega_{\mu[\nu\alpha]}$, just like in (5) where the trace e does not couple to $T_{[\lambda\sigma]\beta}$. So the arbitrariness in both master actions (11) and (27) is related to degrees of freedom which are physically decoupled from the dual field. An important difference between (11) and the theory obtained from (27) after integration over $\omega_{\mu\nu\alpha}$ is the surprising (see Ref. [13]) absence of ghosts in (11) when compared to the latter case where the absence of ghosts is obvious since we have the usual massive Fierz-Pauli theory plus the decoupled term $aB_{\mu\nu}^2$.

Finally, if we compare the usual massless Fierz-Pauli theory (linearized Einstein-Hilbert) to the $m \to 0$ limit of our massive dual model by calculating $A_2(k)$ saturated with symmetric conserved sources, then the same mass discontinuity problems found in Refs. [9,10] for the usual massive Fierz-Pauli theory show up. From (21) we see that only the spin-2 sector can lead to long-range interactions. This gives rise to the same result for the tree-level interacting potential as the massless limit of the massive Fierz-Pauli theory. The remaining (lower spin) terms are contact terms which may be neglected.

III. THE MASSLESS CASE

According to the dualization procedure summarized in the introduction of the last section, it is expected that the $m = 0$ case be singular somehow. At $m = 0$ the functional integral over $e_{\mu\nu}$ leads to a constraint on the spin connection $\omega_{\mu ab}$ instead of quadratic kinetic terms. So the particle content of (1) does not need to reproduce the massless FP theory which describes a massless spin-2 particle. Indeed, it can be proved (see Ref. [20]) that $\mathcal{L}_C(m = 0)$ contains no particle at all.[6] Since our dualization procedure in deriving $\mathcal{L}_m^*$ is also singular at $m = 0$, the particle content of $\mathcal{L}_{m=0}^*$ is not known *a priori*.

First, we note that $\mathcal{L}_{m=0}^*$ is invariant under a gauge transformation, which also acts in the antisymmetric part of $e_{\mu\nu}$, thus enlarging (10), namely,

[6]By using the first-order dual formulation (5) at $m = 0$, an alternative proof which is explicitly covariant and gauge independent can be done [24].

$$\delta e_{\alpha\beta} = \partial_\beta \xi_\alpha + \eta_{\alpha\beta}\phi + \epsilon_{\alpha\beta\mu\nu}\partial^\mu \Lambda^\nu. \tag{28}$$

In terms of dual fields $B^*_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}B^{\alpha\beta}$ we can write $\delta B^*_{\mu\nu} = \partial_{[\mu}\Lambda_{\nu]}$. The importance of this type of symmetry in order to avoid ghosts in nonsymmetric tensor theories has been emphasized in Ref. [25].

We fix the gauge in the antisymmetric sector by imposing

$$\epsilon^{\mu\nu\alpha\beta}\partial_\nu e_{\alpha\beta} = 0. \tag{29}$$

Notice that the antisymmetric gauge transformation is reducible under $\delta\Lambda_\mu = \partial_\mu \Phi$. Consequently, we can only fix three independent degrees of freedom, in agreement with the transverse gauge condition (29).

Regarding the reparametrization and Weyl symmetry, we have found it convenient to choose harmonic gauges (like the Lorentz gauge in electrodynamics and the de Donder gauge $\partial^\mu h_{\mu\nu} - \partial_\mu h/2 = 0$ for symmetric tensors) which have residual gauge invariances under harmonic functions $\Box\xi = 0 = \Box\phi$. Respectively, we define the gauges

$$G_\beta = \partial^\alpha e_{\alpha\beta} + 3\partial^\alpha e_{\beta\alpha} - \partial_\beta e = 0, \tag{30}$$

$$H = \partial^\alpha \partial^\beta e_{\alpha\beta} - \Box e = 0. \tag{31}$$

Each of the gauge conditions (29)–(31) breaks only one of the three symmetries (28). Now we can define the gauge fixed action (disregarding the decoupled Faddeev-Popov term) and obtain an invertible operator $G_{\mu\nu\alpha\beta}$ as follows:

$$\begin{aligned} S &= \int d^4x[\mathcal{L}^*_{m=0}(e) + \lambda_1 G_\mu G^\mu \\ &\quad + \lambda_2(\epsilon^{\mu\nu\alpha\beta}\partial_\nu e_{\alpha\beta})^2 + \lambda_3 H^2] \\ &= \int d^4x e_{\mu\nu}G^{\mu\nu\alpha\beta}e_{\alpha\beta}. \end{aligned} \tag{32}$$

After expanding $G_{\mu\nu\alpha\beta}$ on the basis of spin-s operators $P^{(s)}_{IJ}$ given in the Appendix, we obtain the inverse operator (suppressing indices)

$$\begin{aligned} G^{-1} &= \frac{2P^{(2)}_{SS}}{\Box} - \frac{1}{18\lambda_1\Box}[(1-12\lambda_1)P^{(1)}_{SS} + (1-48\lambda_1)P^{(1)}_{AA} \\ &\quad + (1+24\lambda_1)(P^{(1)}_{AS} + P^{(1)}_{SA})] - \frac{P^{(0)}_{AA}}{2\lambda_2\Box} + \frac{P^{(0)}_{SS}}{3\lambda_3\Box^2} \\ &\quad + \frac{\lambda_1 - \lambda_3\Box}{9\lambda_1\lambda_3\Box^2}P^{(0)}_{WW} - \frac{\sqrt{3}[P^{(0)}_{WS} + P^{(0)}_{SW}]}{9\lambda_3\Box^2}. \end{aligned} \tag{33}$$

We have a gauge-independent massless pole in the spin-2 and spin-1 sectors. Next we deduce the constraints on the sources due to the gauge symmetries (28) and calculate the residue in $A_2(k)$ at $k^2 = 0$. From the invariance of the source term under (28),

$$\int d^4x T^{\mu\nu}\delta e_{\mu\nu} = 0. \tag{34}$$

We deduce

$$T_{\mu\nu} = T_{(\mu\nu)} + \partial_\mu T_\nu - \partial_\nu T_\mu, \tag{35}$$

$$\eta_{\mu\nu}T^{(\mu\nu)} = 0, \qquad \partial_\nu T^{\mu\nu} = 0. \tag{36}$$

In particular, we have $\omega_{\mu\nu}T^{\mu\nu} = 0 = \theta_{\mu\nu}T^{\mu\nu}$. Defining the shorthand notation

$$T^*_{\mu\nu}(P^{(s)}_{IJ})^{\mu\nu\alpha\beta}T_{\alpha\beta} \equiv T^* P^{(s)}_{IJ} T, \tag{37}$$

it is not difficult to check that the spin-0 operators drop out from the saturated propagator:

$$T^* P^{(0)}_{IJ} T = 0. \tag{38}$$

In the spin-1 and spin-2 sectors we have

$$\begin{aligned} T^* P^{(1)}_{SS} T &= \frac{1}{2}T^*\omega T = T^* P^{(1)}_{AA} T, \\ T^*[P^{(1)}_{AS} + P^{(1)}_{SA}]T &= -T^*\omega T, \end{aligned} \tag{39}$$

$$T^* P^{(2)}_{SS} T = T^*_{(\mu\nu)}T^{(\mu\nu)} - \frac{1}{2}T^*\omega T, \tag{40}$$

where

$$T^*\omega T = T^*_{\mu\nu}\frac{k^\mu k_\alpha}{k^2}T^{\alpha\nu}. \tag{41}$$

Collecting all the above results in the formula (20) for $A_2(k)$, the gauge parameters λ_j, $j = 1, 2, 3$ cancel out and we have the gauge-independent result

$$A_2(k) = \frac{2i}{k^2}[T^*_{(\mu\nu)}T^{(\mu\nu)} + T^*\omega T], \tag{42}$$

From the momentum space expressions $T_{\mu\nu} = T_{(\mu\nu)} + i(k_\mu T_\nu - k_\nu T_\mu)$ and $k_\nu T^{\mu\nu} = 0$, we can deduce $k_\mu T^{\mu\nu} = 2i(k^2 T^\nu - k^\nu k_\nu T^\nu)$. Consequently,

$$T^*\omega T = 4[k^2 T^*_\mu T^\mu - |k_\nu T^\nu|^2]. \tag{43}$$

Since we are interested in the residue at $k^2 = 0$, from now on we stick to the lightlike frame $k_\mu = (k, k, 0, 0)$. From $k_\nu T^{\mu\nu} = 0$ we have $T^{(\mu\nu)}k_\nu = i[k^2 T^\mu - k^\mu(k_\alpha T^\alpha)]$, which in the above frame leads to the four equations

$$T^{(02)} = -T^{(21)}, \qquad T^{(03)} = -T^{(31)}, \tag{44}$$

$$T^{00} = T^{11} + 2ik_\alpha T^\alpha, \qquad T^{(01)} = -T^{11} - ik_\alpha T^\alpha. \tag{45}$$

It follows from (44) and (45) that

$$T^*_{(\mu\nu)}T^{(\mu\nu)} = |T^{22}|^2 + |T^{33}|^2 + 2|T^{23}|^2 + 2|k_\alpha T^\alpha|^2. \tag{46}$$

From (43) at $k^2 = 0$ and (46) we finally obtain, for the imaginary part of the residue of $A_2(k)$ at $k^2 = 0$,

$$R_0 = 2(|T_{22}|^2 + |T_{33}|^2 + 2|T_{23}|^2 - 2|k_\alpha T^\alpha|^2) = |T_{22} - T_{33}|^2 + 4|T_{23}|^2 > 0, \tag{47}$$

where we have used $k_\alpha T^\alpha = (T_{22} + T_{33})/(2i)$, which follows from the first equation in (45) and the traceless condition $T_{00} - T_{11} = T_{22} + T_{33}$.

In summary, $R_0 > 0$ and the massless theory is ghost free.

Next we check the equations of motion coming from $\mathcal{L}^*_{m=0}$ at the gauge conditions (29)–(31). Those equations correspond to $K_{\mu\nu} = 0$; see (14). First, the antisymmetric part $K_{[\mu\nu]} = 0$ leads to

$$\partial_\mu(\partial^\alpha e_{\alpha\nu}) - \partial_\nu(\partial^\alpha e_{\alpha\mu}) = 0 \Rightarrow \partial^\alpha e_{\alpha\nu} = \partial_\nu \Phi, \tag{48}$$

where so far Φ is an arbitrary scalar field. In $\partial^\mu G_\mu = 0$, $H = 0$ and $G_\mu = 0$ we have

$$\Box e = 0, \qquad \partial^\mu\partial^\nu e_{\mu\nu} = \Box\Phi = 0, \tag{49}$$

$$3\partial^\nu e_{\mu\nu} = \partial_\mu(e - \Phi). \tag{50}$$

In $K_{\mu\nu} = 0$ we deduce $\Box e_{(\mu\nu)} = 0$. Now we can define the field

$$h_{\mu\nu} = e_{(\mu\nu)} - \frac{\eta_{\mu\nu}}{3}\left(\frac{e}{2} + \Phi\right), \tag{51}$$

which satisfies

$$h_{[\mu\nu]} = 0 = \partial^\mu h_{\mu\nu}, \qquad \Box h_{\mu\nu} = 0, \tag{52}$$

$$h = \frac{1}{3}(e - 4\Phi). \tag{53}$$

All the equations written so far are invariant under residual reparametrization and Weyl gauge transformations with harmonic parameters ($\Box\xi_\mu = 0 = \Box\phi$). Since they imply

$$\delta h_{\mu\nu} = \frac{1}{2}[\partial_\mu\xi_\nu + \partial_\mu\xi_\nu - \eta_{\mu\nu}\partial\cdot\xi], \tag{54}$$

$$\delta\Phi = \phi + \partial\cdot\xi, \qquad \delta e = 4\phi + \partial\cdot\xi, \tag{55}$$

we can use the residual Weyl invariance to get rid of the scalar field Φ, imposing

$$\Phi - e = 0. \tag{56}$$

Since (56) is reparametrization invariant, no further requirement is made on the harmonic reparametrization parameters ξ_μ, which can thus be used to get rid of 4 extra degrees of freedom of $h_{\mu\nu}$. So $h_{\mu\nu}$ contains [see (52)] two helicity states ± 2.

Regarding the antisymmetric part $e_{[\mu\nu]}$, since the solution of the gauge condition (29) leads to a field strength of some vector field,

$$e_{\mu\nu} - e_{\nu\mu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{57}$$

If we plug this back into $\partial^\mu(e_{\mu\nu} - e_{\nu\mu}) = -\partial_\nu h$, we have

$$\Box A_\mu - \partial_\mu(\partial\cdot A) = -\partial_\mu h. \tag{58}$$

We can split the general solution of (58): $A_\mu = A_\mu^{\max} + A_\mu^h$, where $A_\mu^{\max}$ is a general solution of Maxwell equations $\partial^\mu F_{\mu\nu}(A^{\max}) = 0$ while A_μ^h is a specific solution of the nonhomogeneous equation (58). Clearly, A_μ^h does not represent an independent degree of freedom. Moreover, we can get rid of $A_\mu^{\max}$ by using a constrained symmetry of the massless model. Namely, from (24) we see that the massless theory depends on $B_{\mu\nu}$ only through the combination $\partial^\mu B_{\mu\nu}$, which is invariant under $\delta B_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}\partial^\alpha\Lambda^\beta + \partial_\mu C_\nu - \partial_\nu C_\mu$ where the gauge parameter Λ^μ is arbitrary while C_μ must satisfy the free Maxwell equations $\partial^\mu F_{\mu\nu}(C) = 0$. Since the gauge condition (29) does not impose any further constraint on the parameters C_μ, they can be used to cancel $A_\mu^{\max}$. Thus, the antisymmetric part $e_{\mu\nu} - e_{\nu\mu}$ does not contribute to the spectrum of the theory, which consists only of one massless spin-2 particle.

There is another way of checking the particle content of $\mathcal{L}^*_{m=0}$. In fact,[7] $\mathcal{L}^*_{m=0}$ has appeared before in Ref. [26] in a completely different way via the solution of a constraint in a massless master action. As pointed out in a footnote in Ref. [27], it is useful to rewrite $\mathcal{L}^*_{m=0}$ with the help of a nondynamical vector field v_μ as follows,

$$\mathcal{L}^*_{m=0} = \mathcal{L}_0(h) + \frac{1}{3}v^2 + \frac{2}{3}v^\mu(\partial^\alpha B_{\alpha\mu} + \partial^\alpha h_{\alpha\mu}) + h_{\mu\nu}T^{(\mu\nu)} + 2B_{\mu\nu}\partial^\mu T^\nu, \tag{59}$$

where we have added sources satisfying the constraints (35) and (36) and

$$\mathcal{L}_0(h) = -\frac{1}{2}\partial^\mu h^{(\alpha\beta)}\partial_\mu h_{(\alpha\beta)} + \frac{1}{6}\partial^\mu h\partial_\mu h + (\partial^\mu h_{\mu\nu})^2 - \frac{1}{3}\partial^\mu h_{\mu\nu}\partial^\nu h. \tag{60}$$

If we integrate over $B_{\mu\nu}$ in the generating functional, we get a functional delta function which enforces a constraint whose general solution is $v_\mu = 3T_\mu + \partial_\mu\psi$, where ψ is an arbitrary scalar field. Then, after integrating over v_μ we get an effective theory containing $h_{\mu\nu}$ and ψ. It turns out that after the redefinition $h_{\mu\nu} \to \tilde{h}_{\mu\nu} - \eta_{\mu\nu}(\psi + \tilde{h})$ the scalar field ψ disappears and we end up with the linearized Einstein-Hilbert theory with a modified source term, i.e.,

$$\mathcal{L}^*_{\rm eff} = \mathcal{L}^{\rm FP}_{m=0}(\tilde{h}) + \tilde{h}_{\mu\nu}\tilde{T}^{\mu\nu}, \tag{61}$$

where $\mathcal{L}^{\rm FP}_{m=0}$ corresponds to (26) at $m = 0$ and

[7] We thank an anonymous referee for calling our attention to Ref. [26].

$$\tilde{T}_{\mu\nu} = T_{(\mu\nu)} - \partial_\mu T_\nu - \partial_\nu T_\mu + 2\eta_{\mu\nu}\partial\cdot T. \quad (62)$$

The new source is symmetric and conserved, $\partial^\mu \tilde{T}_{\mu\nu} = 0$, thanks to the constraint $\partial_\mu T^{(\mu\nu)} = \partial_\mu F^{\mu\nu}(T)$ which follows from (35) and (36).

In conclusion, as in the usual massless FP theory (linearized Einstein-Hilbert), the symmetric field $h_{\mu\nu}$ couples to a symmetric and conserved source; see Ref. [26] for more comments. Although we have found a pole in both the spin-1 and spin-2 sectors of the propagator, there is only one spin-2 massless particle in the spectrum. The residue calculation is somehow similar to the LEH theory whose propagator contains a gauge-independent massless pole in the spin-2 sector and also in the spin-0 sector, but there is only one spin-2 particle in the spectrum. Indeed, if we saturate the LEH propagator with the effective source (62) and calculate the residue $R_0 = \tilde{T}^*[2P^{(2)}_{SS} - P^{(0)}_{SS}]\tilde{T}$, we have exactly the same result as formula (47).

Since we have been able to get rid of the antisymmetric field $B_{\mu\nu}$ via local field redefinitions, one might try the same manipulations in the massive case. In fact, we can still trade the antisymmetric field $B_{\mu\nu}$ in a vector field v_μ; i.e., if we add $m^2 B^2_{\mu\nu}$ to the right-hand side of (59), after integrating over $B_{\mu\nu}$ we have a Maxwell-Proca theory for the vector field v_μ. However, if we further integrate over v_μ we end up with a nonlocal action for the symmetric field $h_{\mu\nu}$, as we have mentioned in Sec. II. Thus, in the massive case the antisymmetric field is not simply an auxiliary field.

IV. CONCLUSION

Here we have shown that unitarity does not lead to a unique description of massive spin-2 particles in terms of a rank-2 tensor in $D = 4$. In particular, the mass term does not need to fit in the widely used[8] Fierz-Pauli form $e_{\mu\nu}e^{\nu\mu} + ce^2$ with $c = -1$ or, more generally, in the Fierz-Pauli class $c < -1/4$. The other two classes, $c > -1/4$ and $c = -1/4$, also lead to ghost-free theories. The arbitrariness in the mass term is related to the absence of a totally antisymmetric part ($T_{[[\mu\nu]\rho]} = 0$) for the mixed symmetry tensor of the dual theory.

An important ingredient in our model is the use of a nonsymmetric tensor $e_{\mu\nu}$ which naturally appears in the flat space limit of first-order formulations of gravity. Another important point is the nontrivial coupling between the symmetric and antisymmetric parts of $e_{\mu\nu}$ which is also present in the recent suggestion of Ref. [22], where the mass term must be of the usual Fierz-Pauli form.

Unfortunately, the mass discontinuity [9,10] at $m \to 0$ is independent of the arbitrariness in the mass term and coincides (up to contact terms) with the Fierz-Pauli result. However, since we have some freedom in the mass term which does not need to be fine-tuned as in the Fierz-Pauli theory, one may hope to solve the discontinuity problem by adding nonlinear terms without necessarily creating ghosts.

Regarding the massless theory (Sec. III), it has appeared before in Ref. [26] via a different procedure. Its spectrum consists of a massless spin-2 particle. Thanks to the three gauge symmetries (28), the model is ghost-free. The antisymmetric part $e_{[\mu\nu]}$ can be eliminated, and we end up with the usual linearized Einstein-Hilbert theory with modified sources.

[8]One exception is Ref. [3], where the mass of the "would be" ghost is not set to infinity [$c = c(\Box) \neq -1$]. The theory is ghost free due to a phenomenological reason. Namely, the mass of the ghost lies presumably above the energy scale below which the massive gravitational theory is supposed to work. Another exception is Ref. [17]; see comment at the end of Sec. II.

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APPENDIX

From the spin-1 and spin-0 projection operators acting on vector fields, respectively,

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}, \qquad \omega_{\mu\nu} = \frac{\partial_\mu\partial_\nu}{\Box}, \quad (A1)$$

one [13] can build up the projection and transition operators mentioned in the text. First, we present the symmetric operators

$$(P^{(2)}_{SS})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}(\theta^\lambda{}_\alpha\theta^\mu{}_\beta + \theta^\mu{}_\alpha\theta^\lambda{}_\beta) - \frac{\theta^{\lambda\mu}\theta_{\alpha\beta}}{D-1}, \quad (A2)$$

$$(P^{(1)}_{SS})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}(\theta^\lambda{}_\alpha\omega^\mu{}_\beta + \theta^\mu{}_\alpha\omega^\lambda{}_\beta + \theta^\lambda{}_\beta\omega^\mu{}_\alpha + \theta^\mu{}_\beta\omega^\lambda{}_\alpha), \quad (A3)$$

$$(P^{(0)}_{SS})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{D-1}\theta^{\lambda\mu}\theta_{\alpha\beta}, \qquad (P^{(0)}_{WW})^{\lambda\mu}{}_{\alpha\beta} = \omega^{\lambda\mu}\omega_{\alpha\beta}, \quad (A4)$$

$$(P^{(0)}_{SW})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{\sqrt{D-1}}\theta^{\lambda\mu}\omega_{\alpha\beta}, \qquad (P^{(0)}_{WS})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{\sqrt{D-1}}\omega^{\lambda\mu}\theta_{\alpha\beta}. \quad (A5)$$

They satisfy the symmetric closure relation

$$[P^{(2)}_{SS} + P^{(1)}_{SS} + P^{(0)}_{SS} + P^{(0)}_{WW}]_{\mu\nu\alpha\beta} = \frac{\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}}{2}. \quad (A6)$$

The remaining antisymmetric and mixed symmetric-antisymmetric operators are given by

$$(P^{(1)}_{AA})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}(\theta^{\lambda}{}_{\alpha}\omega^{\mu}{}_{\beta} - \theta^{\mu}{}_{\alpha}\omega^{\lambda}{}_{\beta} - \theta^{\lambda}{}_{\beta}\omega^{\mu}{}_{\alpha} + \theta^{\mu}{}_{\beta}\omega^{\lambda}{}_{\alpha}), \tag{A7}$$

$$(P^{(1)}_{SA})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}(\theta^{\lambda}{}_{\alpha}\omega^{\mu}{}_{\beta} + \theta^{\mu}{}_{\alpha}\omega^{\lambda}{}_{\beta} - \theta^{\lambda}{}_{\beta}\omega^{\mu}{}_{\alpha} - \theta^{\mu}{}_{\beta}\omega^{\lambda}{}_{\alpha}), \tag{A8}$$

$$(P^{(1)}_{AS})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}(\theta^{\lambda}{}_{\alpha}\omega^{\mu}{}_{\beta} - \theta^{\mu}{}_{\alpha}\omega^{\lambda}{}_{\beta} + \theta^{\lambda}{}_{\beta}\omega^{\mu}{}_{\alpha} - \theta^{\mu}{}_{\beta}\omega^{\lambda}{}_{\alpha}), \tag{A9}$$

$$(P^{(0)}_{AA})^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}(\theta^{\lambda}{}_{\alpha}\theta^{\mu}{}_{\beta} - \theta^{\mu}{}_{\alpha}\theta^{\lambda}{}_{\beta}). \tag{A10}$$

They satisfy the antisymmetric closure relation (see Appendix B of Ref. [28])

$$[P^{(1)}_{AA} + P^{(0)}_{AA}]_{\mu\nu\alpha\beta} = \frac{\eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\mu\beta}\eta_{\nu\alpha}}{2}. \tag{A11}$$

Adding up (A6) and (A11) we have

$$[P^{(2)}_{SS} + P^{(1)}_{SS} + P^{(0)}_{SS} + P^{(0)}_{WW} + P^{(1)}_{AA} + P^{(0)}_{AA}]_{\mu\nu\alpha\beta} = \eta_{\mu\alpha}\eta_{\nu\beta}. \tag{A12}$$

The reader can check that the operators satisfy the simple algebra

$$P^{(s)}_{IJ}P^{(r)}_{KL} = \delta^{sr}\delta_{JK}P^{(s)}_{IL}. \tag{A13}$$

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