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BEYOND THE PAGES: CULTURAL AND LINGUISTIC ASPECTS OF CANADIAN TEXTBOOKS

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INSTITUTO DE GEOCIÊNCIAS E CIÊNCIAS EXATAS

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UNIVERSIDADE ESTADUAL PAULISTA
"Júlio de Mesquita Filho"
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Beyond the pages: cultural and linguistic aspects of canadian textbooks

Tese de Doutorado apresentada ao Instituto de Geociências e Ciências Exatas do Câmpus de Rio Claro, da Universidade Estadual Paulista Júlio de Mesquita Filho, como parte dos requisitos para obtenção do título de Doutor em Educação Matemática

Orientador: Professora Doutora Rúbia Barcelos Amaral

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IMPACTO POTENCIAL DESTA PESQUISA

A pesquisa visa melhorar o uso de livros didáticos de Matemática em contextos multilíngues, promovendo inclusão e equidade. Ao propor o conceito de bridging, alerta para a ausência de políticas que considerem a diversidade linguística e cultural nas práticas educacionais. Espera-se impulsionar melhorias locais e contribuir globalmente.

POTENTIAL IMPACT OF THIS RESEARCH

The research aims to improve the use of mathematics textbooks in multilingual contexts, promoting inclusion and equity. By proposing the concept of bridging, it draws attention to the absence of policies that consider linguistic and cultural diversity in educational practices. It hopes to drive local improvements and contribute globally.

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to Christina for whom i have always wait

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“...that space of refusal, where one can say no to the colonizer, no to the downpressor... is located in the margins... it offers the possibility of radical perspective from which to see and create, to imagine alternatives, new worlds.” (Hooks, 1989, p. 21).

ABSTRACT

This dissertation examines how mathematics textbooks produced in Canada - specifically the Mathematics Makes Sense (MMS) collection - can be analyzed and interpreted in Brazilian educational contexts, with an emphasis on multilingual schools. This is a case study with a qualitative documentary approach, centered on a review of relevant materials and based on theoretical perspectives that connect the socio-political and linguistic dimensions of Mathematics Teaching. Content analysis was used as a technique to organize, categorize and interpret the textual and visual data of the four volumes analyzed, from 6th to 9th grade, allowing us to identify cultural references and their possible resonance or distancing in relation to local realities. The results indicate that the MMS collection includes examples related to Canadian climates, festivals and sports, such as winter festivities, ice hockey and references to indigenous groups, which may seem distant to Brazilian students. Although these characteristics present social and cultural elements that can broaden students' repertoires, there is a risk of disconnection if teachers do not guide students in articulating these contents with their own contexts. In this sense, the concept of “bridging” is proposed as a process of cultural mediation: the teacher moves between foreign examples and the students' experiences, adapting the material, for example, by replacing field hockey with local sports or relating Canadian celebrations to regional festivities. Activities with games, arts or mentions of indigenous peoples also require attention to the Brazilian context, although this adaptation should not be the sole responsibility of the teacher. It is argued that public policies and educational managers should provide the basic conditions - such as continuing education, support for research and flexible curriculum - for this process to take place in a structured way, avoiding teacher overload and the difficulties associated with complex pedagogical decisions. Finally, it is recognized that the use of foreign textbooks is not neutral, as it involves symbolic and power disputes that legitimize certain scenarios to the detriment of others. The conclusion is that the MMS collection can enrich the school experience, provided it is used with critical mediation and institutional support, so that mathematics is understood as a socio-cultural practice - historical and diverse - which enables dialogue between different worlds.

Keywords: Cultural Bridging; Cultural Recontextualization; Foreign Textbooks; Sociopolitical Dynamics; Multilingual Recontextualization.

RESUMO

Esta tese examina como os livros didáticos de matemática produzidos no Canadá – especificamente a coleção Mathematics Makes Sense (MMS) – podem ser analisados e interpretados nos contextos educacionais brasileiros, com ênfase nas escolas multilíngues. Trata-se de um estudo de caso com abordagem qualitativa documental, centrado em uma revisão de materiais relevantes e fundamentado por perspectivas teóricas que conectam as dimensões sociopolíticas e linguísticas do Ensino de Matemática. A análise de conteúdo foi utilizada como técnica para organizar, categorizar e interpretar os dados textuais e visuais dos quatro volumes analisados, do 6º ao 9º ano, permitindo identificar referências culturais e sua possível ressonância ou distanciamento em relação às realidades locais. Os resultados indicam que a coleção MMS inclui exemplos relacionados a climas, festivais e esportes canadenses, como festividades de inverno, hóquei no gelo e referências a grupos indígenas, que podem parecer distantes para estudantes brasileiros. Embora essas características apresentem elementos sociais e culturais que possam ampliar o repertório dos alunos, existe o risco de desconexão caso os professores não orientem os estudantes na articulação desses conteúdos com seus próprios contextos. Neste sentido, propõe-se o conceito de “ponte” como um processo de mediação cultural: o professor transita entre exemplos estrangeiros e as vivências dos alunos, adaptando o material, por exemplo, ao substituir o hóquei por esportes locais ou relacionar celebrações canadenses com festividades regionais. Atividades com jogos, artes ou menções aos povos indígenas também demandam atenção ao contexto brasileiro, embora essa adaptação não deva ser responsabilidade exclusiva do professor. Argumenta-se que políticas públicas e gestores educacionais devem oferecer condições básicas – como formação continuada, apoio à pesquisa e currículo flexível – para que esse processo ocorra de maneira estruturada, evitando a sobrecarga docente e as dificuldades associadas a decisões pedagógicas complexas. Por fim, reconhece-se que o uso de livros didáticos estrangeiros não é neutro, pois envolve disputas simbólicas e de poder que legitimam determinados cenários em detrimento de outros. Conclui-se que a coleção MMS pode enriquecer a experiência escolar, desde que utilizada com mediação crítica e apoio institucional, de modo que a matemática seja compreendida como uma prática sociocultural – histórica e diversa – que possibilita o diálogo entre diferentes mundos.

Palavras-chave: Ponte Cultural; Recontextualização Cultural; Livros Didáticos Estrangeiros; Dinâmicas Sociopolíticas; Recontextualização multilíngue.

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LIST OF ABBREVIATIONS

CNN Cable News Network
BC British Columbia
BNCC Base Nacional Comum Curricular
ICT Information and Communication Technology
IBGE Instituto Brasileiro de Geografia e Estatística
MMS Mathematics Makes Sense NWT Northwest Territories
NCTM National Council of Teachers of Mathematics
NHL National Hockey League
NWT Northwest Territories
PCN Parâmetros Curriculares Nacionais
PISA Programme for International Student Assessment
PNLD Programa Nacional do Livro e do Material Didático
TIMSS Trends in International Mathematics and Science Study
SIDRA Sistema IBGE de Recuperação Automática
UNESP Universidade Estadual Paulista "Júlio de Mesquita Filho"
WNCP Western and Northern Canadian Protocol

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1 INTRODUCTION

When I heard Invisível (PassaPusso et al., 2017) for the first time, I felt each verse reverberate like a call to look deeper, to notice what so often goes out of focus and therefore seems non-existent.

> “You passed me by. And didn't even look at me. You've passed me by. And didn't even look at me. You think I don't attract attention (you think). Fool your heart. You think I don't attract attention. There's no color, there's no face, it started, and it won't stop. Your heart will soar, there's no way to tell. Nobody saw, nobody saw. No one can find you. Invisible, invisible. Nobody finds you. There are ways of seeing. Ways of being. Ways of having. Way of seeing. Better half. Better half. I want to see you in the city. Everyone's looking. Nobody finds you. The city's scary, but it'll dawn. You've already passed me by. And didn't even look at me. You've passed me by. And didn't even look at me. You think I don't attract attention (you think). You fool your heart. You think I don't attract attention. Fool your heart. No more talking, no more explaining so you can understand me. Babylon, many ways to try to trap you. Hit bass drum hit snare hit thigh boom-bum. Without rushing, the talk is good enough. Every word you keep in your mouth turns to drool. Philosophy I get from reading Gandhi and Sai-Babba. Ragga, I listen to Shabba, in the hinterland, Luiz Gonzaga. The battle is about to double, here it is done, here it is paid, yes. You've already passed me by. And didn't even look at me. You've passed me by. And didn't even look at me. You think I don't attract attention (you think). You fool your heart. You think I don't attract attention. You passed me by. Passed me by. Passed me by. Looked at me. Looked at me. Looked at me. It caught my eye. Got my attention. You caught my eye. Your heart. Your heart. Your heart (people.” (PassaPusso et al, 2017, our translation).

This reflection traces back to early experiences in Jundiapeba, a peripheral neighborhood of Mogi das Cruzes (SP), where a mother – originally from the semi-arid region of Pernambuco – overcame countless challenges before finally earning degrees in Pedagogy and Arts. Within that home, knowledge was upheld as a powerful instrument of transformation, even amid financial hardships and daunting routines.

An affinity for numbers and a concern for public policies guided initial educational pursuits, leading to Engineering studies as well as later engagement in Mathematics Teaching. Over time, practical work in industries merged with academic goals, culminating in a Master’s investigation on educational indicators in the city of origin. Yet, the turning point arrived during doctoral studies, while teaching Mathematics in a multilingual school that adopted foreign textbooks, specifically Mathematics Makes Sense (MMS) from Canada.

Observing the contrasts between the local diversity and references to winter festivals, sports on ice, and extreme negative temperatures spurred a deeper inquiry: how can cultural elements from a Canadian context, featured in Mathematics textbooks, be reinterpreted so they do not become invisible to learners in Brazilian multilingual settings? This question echoes themes in the song "Invisível" (Passapusso et al., 2017), which suggests an urge to see beyond the surface. It also highlights how certain references – such as images of snow or Indigenous Canadian festivals – may sound distant for students unacquainted with such realities. The risk of disconnect arises: what if local experiences and identities remain unseen within the pages of an imported curriculum? Instead of sparking excitement, a sense of mismatch may take hold, reinforcing the notion that not everyone or everything is truly being acknowledged.

In this spirit, the present research formulates a guiding concern: Which cultural aspects of the Canadian context are made explicit in Mathematics textbooks, and in what ways can those elements be analyzed and recontextualized in Brazilian multilingual schools without rendering them invisible to students? The hypothesis suggests that, while Canadian materials offer robust approaches and compelling visuals, their meaningfulness in local classrooms rests heavily on teacher mediation.

Drawing inspiration from the idea of bridging, this perspective builds on sociocultural theories, such as those of Vygotsky (1978) – who emphasizes the role of historical and social contexts in concept formation – and on critical analyses from Valero (2004, 2012, 2023) and Apple (2018), who interpret textbooks as cultural and political artifacts that privilege certain voices while silencing others. Additionally, the work of Moschkovich (2010, 2015) highlights the complexities and possibilities of Mathematics teaching in multilingual settings, where language must be seen not merely as direct translation but as a negotiation of meaning.

A qualitative documentary approach was adopted, centering on the examination of four volumes from the MMS collection using the Content Analysis (Bardin, 2016). This analytical framework makes it possible to identify sociocultural references – festivities, sporting traditions, Indigenous elements – and to explore whether these examples do or do not resonate with the experiences of Brazilian students.

Earlier research on public education indicators carried out at the Master's level, illuminated the often-disconnected nature of proposed governmental goals. Now, at the doctoral level, the focus shifts to the cultural side of Mathematics textbooks, especially in a country with vast linguistic, ethnic, and regional diversity. By confronting a material shaped for another location, the investigation seeks to reinforce the notion that sociolinguistic awareness is crucial. If a text remains unadapted or unexamined, there is a risk of marginalizing or ignoring learners' worldviews.

Throughout all the chapters, "Invisível" remains a central metaphor that calls attention to the subtle ways in which Mathematics – sometimes perceived as purely universal or neutral – carries an array of cultural assumptions. Adopting foreign textbooks in Brazilian multilingual contexts goes well beyond linguistic conversion or the introduction of so-called innovative methods; it entails acknowledging that, without appropriate mediation, numerous local elements may be overshadowed.

Yet, this is not intended as a wholesale critique of the Canadian materials; rather, it emphasizes how they can be enriching resources once educators actively reinterpret examples, contextualize them to local realities, and foster comparisons between different cultures. Through such intercultural interplay, the concept of bridging becomes tangible: an ongoing effort to ensure that content originally conceived elsewhere finds resonance in Brazilian realities. Stepping into these sections, therefore, is an invitation to perceive how bridging worlds can enable Mathematics teaching to be more inclusive, culturally aware, and consistently attuned to the learners who engage with it.

1.1 Research objectives and structure

Over the past decade, a noticeable increase in Canada-based bilingual school franchises has been observed in Brazil (Souza, 2020, our translation). According to Silva and Martins (2022, our translation), one such network – introduced in the mid-2000s – already operates over 150 units across multiple

Brazilian states. This expansion highlights the likelihood that textbooks originally developed for the Canadian context will appear in local daily practice, particularly in multilingual learning environments. Such growth in the editorial market (Michetti, 2022, our translation) also responds to and reinforces a perception that countries performing well on international education indexes produce “superior” content (Brito, 2020, our translation).

As a result, many Brazilian schools have turned to these materials to access “more advanced” resources. Amid these trends, it becomes increasingly relevant to examine how Canadian-produced textbooks reflect specific sociocultural features and to what extent these align – or not – with the realities of Brazilian classrooms.

This investigation originated from an initial plan to compare Brazilian and Canadian mathematics textbooks, focusing on Geometry. Early evaluations of that project emphasized a broader need: beyond purely content-driven analysis, it was important to explore the cultural markers embedded in foreign textbooks and how such markers might (or might not) resonate with Brazil. One suggestion was to consider these foreign materials not merely as educational resources but also as potentially colonizing models – often adopted because their countries of origin feature prominently in international rankings.

Building on these insights, I decided to center the research on the Mathematics Makes Sense collection, a set of Canadian textbooks widely acknowledged abroad. With the expansion of Canadian-franchised schools in Brazil, there is a clear possibility that local educators, especially in bilingual contexts, will be using or adapting these materials.

Thus, the focus shifted from “How is Geometry represented in foreign books?” to “What cultural aspects of the Canadian context are embedded and made explicit in mathematics textbooks, and how can these elements be interpreted in multilingual Brazilian schools?” In discussions with my supervisor and the TeorEMa group, we also recognized that any analysis of textbooks should account for historical and sociopolitical dimensions – rather than purely methodological factors – given that language, culture, and curriculum inevitably intersect.

From these developments, the final guiding question emerged: What cultural aspects of the Canadian context are embedded and made explicit in mathematics textbooks now circulating in Brazil, and how can these elements be interpreted in multilingual schools?

The general objective of this research is:

To explore and understand how Canadian-produced textbooks shape, and are shaped by, the multilingual and sociocultural specificities of the Brazilian educational context.

To support this broader aim, three specific objectives guide the work:

a) Analyze the sociocultural differences present in Canadian mathematics textbooks and how these differences can affect the interpretation of these materials in Brazilian educational contexts.
b) To investigate the cultural and linguistic aspects explicit in Canadian textbooks and how they can dialog with Brazilian sociocultural contexts.
c) To discuss the potential sociopolitical implications of adopting foreign textbooks in Brazil, with a focus on how these materials align with or challenge local educational policies and practices.
d) Examine how Canadian textbooks approach the teaching of integers, fractions, and language, in light of existing research on these topics.

By weaving together my own teaching experiences with these four analytical dimensions, this dissertation positions itself at the intersection of practical educational challenges and academic reflection. My personal trajectory – marked by observing how foreign examples often failed to connect with local students – pushed me to move beyond content-driven analysis and consider how textbooks can embody cultural perspectives and hidden assumptions. Consequently, this study aims not only to show “what” these textbooks contain but also to unveil the sociocultural stories they bring, along with the possible silences or mismatches encountered in Brazilian schools.

It is also a tribute to the journey that has brought me here – rooted in my mother’s devotion to education and my own challenges with culturally diverse classrooms. Every page represents an attempt to turn personal insights into a collective dialogue that fosters more context-aware and inclusive teaching practices.

This dissertation is organized into seven sections that connect the research question, objectives, and final argument in a coherent flow. The first section introduces the personal motivation and context that led to the guiding question – what cultural aspects of the Canadian context are embedded in textbooks and how these might be reinterpreted in multilingual Brazilian schools – while also presenting the general and specific objectives. The second section surveys the scholarly works and theoretical discussions on sociocultural, linguistic, and political perspectives in Mathematics Education, clarifying the gap this investigation addresses.

The third section explains the framework that underpins the textbook analysis, discussing how textbooks function as cultural and political artifacts and describing the three main dimensions of interest: sociocultural, sociopolitical, and linguistic. The fourth section describes the historical and structural features of the Canadian mathematics curriculum, providing context for the textbooks and highlighting the curricular environment from which they emerge. The fifth section defines the methodological approach, detailing the qualitative procedures, the rationale for selecting the Mathematics Makes Sense collection, and the analytical steps guided by the contente analysis.

The sixth section presents and discusses the data, illustrating how Canadian textbooks reveal particular cultural references, linguistic elements, and sociopolitical assumptions that may align or diverge from Brazilian realities, thereby speaking to the research question and objectives. The seventh and final section offers an overview of findings, addresses limitations, suggests directions for future studies, and consolidates how the evidence meets the dissertation's overall argument regarding the interplay of foreign textbooks and local classroom practice in multilingual contexts.

By investigating the cultural import of these materials – promoted by Canada-based franchises and the widespread assumption of Canadian educational superiority – this dissertation aims to illuminate the power dynamics and hierarchical discourses embedded in the complex process of curriculum transfer between distinct educational contexts. Rather than viewing foreign textbooks as neutral assets, I underscore the sociopolitical tensions that arise when defining whose knowledge is deemed authoritative, whose voices shape the material, and who stands to benefit from their adoption.

This research hopes to encourage a more critical and context-sensitive use of such resources in Brazil, taking into account linguistic plurality, diverse cultural identities, and the subtle ways in which power relations can either validate or marginalize local practices. Just as the song “Invisible” reminds us that “what is not seen must be revealed,” this dissertation takes a critical lens to how foreign textbooks operate as cultural and political artifacts in new arenas.

By shedding light on the hidden power imbalances and cultural silences that can unfold in multilingual school settings, I aim to foster discussions that value diversity and inclusion in Mathematics Education. Ultimately, the intent is to spark debates on teacher agency and equitable knowledge production, challenging any singular notion of “advanced” content and advocating for dialogical approaches that genuinely respect local realities.

2 LITERATURE REVIEW

The aim of this section is to present the literature review that investigates the relationship between mathematics textbooks originating from the Canadian context and how they may be reframed or reinterpreted elsewhere. Guided by the identified gap in the literature, this review seeks to understand how previous studies have approached or analyzed the use of foreign textbooks, emphasizing cultural, linguistic, and political dimensions that emerge when such materials are adopted in different educational contexts.

Therefore, two main objectives guide this review. First, to identify and compare studies that address cross-country influences of textbooks, focusing on cultural and linguistic factors. Second, to examine research that discusses sociocultural and political impacts linked to the use of foreign or local materials in teaching environments.

When selecting references, particular attention was given to works exploring how textbooks integrate or highlight values and perspectives from outside their target audiences, as well as how these materials may be reshaped by local realities. As a result, the set of texts analyzed covers discussions on content, methodologies, assessments, curriculum construction, and inclusion, forming a diverse and complementary panorama.

By situating this study at the confluence of multiple perspectives, these works broaden the debate and enrich the analysis, pointing to possibilities for critical reflection and future interventions regarding the integration of foreign textbooks, even though they are not the direct focus of the final discussion.

2.1 Presentation of the review process and selection of references

Following Creswell's (2014) guidelines for systematic literature reviews, an initial broad bibliographic survey was carried out to identify studies related to the analysis of mathematics textbooks at both national and international levels. This process entailed defining clear research questions and selection criteria, as well as mapping out the databases and descriptors that would guide the search.

Through this systematic approach, it was possible to identify different strands of inquiry on textbooks, ranging from investigations focused on the structure of mathematical content to those that address cultural, political, and linguistic dimensions. In order to delineate the set of publications, the period from 1980 to 2023 was established, despite initial results returning works from as early as 1932. This range was chosen to capture a sufficiently broad set of studies considered most pertinent. Next, the review was refined by limiting the analysis to studies published from 2010 onward, ensuring a focus on more current discussions and allowing for a deeper examination of contemporary trends.

Finally, the literature produced was screened and evaluated through an iterative reading and coding process, which facilitated the identification of recurring themes and gaps in the field. This step-by-step procedure – encompassing search, selection, analysis, and synthesis – ensured a rigorous foundation for the subsequent discussion, aligning with Creswell’s systematic recommendations and situating the present research within the ongoing scholarly debate.

The descriptors used included generic combinations such as “analysis” AND “textbook” AND “mathematics”, “methodologies” AND “teaching” and “textbook” AND “pedagogical practices”.

In addition, the proceedings of significant events in mathematics education were examined, including the National Meeting of Mathematics Education (ENEM), Psychology of Mathematics Education (PME), Congress of the European Society for Research in Mathematics Education (CERME) and the International Conference on Research and Development of Mathematics Textbooks (ICMT).

Table 1 shows the distribution of these publications among dissertations, theses, publications in events, book chapters, books and articles.

Table 1 - Number of academic productions dealing with the analysis of textbooks between 1980 and 2023.

		1980 to 1999	**2000 to 2009**	**2010 to 2019**	**2020 until 2023**
Articles	International	52	47	235	112
	National	1	33	13	0

		1980 to 1999	2000 to 2009	2010 to 2019	2020 until 2023
Book chapter	National	0	0	0	2
Dissertation	International	1	0	2	0
	National	2	17	5	3
Books	International	8	2	2	1
	National	0	0	2	0
Thesis	International	1	1	4	0
	National	0	1	1	0
Conference Proceedings	International	1	9	64	21
	National	0	2	12	8
Total		66	112	340	144

Source: The author (2023).

As mentioned, the search returned publications from the first decades of the 20th century to recent productions. After reading the titles and abstracts, it was decided to delimit the interval between 1980 and 2023 to maintain consistency with contemporary transformations in math teaching and public policies related to textbooks.

Then, considering the volume and the desired contemporaneity, it was decided to adopt 2010 as the time frame for the detailed examination of texts, reducing the initial set to 485 studies. We then began a second stage in which we looked more specifically at the 485 studies found, using descriptors related to the central objectives of the research, such as “cultural aspects” AND “textbooks” AND “mathematics”, “multilingualism” AND “education” AND “textbooks” and “cultural representations” AND “mathematics”.

The aim was to identify studies that highlighted cultural, linguistic or political dimensions in the analysis of mathematics textbooks.

In all, 57 studies were pre-selected for exploratory reading of the abstracts and, based on criteria related to adherence to the research problem (presence of cultural, linguistic or political dimensions in the analysis of mathematics textbooks) and the depth of the methodological and conceptual discussion, 7 papers (Chart 1) were selected for full reading, with the aim of gaining a more precise understanding of their results and approaches.

In other words, priority was given to choosing investigations that provided empirical evidence or in-depth reflections on the use of foreign textbooks (particularly Canadian ones) and their dialogue with Brazilian contexts, especially in dimensions such as multilingualism and multiculturalism.

The exploratory reading of the studies indicated different perspectives, all of which have in common the examination of mathematics textbooks as artifacts that are not limited to the transmission of content, but also convey cultural, linguistic and political values. From this movement, two main axes emerged to group the research identified: (i) Content and Methodologies and (ii) Sociocultural and Political Relations.

The decision to group the studies into two categories is related to the recurring observations in the selected publications. Most of the authors emphasize the way content is structured, organized and presented, as well as the social, cultural and political impacts and meanings associated with textbooks. In this way, the two categories seek to differentiate:

a) Content and Methodologies: focus on issues of conceptual organization, teaching and learning strategies, types of exercises, mathematical language, didactic sequences and methodological approaches present in the books.
b) Sociocultural and Political Relations: includes research focusing on cultural, linguistic and political aspects, including elements related to inclusion, representations of social groups or the influence of educational policies on the development and adoption of teaching materials.

This division of Chart 1 seeks to simplify the grouping and discussion of references, preserving the most recurrent dimensions in the literature reviewed, and dialogues directly with the guiding question of this research and its specific objectives.

Chart 1 - Productions selected and classified by Category

Category	Source	Type	Year	Author(s)	Title
Contents and Methodologies	Interfaces Magazine	Article	2020	Attie and Krpan	Argumentation in mathematics textbooks: Brazil and Canada
	JIEEM Magazine	Article	2021	Evangelista et al.	Proposals for Activities with Tables in Mathematics Textbooks
	UFPE Repository	Dissertation	2013	Santos Junior	Strategies used by 7th, 8th and 9th grade students in solving sharing problems
	USP Repository	Dissertation	2019	Souza	Mathematics textbooks from Brazil and Canada: a focus on geometric construction
	UNICAMP Repository	Dissertation	2023	Hernandes	Isometries in mathematics textbook collections
Sociocultural and Political Relations	CAPES Repository	Article	2018	Garcia et al.	Australia, Brazil and Canada: the impact of assessments on science teaching

Category	Source	Type	Year	Author(s)	Title
	Blucher Magazine	Article	2019	Brito	Design, education and inclusion in the Province of Québec

Source: The author (2023).

Although some of the works in Chart 1 could move between the two categories, we tried to choose the one in which each study had the greatest emphasis.

Research in this category includes:

a) Attie and Krpan (2020) which discusses how mathematical argumentation is approached in textbooks from Brazil and Canada, highlighting methodological and terminological differences;
b) Evangelista da Silva et al. (2021) which examines the way the tables are presented and the pedagogical objectives involved;
c) Santos Junior (2013) which investigates strategies for solving sharing problems and the possible implications for the selection of content in teaching materials;
d) Souza (2019) who compares geometric constructions in Brazilian and Canadian books, highlighting differences in sequences and conceptual emphases;
e) Hernandes (2023) who studies the treatment of isometries in textbook collections, highlighting the variations in the way this topic is approached.

As for the Sociocultural and Political Relations category, there are studies that focus on the presence (or absence) of cultural, linguistic, identity and political elements in mathematics textbooks, as well as how educational policies can influence the design and adoption of these materials.

In this category:

a) Garcia et al. (2018) examine different national guidelines (Australia, Brazil and Canada) and their possible repercussions on the formulation of educational proposals;
b) Brito (2019) describes the dynamics of design, education and inclusion in Quebec, highlighting issues related to multiculturalism and the guidelines that guide the production of materials in the Canadian context.

The analysis of content structures and teaching strategies (Contents and Methodologies) can reveal cultural, linguistic and conceptual choices related to the context of origin, while the study of policies and values (Sociocultural Relations and Policies) shows the extent to which Canadian guidelines can diverge from Brazilian ones, including considerations about the limits of transposing these contents to another setting.

This dual classification is associated with the recurrence of the approaches found in the literature and their relevance to the proposed research, since several studies emphasize the form and structure of mathematical content, while others delve into aspects linked to culture, language and institutional guidelines.

2.2 A more comprehensive look at the selected works

This subsection organizes the selected studies into two main subsections - Content and Methodologies and Sociocultural and Political Relations in Mathematics Textbooks - to highlight how each research group addresses different (but interconnected) dimensions of the use of textbooks from Brazil and Canada. The first subsection discusses the conceptual and methodological organization of mathematical content (Attie & Krpan, 2020; Santos Junior, 2013; Souza, 2019; Hernandes, 2023), including aspects such as rigor, autonomy and teaching practices. Finally, the second subsection emphasizes the cultural, linguistic, political and identity elements present in the books and their re-signification in different contexts (Garcia et al., 2018 and Brito, 2019).

By consolidating the methodologies, findings and implications of these works, the subsection establishes links between varied approaches and highlights the relevance of understanding both the disciplinary content and the underlying cultural values in the transposition of teaching materials to multilingual realities.

2.2.1 Contents and Methodologies

The organization of mathematical content in textbooks is essential for promoting contextualized learning. This segment analyzes how different studies have investigated and presented data related to this topic.

Attie and Krpan (2020) explore the differences in argumentation processes in mathematics textbooks from Brazil and Canada, based on the explanatory and justificatory categories proposed by Balacheff (1988). Using content analysis as a methodology, as described by Bardin (2010), the study investigates how pedagogical approaches in these two contexts reflect distinct sociocultural characteristics and impact learning in mathematics.

The books analyzed were selected for their official adoption in Sergipe, Brazil, and Ontario, Canada, based on their recommendations by the main national programs: the Programa Nacional do Livro Didático e do Material Didático (PNLD) and the Trillium List[1] respectively. The central hypothesis is that structural and cultural differences in educational systems contribute to the unequal performance observed in PISA between the two countries. In this context, Brazil ranks lower than Canada, one of the leaders in the international ranking of educational performance in mathematics.

The analysis focused on topics considered challenging for students, such as divisibility, numerical expressions, division of fractions and calculation of areas. The research sought to understand how argumentation processes are used to present and justify mathematical content, highlighting the differences between an explanatory approach, which emphasizes the clarity of the process, and a justificatory approach, which privileges logic and mathematical rules. In

[1] The Trillium List contains the titles of current textbooks approved by the Minister of Education that educators can use to teach students in Kindergarten to Grade 12 in Ontario's publicly funded schools. Trillium List. (2005). *Trillium List of Approved Textbooks*. Ministry of Education, Ontario, Canada. Disponível em https://www.ontario.ca/page/trillium-list.

both contexts, a predominance of normative practices was observed, to the detriment of a more detailed exploration of the processes underlying mathematical learning.

The results indicate that Canadian textbooks are more concerned with promoting autonomy and the search for patterns, in line with a more student-centered investigative practice. However, the inclusion of cultural elements in Canadian textbooks, such as the incorporation of indigenous perspectives, often occurs superficially, without significant integration into the core mathematical content. On the other hand, Brazilian textbooks have a stronger focus on rules and algorithms, with less emphasis on explaining processes. This contrast reflects the differences in pedagogical priorities and educational contexts in each country.

Although Canadian textbooks stand out for encouraging autonomy, both contexts show limitations in the use of argumentation as a tool for exploring complex mathematical processes. As Attie and Krpan (2020, p. 2) state, “The research showed that, in general, Canadian textbooks are more concerned with encouraging aspects such as autonomy and the search for patterns. However, in both contexts, there is an attachment to rules to the detriment of exposing the process involved in developing those same rules.”

In summary, the analysis revealed that, despite cultural and methodological differences, both Brazilian and Canadian materials face challenges in adopting more investigative and less prescriptive practices, limiting the potential for a truly exploratory and contextualized approach to teaching mathematics.

Evangelista, Guimarães and Oliveira (2021) analyze how the content of tables in mathematics textbooks for the initial years of elementary school - based on central curricular documents for mathematics education in the two contexts analyzed: the National Curriculum Parameters (PCNs) in Brazil and the Progression des Appointments (PAP) in the United States.[2] in Brazil and the Progression des apprentissages: Mathématique no Quebe (MELS)[3].

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[2] Brasil. (1997). *Parâmetros Curriculares Nacionais: Matemática*. Brasília: MEC/SEF.
[3] Ministère de l'Éducation, du Loisir et du Sport (MELS). (2009). *Progression des apprentissages: Mathématique*. Gouvernement du Québec.

The PCNs, introduced in 1997, highlight the importance of teaching practices that promote the collection, organization, analysis and communication of statistical data through representations such as tables and graphs. According to the document, the teaching of mathematics should be related to the real world, using visual representations to help understand mathematical principles and concepts. Thus, tables are conceived as "learning objects that allow the display of collected data and the critical analysis of information, enabling students to infer, evaluate and avoid hasty interpretations"[5].

In Quebec, the MELS presents an approach that organizes statistical learning into three cycles, covering six years of elementary school. This document emphasizes the development of students' statistical thinking through activities that lead them to collect, organize, interpret and represent data in tables or graphs, including pictograms and bar graphs. These skills are "progressively structured so that, at the beginning of each cycle, students carry out the activities with the support of the teacher, while at the end of the cycle they are expected to be autonomous in their execution"[6].

In the article, the authors base their analysis on relevant studies in the literature. For example, to discuss the acquisition of mathematical skills in the use of tables and graphs[4] highlighting the practical perspective in developing these skills. In addition, to justify the importance of the investigative cycle as a tool for developing statistical thinking and the integration of school activities that promote data collection, organization and analysis[5].

The research showed that although the two curriculum documents value the use of tables as teaching tools, they differ in their emphasis and detail. The PCNs are explicit in distinguishing between simple and double-entry tables, as well as stressing the importance of collecting real data and constructing tables as part of the educational process.

However, the lack of clear gradations between skills throughout the cycles leaves gaps in the progression of complexity, burdening teachers with the task of

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[4] Martí, E., Sedano, S., & La Cerda, I. (2010). La adquisición de competencias matemáticas en el uso de tablas y gráficos: Un análisis desde la perspectiva de la práctica. *Revista Educación Matemática*, 22(3), 187-205.
[5] Guimarães, G., & Gitirana, V. (2013). Ciclo investigativo e o desenvolvimento do pensamento estatístico. *Ensaio: Avaliação e Políticas Públicas em Educação*, 21(80), 465-490.

introducing different types of variables and fundamental elements of tables. In contrast, the Quebec document, despite emphasizing the progressive development of skills, lacks specificity regarding the types of tables or variables to be worked on, limiting itself to the general organization of the skills to be acquired in each cycle (Martí, Sedano & La Cerda, 2010; Estrella, 2014).

Integrating these references into teaching is essential for strengthening statistical skills and promoting meaningful learning. However, the research found that the textbooks analysed, both in Brazil and Quebec, still have weaknesses in terms of contextualizing data and exploring tables as tools for developing critical thinking (Guimarães & Gitirana, 2013; Silva & Guimarães, 2013). These limitations underline the need for a review of editorial and pedagogical practices, as well as greater alignment with the curricular objectives described.

Hernandes (2023) analyzes the treatment of Isometries in collections of mathematics textbooks from Brazil and Canada, highlighting conceptual and methodological differences in the teaching of Geometric Transformations in different educational contexts. Based on a qualitative approach, the research used two textbook collections as a corpus of analysis: one Brazilian, from the 2020 National Textbook and Teaching Material Program (PNLD), and the other Canadian, belonging to the Trillium List, published in 2005 and widely adopted in Canada. The choice of topic is based on the growing relevance of Isometries in national curricula, as indicated by the Common National Curriculum Base (BNCC).

The main objective of the research was to understand how Isometries are presented in the textbooks of these two countries, investigating the conceptual rigor, the examples and the tasks proposed. To this end, the analysis was structured into three main units: Conceptualization, Examples and Tasks. The study adopted documentary analysis as its research method, allowing a detailed understanding of the approaches used in each collection.

The theoretical foundation is based on works such as those by Lima[6]who presents the concept of Isometries with mathematical rigor, and Euclid[7]whose Plane Geometry offers historical foundations for teaching. In addition, the work is

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[6] Lima, E. L. (1996). *Geometria: Um Tratado Elementar*. Rio de Janeiro: SBM.
[7] Euclides, em Bicudo, M. A. V., Borba, R. E., & Saito, L. (2009). *Geometria Euclidiana: Fundamentos e Aplicações*. São Paulo: Editora da UNESP.

aligned with the pedagogical discussions proposed by the BNCC and Canadian curriculum documents.

The analysis revealed that the concept of symmetry is inconsistent in both collections. In Brazil, the concept is often associated with geometric figures, while in Canada there is a greater emphasis on the relationship between points and their images. There was also limited use of color to explore the aesthetic potential of the topic, although this approach is suggested in both contexts.

The approach to translations in the Brazilian books shows a modest use of vectors, concentrating on the practical application of geometric figures. The Canadian books, on the other hand, are more mathematically rigorous, integrating the concept of vector with tasks that stimulate geometric reasoning.

Both collections explore the fixed point of rotations, but the Canadian collections highlight clockwise and anti-clockwise orientation more clearly, helping to develop a more solid understanding of this concept. The tasks in the Brazilian books, although practical, lack a more robust articulation with theory.

Reflections around axes or points are explored in both collections, but the analysis identified difficulties in representing overlaps, particularly in the Brazilian books. The Canadian books present a greater diversity of examples, including more complex cases such as reflections around inclined lines.

The results show that although both countries treat Isometries in a manner consistent with their respective curricula, there are significant differences in the depth and conceptual rigor of the approaches. The research suggests that Brazilian textbooks could benefit from a more robust integration of theory and practice, in line with the recommendations of the BNCC. In the Canadian context, the greater emphasis on mathematical rigor provides a solid foundation, but could be complemented by approaches that incorporate more cultural contexts and practical applications.

This analysis contributes to the discussion on the role of textbooks in mathematical education, proposing ways to improve the teaching of Isometries in different cultural and educational contexts.

Souza's research (2019) carries out a comparative documentary analysis of the approach to geometric construction in mathematics textbooks in the final years of elementary school in Brazil and Canada, seeking to identify methodological, curricular and cultural similarities and differences. The books analyzed include Projeto Teláris[8] and Matemática na Medida Certa[9]from Brazil, and Nelson Mathematics[10] and Math Makes Sense[11]from Canada. These materials were studied in the light of the National Curriculum Parameters (PCN) and the "Ontario Curriculum[12]", which structure the pedagogical bases of each country.

The documentary analysis methodology was based on a detailed review of the content in the books, highlighting how geometric themes are explored. The analysis included identifying the concepts presented, the exercises proposed and the integration of technological resources, such as dynamic geometry software. Souza (2019) emphasizes that comparing educational contexts reveals the relationship between teaching materials and national curricula. He also tries to show how comparison is a central process in intellectual activity, helping to understand the specificities of different educational systems[13].

The results indicated that the Brazilian and Canadian collections have different approaches, although they share some characteristics. In Brazil, the books Projeto Teláris and Matemática na Medida Certa are aligned with the PNLD, but the geometric construction content is often incomplete and limited. In Canada, the Nelson Mathematics and Math Makes Sense collections stand out for their alignment with the Ontario Curriculum and their inclusion of exploratory activities and use of technology, although they similarly have gaps in depth and breadth.

The research revealed that only circumference and triangle are consistently addressed in the four collections analyzed. This limited choice reflects a narrow focus on constructive geometry in both contexts. In addition, the Canadian materials show greater integration of educational technologies and exploratory activities, in line with the growing use of tools such as dynamic geometry software. In Brazil, the materials reflect structural challenges and

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[8] Ática. (2019). *Projeto Teláris: Matemática*. São Paulo, Brasil: Editora Ática.
[9] Leya. (2019). *Matemática na Medida Certa*. São Paulo, Brasil: Editora Leya.
[10] Thomson Nelson. (2005). *Nelson Mathematics*. Toronto, Canadá: Thomson Nelson.
[11] Pearson. (2005). *Math Makes Sense*. Toronto, Canadá: Pearson Education Canada.
[12] Ontario Ministry of Education. (2005). *The Ontario Curriculum Grades 1-8: Mathematics*. Toronto, Canadá: Queen's Printer for Ontario.
[13] Ciavatta, M. (2009). *História e memória da educação profissional no Brasil*. Autores Associados.

resource limitations, typical of the Brazilian educational context[14]which highlights the potential of the textbook as an essential resource in the teaching of mathematics.

These findings reinforce the need to improve teaching materials so that they represent geometric construction in a more complete and methodologically consistent way. Souza (2019) concludes that there is room to improve the approach to geometric construction in the textbooks analyzed, aligning them better with curricular demands and specific cultural contexts. The research highlights the role of the textbook as a mediator between national curricula and teaching practices, a perspective corroborated by authors such as Pereira dos Santos and Tolentino-Neto (2015), who emphasize the impact of teaching resources on student performance.

Santos Junior (2013) aimed to investigate the strategies employed by Brazilian students to solve sharing problems. Conducted in three schools in the state of Pernambuco, the research was based on a theoretical framework that included works connecting Brazilian and Canadian experiences in teaching mathematics.

The research's theoretical framework was based on the chaining model[15] which categorizes sharing problems into four types: starting, source, composition and well. This model was developed from studies of Canadian mathematics textbooks, providing a consistent methodological framework for assessing student performance and strategies and investigating the registers and strategies mobilized by Brazilian and Canadian students in algebraic structure problems.[16]. These connections between Brazilian and Canadian studies brought a comparative perspective to the research, broadening its scope and relevance.

Data collection involved 251 students in the 7th, 8th and 9th grades of elementary school, who took two tests (A and B) with seven questions each. The questions were designed based on the categories defined by the theoretical

[14] Dante, L. R. (1996). *Didática da matemática: Uma análise da influência do livro didático*. Ática.
[15] Marchand, P., & Bednarz, N. (1999). Étude sur la résolution de problèmes algébriques dans des manuels scolaires: Une perspective comparative.
[16] Câmara, M. C., & Oliveira, R. M. (2008). Estratégias e registros mobilizados por alunos do 6º ano do ensino fundamental do Brasil e do Canadá na resolução de problemas de estrutura algébrica tipo partilha.

framework[17]allowing us to analyze the cognitive strategies used by the students. These categories included graphic, algebraic and numerical records, which were used to understand the predominant forms of reasoning.

The results showed that the problems classified as “well type” were more difficult for the students, due to the high level of abstraction and logical linking they required. In addition, it was observed that the strategies adopted by the students remained consistent between school years, indicating a limited progression in the development of analytical skills. The research also revealed a significant reliance on basic methods, such as direct calculations, even in problems that required greater conceptual sophistication.

Although data collection was restricted to the Brazilian context, Canadian references provided a theoretical basis that enriched the analysis. The study showed that mathematical strategies and learning difficulties share similarities in different educational contexts, highlighting the relevance of a pedagogical approach that encourages the use of diverse registers and the development of more advanced analytical skills.

Santos Junior (2013) concludes that pedagogical interventions are needed to strengthen algebraic reasoning and to more effectively explore the use of multiple registers in mathematics teaching. The inclusion of Canadian references in the research reinforces the value of comparative studies and international methodologies to strengthen mathematics education in Brazil, while highlighting the local challenges faced by students in the learning process.

In short, this category seeks to show how the textbooks analyzed, despite presenting methodological and structural differences between Brazil and Canada, share limitations related to the integration of investigative and contextualized approaches. While Canadian materials highlight practices that foster autonomy and the search for standards, cultural elements, such as indigenous perspectives, are often treated superficially.

On the other hand, Brazilian books remain rooted in an approach centered on algorithms and rules, which restricts the development of a critical and investigative understanding. These findings pave the way for the next subsection, where Sociocultural Perspectives in Mathematics Textbooks will be explored,

[17] Marchand, P., & Bednarz N. Reference 18, p. 19.

seeking to understand how cultural and social dimensions influence educational content and practice in diverse contexts.

2.2.2 Sociocultural and political relations

The article by Garcia, Fazio, Panizzon and Bizzo (2018) seeks to understand the political implications of the global culture of testing in science teaching in Brazil, Canada and Australia by correlating the results of the large-scale assessments of the Program for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS) with the public educational policies in force in each country and their curricula.

They use references that corroborate and give continuity to studies within the areas in the three realities, with, in this case, the challenges of distance continuing education for teachers working in science, and, in this case, educational evaluation, as a reference for the training of science teachers.

The researchers collected evidence from recently published studies and research in the three countries. From Australia, they used evidence from the TIMSS 2015 report: "A first look at Australia's achievements by the Australian Council for Educational Research" (ACER). This study sought to promote research-based knowledge to improve lifelong learning. From Canada, the Ontario region, the evidence collected in the survey "Exploring the current state of science education from grades 4 to 8 in Ontario", a mixed-methodology study with more than 200 science teachers in the region, found that science takes a back seat and, in addition to interviewing teachers, also analyzed TIMSS data from 2015 to support the analysis.

From Brazil, data from the Greater ABC region was used[18] the following studies: "Educational policies and science education in Brazil: a case study", which analyzed young people's perception of the importance of the subjects Portuguese, Mathematics and Science studied daily in a context marked by public policies that emphasize Language and Mathematics exclusively. In addition, the effects of the Basic Education Development Index (IDEB) in this region on science teaching are carried out by the Observatório da Educação do Grande

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[18] The Greater ABC Paulista region consists of the municipalities of Santo André, São Bernardo do Campo, São Caetano do Sul, Diadema, Mauá, Ribeirão Pires, and Rio Grande da Serra.

ABC (Greater ABC Education Observatory)[19] and the Grande ABC Evaluation and Accountability Policies[20]. Finally, PISA data from the three countries on Science from 2006, 2009, 2012 and 2015 were used and evaluated to broaden the analysis and understanding of the phenomenon.

Garcia et al. (2018) conclude that educational policies in the three countries have similar flaws, especially in prioritizing assessed subjects to the detriment of a balanced education. They argue that the over-reliance on large-scale assessments contributes to the unequal distribution of educational resources and reinforces systemic inequalities. This scenario points to the need for more comprehensive policies that recognize the importance of all subjects in student development, with an emphasis on interdisciplinary approaches that promote the integration of science, mathematics and language.

In the Australian context, the performance of TIMSS science students in the fourth and ninth grades, even after 20 years of implementing these policies, has stagnated remarkably.

These findings reinforce the impact of curriculum guidelines on pedagogical practices and textbook design, highlighting the importance of consistent alignment between policies, curricula and educational materials.

The next subsection will look at the organization of mathematical content, exploring how these elements are presented in different educational contexts.

Brito's (2019) study investigates the relationship between the sociocultural transformations that took place in Quebec between the 1960s and 2010s and the way in which these changes are represented in elementary school textbook illustrations. The analysis focuses on how the teaching materials reflect values of inclusion and diversity, proposing a critical look at advances and limitations in visual representation.

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[19] The Observatório da Educação do Grande ABC is an initiative aimed at monitoring and critically analyzing educational indicators in the Grande ABC region, promoting debates on public policies and pedagogical practices. It acts as a forum for managers, educators and researchers to formulate evidence-based proposals for improving basic and vocational education.

[20] The Evaluation and Accountability Policies of the Greater ABC region refer to initiatives aimed at monitoring educational performance in the region, emphasizing the application of standardized evaluations and *accountability* strategies. These policies seek to align school results with established quality standards, promoting debates on the impacts of such practices on teaching and school management, as well as their influence on the equitable development of public education networks.

Between the 1960s and 1970s, Quebec underwent profound sociocultural changes, marked by a move away from traditional religious influences and the adoption of multiculturalism and gender equality as core values of society[21]. This context has been reinforced by local policies, such as those promoted by the Quebec Ministry of Immigration, Diversity and Inclusion.[22,23]which emphasize the appreciation of cultural diversity.

The main objective of the study is to analyze whether these transformations have been incorporated into the visual representations of textbooks, with a focus on gender equality and ethnic inclusion. The research compared seven textbooks published in the 1960s (4 math and 3 history) with seven books from the 2010s, looking at visual aspects related to the human figure and its relationship to the multicultural context in Quebec.

The analysis included a sample of 14 textbooks, 7 from the 1960s and 7 from the 2010s. The books were selected because they were produced and published locally in Quebec, ensuring a link with the cultural context of the region. The methodology considered categories such as the presence of human figures, gender equality and ethnic inclusion.

The books were evaluated based on the proportion of pages containing illustrations: 62% of the pages in books from the 2010s had some kind of illustration, compared to 48% in books from the 1960s. In addition, the illustrations were classified according to the ethnic diversity represented and the role of female figures in the scenarios presented[24].

The results revealed significant progress in terms of inclusion in recent decades. In books from the 2010s, 45% of human figures represented ethnic diversity, in contrast to only 18% in books from the 1960s. Female representations increased from 30% to 55%, but there is still a predominance of

[21] Després-Poirier, C. (1995). Sociocultural transformations in Québec.

[22] Immigration, Diversité et Inclusion Québec. (2018a). Immigration policy. Gouvernement du Québec.

[23] Immigration, Diversité et Inclusion Québec. (2018b). Diversity and inclusion in Québec. Gouvernement du Québec.

[24] McCaffrey, T., & McCaffrey, C. (2017). Content analysis in children's media.

male figures in leadership roles or technical activities, especially in mathematics books[25].

Despite these improvements, the study identified persistent challenges. In many cases, representations of minority groups remain superficial, with visual stereotypes that do not fully reflect Quebec's multicultural reality. This limitation is especially evident in illustrations that relegate non-white characters to secondary or decorative roles[26].

In addition, although textbooks from the 2010s include greater ethnic and gender diversity, some areas, such as technical illustrations, continue to reflect the predominance of dominant groups, indicating that multicultural values have not yet been fully integrated (2018b).

Although Quebec textbooks have made progress in terms of inclusion, the results indicate the need for a continued effort to overcome structural inequalities. The study concludes that designers, authors and publishers need to work collaboratively to create materials that not only reflect, but also promote values of diversity and social equity [27,28].

The research reinforces the importance of more engaged editorial practices, proposing that textbooks become critical tools for shaping a truly inclusive society. In addition to expanding visual representations, the study points to the need to contextualize these representations in narratives that challenge prejudices and promote inclusion.

2.3 The contributions of the literature review to this research

By bringing together the results of national and international studies on mathematics textbooks in Brazil and Canada, this literature review systematizes trends, problems and perspectives that permeate the field of studies on the

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[25] Bishop, R. S. (1990). Windows, mirrors, and sliding glass doors. *Perspectives: Choosing and Using Books for the Classroom*.
[26] Martínez-Bello, V. E., & Martínez-Bello, A. (2016). Diverse representations in children's education.
[27] McCoy, K. (2003). Design ethics and social responsibility.
[28] Berman, D. (2013). *Do Good Design: How Designers Can Change the World*.

analysis of mathematics textbooks, providing an understanding of the educational dynamics under investigation.

A he methodological diversity found in the review, ranging from document analysis and comparisons of collections from different countries to the use of qualitative approaches, shows the breadth of research paths and resources available to scholars of mathematics textbooks. In this sense, Souza (2019) carries out a documentary analysis of collections from Brazil and Canada; while Attie and Krpan (2020) and Santos Junior (2013) promote comparative studies based on different dimensions; while Garcia et al. (2018) develop qualitative investigations.

It is interesting to note that some works combine these paths simultaneously: Hernandes (2023), for example, when investigating isometries in textbooks from Brazil and Canada, adopts a qualitative and comparative approach, supported by documentary analysis, as the author explains:

> "Given that the central objective of this work is to understand the treatment of isometries in Mathematics Textbooks from Brazil and Canada in collections for the Final Years of Elementary School, the qualitative paradigm contributed to what was intended to be achieved. This is because the data produced emerged from the description of what was observed on the pages that contained the desired content and then underwent interpretation, critical reflection and theoretical analysis by the researcher." (Hernandes, 2023, page 64, our translation).

In this context, both quantitative procedures (for example, mapping and statistics on the frequency of publications) and qualitative strategies (such as content interpretation, discourse analysis and cultural representations) are identified. In the documentary studies by Souza (2019) and Hernandes (2023), the way in which mathematical content is organized in Brazilian and Canadian collections suggests that the development of chapters, tasks and examples is not limited to technical approaches, but reflects curricular and cultural choices.

Although Souza (2019) points out that Canadian curricula encourage investigative practices and autonomous learning proposals, his analysis shows that the exploration of cultural references is still not deepened in textbooks.

Similarly, Hernandes (2023) indicates restrictions in the integration of more contextualized content, even where greater alignment with investigative practices would be expected. When relating these observations to the aim of this work - to understand how Canadian values and perspectives can be reinterpreted in Brazilian multilingual schools - it becomes clear that such limitations influence the effective use of foreign material in contexts of great linguistic diversity, requiring pedagogical approaches that are more situated in the local reality.

The comparisons made by Attie and Krpan (2020) and Santos Junior (2013) show evidence of how sociocultural differences affect the teaching of mathematics, whether in the processes of argumentation or in solving sharing problems. In particular, Attie and Krpan (2020) point out that Canadian textbooks tend to propose strategies that favor student autonomy, although they don't necessarily delve into cultural references. This characteristic resonates with the guiding question of this research, as it suggests that the emphasis on investigative practices and the superficial approach to ethnic or linguistic aspects can coexist.

In the Brazilian case, Santos Junior (2013) shows that the use of Canadian theoretical references does not dispense with adjustments to the local reality, which reflects the need to investigate whether multilingual schools are able to re-signify elements of foreign books or whether they only incorporate the values of the original context in a limited way. Inspired by this reflection, this research proposes to analyze, at a documentary level, how Canadian values and perspectives present in textbooks can be reinterpreted in Brazilian multilingual schools - and whether these multilingual schools could promote adaptations more in line with their linguistic and cultural reality, or whether the adoption of these foreign textbooks would tend to reproduce values from the context of origin - an aspect that remains little explored in the literature analyzed.

The work by Garcia et al. (2018) highlights the impact of large-scale assessment policies in Brazil, Canada and Australia. According to the authors, these policies can induce the prioritization of certain contents or skills, influencing the way publishers and managers compose textbooks. This analysis is relevant to this research, which examines how the contents and values of the Canadian context reach Brazilian classrooms. If the pressure to perform well on tests has a significant impact on the development of materials, it is possible that cultural references, even if they are included in official documents, are left in the background.

Although some authors (e.g. Garcia et al., 2018; Brito, 2019) mention the influence of policies and cultural factors on the production and selection of foreign materials, none of the studies reviewed deals directly with the repercussions of transferring these books to multilingual schools, where values, languages and experiences are intertwined. Against this backdrop, this paper carries out a documentary analysis of Canadian textbooks, considering Brazilian institutions marked by diversity.

The intention is to investigate, based on the content and organization of these materials, the extent to which they can promote transpositions that are consistent with local plurality, or whether they end up reflecting a more limited transposition, without fully recognizing the cultural and linguistic particularities of the target audience. In this way, they do not empirically investigate the actual practices of use in the classroom, but examine the potential (and possible limitations) of these books in multilingual contexts.

For this research, the studies analyzed highlight factors that look beyond the mathematical content, highlighting the relevance of understanding how Canadian values and perspectives are appropriated - or left aside - although this research does not empirically monitor how such values are transported to everyday school life, it corroborates Brito (2019) and Garcia et al. (2018) that environments with linguistic and cultural plurality demand pedagogical practices that are attentive to diversity, while at the same time facing curricular and political restrictions that can influence how foreign materials are used. Therefore, instead of assuming a direct incorporation of Canadian references, we consider the possibility that, given these limitations, there may be only a partial or poorly contextualized adoption of these values.

Finally, this review not only suggests appropriate approaches to analyzing the presence of sociocultural elements in Canadian textbooks, but also provides clues as to what happens when these references cross borders and are transported to different realities. Authors such as Brito (2019) show that, even in contexts that value multiculturalism, representations of minority groups can be restricted to a superficial level. Garcia et al. (2018) warn of the role of large-scale assessment policies, which, by emphasizing quantitative results, can relegate cultural and inclusive aspects to the background.

Regarding Regarding the construction and organization of content, Souza (2019) notes that although Canadian textbooks are more mathematically rigorous and encourage student autonomy, they do not always delve into cultural aspects recommended in national guidelines, such as the Western and Northern Canadian Protocol (WNCP, 2006) or the Trillium List guidelines (Ontario Ministry of Education, 2020). Attie and Krpan (2020) corroborate this perception by pointing out that, even though there is openness to investigative practices, references to indigenous perspectives and other cultural dimensions are still limited in some teaching proposals.

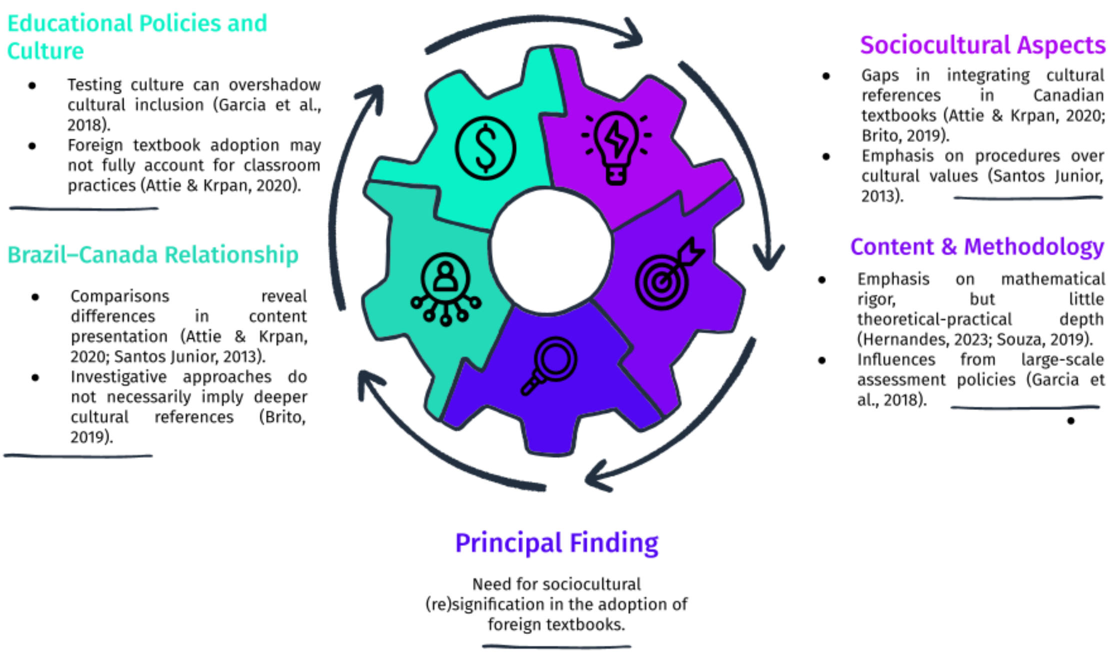
Santos Junior (2013) points out that, when using foreign theoretical bases or teaching materials in Brazilian classrooms, adjustments to the local context are necessary, which brings up the possibility of reframing these contents in line with the National Common Curricular Base (BNCC, Brazil, 2017).

Attie and Krpan (2020) reinforce this finding by observing that, although there is openness to investigative practices, indigenous references and other cultural perspectives appear in a limited way. Santos Junior (2013), on the other hand, emphasizes the need to adjust to the local context when using foreign theoretical bases or teaching materials, raising questions about a reframing of this content that actually works in Brazilian classrooms.

These observations suggest that, even if certain curricula are recognized for their solid organization, the implementation of investigative proposals and the deepening of cultural aspects may not take place uniformly in all materials and practices, varying according to the approach adopted.

The gap that runs through these studies - and which this research aims to investigate at a documentary level - refers to the possibility that sociocultural values and perspectives from the Canadian context may (or may not) be re-signified in the use of textbooks in Brazilian multilinguals, where the multiplicity of languages and cultures tends to produce challenges (such as the need for translation, pedagogical adaptation and negotiation of meanings).

Figure 1 - Thematic-conceptual synthesis of studies on mathematics textbooks (Brazil-Canada)



Source: The author (2025)

As summarized in Figure 1, the research gathered focuses on the relationship between educational policies, sociocultural aspects and methodologies for adopting foreign textbooks.

Although they discuss strategies for integrating cultural references and point to the possible impact of standardized assessments, they do not claim that there are determining gaps in these materials, nor that Canadian textbooks are destined for Brazilian schools. What stands out is the importance of investigating how such proposals are recontextualized in multilingual environments in Brazil, ensuring that cultural and linguistic dimensions are effectively considered in the use of these resources.

Even though studies have already pointed out the superficial presence of indigenous cultural elements in Canadian textbooks (Attie & Krpan, 2020; Brito, 2019) or questioned the excessive emphasis on procedures (Santos Junior, 2013), there is still no in-depth analysis of how, in practice, these references can be translated, adapted or even ignored in highly diverse educational settings.

This gap reinforces the originality of this investigation, which directly examines the structure of these materials and their Canadian multicultural perspectives, seeking to contribute to a more critical and contextualized use of foreign works in Brazil. In this sense, the song "Duas Cidades" (Passapusso & Barreto, 2016) evokes the richness and tension of overlapping cultural contexts, something that can also be observed in multilingual school environments:

> "Every day you wake up early for work
> Put on your garlic string
> And go into battle
> An eye for an eye. Tooth for tooth
> Spread it. Babylonian law is different
> Whoever watches buys scapular clover
> Put on your garlic cord
> And go into battle
> An eye for an eye. Tooth for tooth
> Spread. The law of Babylon is different
> Already on the way down and doesn't know how to go down dancing
> You know how to go up in life but you don't know how to go up sambaing
> Already on the way down and don't know how to go down dancing
> You know how to go up in life and you don't know how to go up sambaing
> Saudade arrives, saudade leaves messing up
> And when I leave town I'll tell you about it
> Dividing Salvador
> Dividir salvador says in
> Which city you fall into
> Uptown downtown
> Tell me which city you're in
> Oh oh oh bagda..."
> (Passapusso & Barreto, 2016, our translation)

The poetic force of these verses metaphorically highlights the multiple cultural layers in confrontation and coexistence, elements that can be reflected in the adoption (or recontextualization) of textbooks from other social realities.

3 THE THEORETICAL FRAMEWORK

Research in the area of mathematics teaching highlights that this subject carries cultural values and social implications, giving rise to proposals that range from traditional approaches to those that prioritize contextualization and critical education (Amorim, 2012; Cavalcante, 2021). In this scenario, the textbook is not just a receptacle for content, but a cultural and political artifact (Apple, 2013; Munakata, 2016).

In stating that the textbook is a cultural artifact, it is recognized that, by selecting, organizing and presenting certain themes and activities, it reproduces worldviews and interpretations that reflect the values of a historical and social context. With regard to the political character, it is understood that the choices made in the production of the book - such as which contents to emphasize, which voices to represent or silence, what type of language to use - are not neutral. They express interests, ideologies and power relations that can reinforce or question certain positions within the educational field (Apple, 2018; M. A. da Silva, 2019).

Thus, when textbooks are used that are produced in another country, such as Canadian textbooks in Brazil, they become even more complex: they bring with them cultural, linguistic and political references that are alien to Brazilian realities, requiring the teacher to take a critical look so that they are suitable for the different school communities. (Andrade, 2024; Perovano, 2022). It is in this mediation that the active role of educators is revealed in the interpretation, adaptation and eventual re-signification of these materials, with the aim of not only transmitting mathematical content, but also promoting dialog between cultures and multiple perspectives.

Studies by Banks (2014) and Candau (2012) show, however, that this distance can become a source of cultural enrichment if the teacher promotes mediation work that values similarities, problematizes differences and encourages comparison between realities. However, merely changing the language is not enough. The transposition of examples, terminologies and cultural references requires careful analysis of their validity in the new school environment (Moschkovich, 2010; Planas & Setati, 2009).

In addition to linguistic issues, there are political dimensions at play. Large publishing houses, often transnational, tend to monopolize the distribution of collections, defining what is considered legitimate knowledge (Apple, 2018; Ball, 2012). The internationalization of textbooks can reinforce teaching models that are more aligned with large-scale assessments or neoliberal agendas, to the detriment of critical and contextualized proposals. (Valero, 2012, 2018, 2023).

On the other hand, in schools that adopt multilingual practices, the use of foreign books can expand the students' repertoire, as long as there is planning that includes, for example, the legitimacy of the simultaneous use of different languages. (Planas & Setati, 2009) and the emphasis on semiotic mediation, which seeks to construct meanings from symbols and various social practices (Radford, 2010).

It is in this context that this research is developed, with the aim of analyzing the sociocultural differences present in textbooks from the Canadian context and understanding how these elements dialogue with Brazilian plurality. It also seeks to investigate linguistic and socio-political aspects and discuss the role of the teacher in integrating foreign content. The emphasis is on the possible connections, but also on the gaps that can emerge when using a textbook designed in another reality (Valero & Knijnik, 2016).

In this sense, it is assumed that the discipline of mathematics, far from being neutral, carries cultural meanings and is a locus of ideological disputes (Apple, 2018; Skovsmose, 1994, 2023). Thus, the textbook is not just an exercise guide, but a mediator of knowledge and culture, whose uses and appropriations depend on teacher mediations and the positioning of teaching policies.

By exploring the relationship between the centrality of the textbook, teacher training and the presence of foreign materials, this theoretical framework is situated at the intersection of the global and the local.

The bet is that the mathematics taught can accommodate a varied repertoire of cultural references, strengthening the student's critical sense and their ability to move between different codes and worldviews ((D'Ambrósio, 2005; Ladson-Billings, 2014). Ultimately, it is a question of problematizing the curricular and methodological choices that underpin the adoption of international textbooks, so that these resources can be examined from a perspective that privileges diversity, contextualization and social transformation - essential elements for a democratic and inclusive mathematics education.

3.1 Mathematics Education, Contextualization and the Role of the Textbook as a Cultural and Political Artifact

Mathematics teaching, due to its cultural values and social implications, goes through different teaching proposals ranging from more traditional approaches to perspectives that emphasize contextualization and the critical development of students (Amorim, 2012; Cavalcante, 2021).

In this scenario, the textbook is not limited to functioning as a simple repository of content: it is also a cultural and political artifact (Apple, 2013; Munakata, 2016). Understanding the textbook in this way means recognizing that there are values, ideologies and worldviews underlying the selection and organization of content, as well as the way in which this content is presented. In other words, the textbook actively participates in the construction of meanings and the legitimization of certain visions of reality, influencing the way students interpret school knowledge and their relationship with the world.

This political-cultural dimension becomes particularly evident when we look at the role of the PNLD in Brazil. While this program reinforces the centrality of these materials in the teaching process, its guidelines and evaluation criteria also reveal tensions related to the integration of collections that take into account different sociocultural contexts and realities. (Melo, 2016). Thus, the choice and use of textbooks are not neutral processes: they involve disputes over which contents, approaches and representations of the world will be prioritized in the classroom.

3.1.1 Reflections and perspectives on textbooks in the Brazilian context

The selection and use of textbooks are important elements in planning and conducting math classes. However, its daily influence depends, in essence, on how the teacher interprets and transforms the material to the specificities of the school context.

When investigating the link between teacher training and the choice of mathematics textbook in the context of Brazilian public schools, Andrade (2024) points out that this process involves different types of knowledge, including those related to mathematical mastery, pedagogical and didactic knowledge, an understanding of the official curriculum, the teacher's experience and an attentive eye to the characteristics of the students and their sociocultural environment.

Although this movement occurs in any textbook integration, it takes on a different shape when the material is foreign, as it introduces sociocultural and linguistic references from different realities.

Andrade (2024) e Perovano (2022) when investigating the dynamics between teachers and textbooks in the Brazilian context, highlight how these materials influence teaching actions and pedagogical conceptions in the classroom. Although their analyses focus on the national scenario, they offer important avenues for broader reflections, including international contexts, by highlighting the multiple factors - cultural, linguistic and political - that permeate the integration of textbooks.

Along these lines, Perovano (2022) points out that the textbook, in addition to compiling content, can serve as a source of learning for the teacher, by presenting reflections that lead to a review or expansion of their actions. This potential, however, does not arise automatically: it depends on a reflective attitude and a careful reading of the limitations and possibilities of the material (Valero & Knijnik, 2016). When the textbook is produced in another country, such an analysis requires extra attention, since it is necessary to consider cultural values, habits and expressions that may not be in immediate dialog with the experiences of Brazilian students.

For decades, the textbook has acted as a reference resource in many math classrooms, both in Brazil and in other countries (Apple, 2013, 2018; Munakata, 2016). In many cases, it becomes the real curriculum, even though there are official documents and curriculum guidelines, the rhythm and content of classes end up following what is structured in its chapters (Ball, 2012). This centrality is not limited to the logistical aspect; it involves pedagogical, sociocultural and political conceptions that shape both the teaching routine and the way students learn (Apple, 2018; Munakata, 2016).

One of the reasons for this is the need to organize and systematize content in an order that seems progressive (Pinheiro et al., 2021). Often, teachers have only a few hours to plan and turn to the book for guidance on what to teach each semester. This master line of chapters, exercises and examples can help beginners or teachers with many classes, helping to coordinate the pedagogical work.

At the same time, the book acts as a filter between mathematical knowledge and the classroom, selecting what should be covered, the examples to be discussed and the languages adopted (Apple, 2018). This selection is not neutral, as it involves decisions aligned with visions of teaching, learning and society (Cassiano, 2007). For example, a book that favors repetitive exercises can reinforce the image of mathematics as mere memorization of algorithms, while another that proposes open-ended problems can encourage a more reflective and investigative attitude (Munakata, 2016).

In general, in math textbooks aimed at various contexts, there is a strong presence of exercises aimed at practicing algorithms and procedures. Although this approach is necessary, Reeder and Bateiha (2016) and Rubin et al. (2014) warn of the risk of mechanical repetition taking precedence over reflection and problem-solving. In this scenario, students can lose the meaning of the content, as they often don't understand the reasons for the operations carried out or how to relate them to other areas of knowledge.

In addition, the way the book organizes chapters and sequences points to a conception of a hierarchy of mathematical knowledge (Munakata, 2016). It usually follows a linear logic: fundamental operations, then fractions, percentages, equations, functions, etc. This progression may not meet the needs and interests of a particular group of students, generating tensions between what the textbook proposes and what the teaching team considers to be a priority (Munakata, 2016).

Another factor that adds to the handling of the textbook is the teacher's training (Perovano, 2022). If, on the one hand, they may perceive the book as an ally, on the other, there is a lack of training to help them critically analyze the proposed activities and integrate the material into the reality of their classroom

(Andrade, 2024). Teachers may hesitate to propose debates or investigations that differ from what is in the textbook, due to insecurity or lack of time. (Cavalcante, 2021).

The textbook is not merely a collection of exercises or a compilation of content, but a cultural, political and pedagogical artifact (Apple, 2018; Munakata, 2016). Their ability to mediate learning depends on the teacher's attitude and the institutional context (Andrade, 2024). It can serve as a guide for organizing and accessing knowledge, or it can limit possibilities when handled uncritically. In this sense, the centrality of the textbook and its cultural and political dimensions are issues that cut across the proposed framework.

This global influence is particularly marked in subjects such as mathematics, which has been considered by some to be universal, as pointed out by (Bishop, 1988). However, contemporary studies such as Skovsmose (2023) and Valero (2023) emphasize that the subject of mathematics at school is impregnated with cultural values and practices, and is not reduced to a pure and neutral body of knowledge.

(Fan et al., 2018) discuss the role of cultural issues in the formation and integration of textbooks, investigating cases in which the transposition of a Chinese material into the English context required more than mere language changes; it required profound epistemological and methodological adaptations. Similarly, when foreign materials are introduced into Brazil without proper contextualization, there is a risk of presenting examples and problems that, although coherent in their country of origin, have little to do with the local reality and imaginary.

In addition to the cultural issue, there is also the economic dimension. Ball (2012) argues that, as part of a neoliberal logic, high value-added teaching materials are sold to public and private education networks as ready-made solutions, packaged in discourses of efficiency and modernity.

Under a neoliberal and standardized logic, teaching materials of foreign origin are often associated with supposed excellence or methodological neutrality, attributions which, in general, lack a more robust foundation (M. A. da Silva, 2019). This perception can deepen existing hierarchies, as the adoption of imported books often becomes a symbol of distinction or status (Klees, 2008).

On the other hand, some argue that, in multilingual contexts, such materials can strengthen intercultural exchange and the appreciation of diversity. Akkari & Radhouane (2022)for example, argue that the presence of books in different languages favors the building of linguistic bridges, while educators from schools with an international profile point out that interaction with plural teaching resources broadens students' cultural background (Kirsch & Duarte, 2020). However, this potential will only be consolidated if there is careful pedagogical mediation, capable of articulating the global and the local, ensuring that references from other countries establish meaningful dialogues with the realities in the classroom. (Andrade, 2024; Perovano, 2022).

From this perspective, it is important to consider the role of the teacher as a cultural agent, who not only applies the book, but adapts it, reinvents it and, when necessary, challenges it (Andrade, 2024; Valero, 2004). The critical appropriation of books of foreign origin can become an opportunity for intercultural comparison and reflection on different mathematical and educational actions. However, if adopted uncritically, these books can reinforce the metaphor of the upper city and the lower city of Baiana System (Passapusso & Barreto, 2016)and crystallizing a hegemonic vision that privileges groups closest to the dominant cultural references (Giroux, 2011).

3.2 The three dimensions: sociocultural, sociopolitical and linguistic

This subsection discusses three fundamental dimensions for this research into mathematics textbooks - the sociocultural, sociopolitical and linguistic - considering that this resource goes beyond the mere transmission of content. Globalization and the growing exchange of curricula make it urgent to reflect on the cultural translations that take place in the school environment, especially in multilingual institutions, either because they welcome students from different linguistic backgrounds or because they adopt bilingual or international pedagogical proposals.

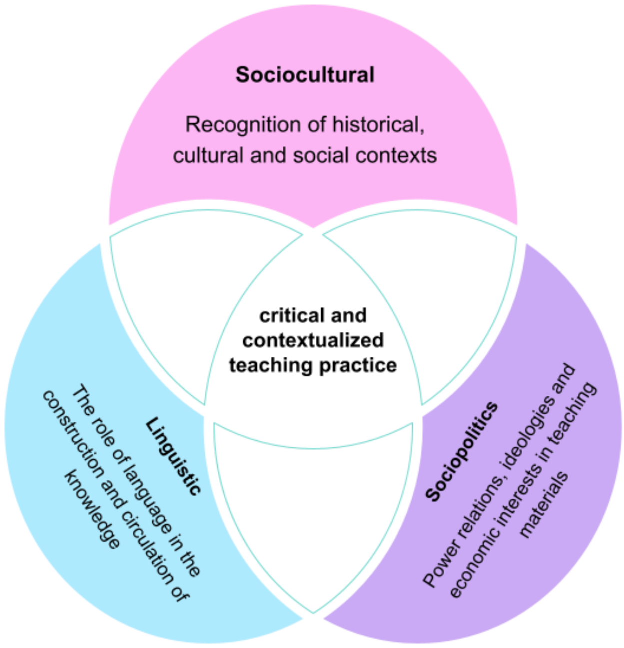
In this scenario, the textbook acts as a key element: it carries representations that go beyond simple equations or mathematical exercises, projecting worldviews, values and identities.

Thus, throughout the three blocks of this section, each of which dialogues more directly with a specific objective (sociocultural, linguistic and sociopolitical), we seek to demonstrate how the textbook reveals itself as an artifact that not only reflects, but also shapes school customs.

From an integrative perspective, the sociocultural, linguistic and sociopolitical dimensions are interwoven throughout the theoretical reflections that underpin this study, as indicated by the contributions of Ladson-Billings (1995, 2014); Moschkovich (2010, 2015); Planas & Setati (2009); Radford (2000, 2009, 2010, 2012); Tan and Xun (2023); Valero (2004, 2012, 2018, 2023); Vygotsky (1978, 1998).

While Vygotsky (1978, 1998) emphasizes the development of the subject in interaction with the historical and social context, Ladson-Billings (1995, 2014) advocates for a culturally relevant pedagogy that dialogues with student identities. Research on bilingualism and multilingualism in mathematics Moschkovich (2010, 2015); Tan and Xun (2023); Planas & Setati (2009) highlight the importance of strategies that transcend literal translation to promote meaningful understanding, consistent with the semiotics of learning proposed by Radford (2000, 2010, 2009, 2011, 2012) with Valero's (2004, 2012, 2018, 2023) socio-political reflections on power relations in the construction and circulation of mathematical knowledge.

Figure 2 - Diagram illustrating the integration of international textbooks in multilingual schools



Source: The author (2025)

In this context, Figure 2 proposes a synthesis of the interconnections between the sociocultural, linguistic and sociopolitical in the integration of international textbooks in multilingual schools. It includes authors such as D'Ambrósio (2005) and Skovsmose (1994, 2023), who emphasize the relevance of context, teaching knowledge and the critical role of language in the construction of knowledge, along with (Apple, 2013)which exposes the issues of power and ideology present in the production and distribution of teaching materials.

3.2.1 Sociocultural dimension: Reflections on cultural influences and the internationalization of textbooks

The sociocultural perspective of learning, proposed by Vygotsky (1978, 1998), recognizes that the formation of concepts results from human interactions, mediated by symbolic tools such as language. According to Vygotsky (1978, p. 86), “learning awakens internal developmental processes that are able to operate only when the child interacts with people in his environment.” Applied to math teaching, this understanding suggests that algorithms, procedures and representations are not assimilated in isolation, but emerge from negotiations of meaning that take place in the classroom.

This understanding, applied to math teaching, suggests that algorithms, procedures and representations are not assimilated in isolation, but emerge from negotiations of meaning that take place in the classroom.

From this perspective, the textbook is a cultural artifact whose interpretation depends on teacher mediation and dialog with students' experiences. Rather than transmitting absolute truths, mathematics teaching involves social practices that recognize the influence of the historical, linguistic and cultural context - which becomes even more evident when foreign textbooks are integrated.

Criticism of a linear conception of mathematics teaching coincides with the reflection of D'Ambrósio (2005, p. 100), for whom “mathematics is not a particular cultural value of the West [...], but is expressed in the way different peoples count, measure and explain phenomena.” When the teaching material comes from another country, these cultural differences can be accentuated, as “there is always the risk of imposing a mathematical ethnocentrism” (D'Ambrósio, 2005, p. 101).

At the same time, these discrepancies can be transformed into opportunities for enrichment, if the teacher builds bridges with the students' daily lives, in a dynamic that Ladson-Billings (1995, p. 477) calls “culturally relevant pedagogy”, that is, “a set of practices that recognize, validate and use students' cultural references to develop academic effectiveness.”

Vygotsky's (1978, p. 86) concept of the Zone of Proximal Development (ZDP) reinforces the importance of mediation in this process, showing that reaching new levels of knowledge depends on the interaction between teacher and student. The foreign textbook offers a set of possibilities that need to be (re)interpreted by the teacher and the students themselves, in an exercise of negotiating meanings.

The ZPD reinforces the importance of mediation by showing that reaching new levels of knowledge depends on the interaction between teacher and student. For Freire (2000, p. 34)“Reading the world always precedes reading the word”, highlighting the social and contextual nature of learning. For this reason, the textbook offers a range of possibilities that require (re)interpretation and discussion.

Radford (2010, p. 2) points out that mathematical understanding is “a semiotic process that involves the interpretation of historical and cultural signs”. It is therefore up to the teacher, through analogies, debates and adaptations, to facilitate the translation of these signs so that they make sense in the local reality.

The impact of this linguistic dimension is explored in more detail below, considering the multiple variables involved in the use of additional languages in the teaching and learning of mathematics. The teacher's mediation in this scenario is not limited to occasional substitutions of terms, but includes creating analogies, problematizing contexts and opening up space for students to recognize their own cultures as a legitimate part of the training process. These strategies, although challenging, provide opportunities to value the plurality of perspectives, reinforcing the importance of a critical and dialogical pedagogical practice.

In summary, the sociocultural discussion about foreign textbooks in multilingual schools shows that mathematics, far from being neutral or universal in itself, is inevitably embedded in historical and cultural relations. As indicated by Vygotsky (1978, 1998), knowledge emerges from social interaction; and, according to D'Ambrósio (2005) and Ladson-Billings (1995, 2014), this construction process must consider local identities and cultures in order to be effectively meaningful.

From Radford's (2010) semiotic-cultural perspective, learning mathematics is conceived as a continuous process of producing meanings, in which each classroom interaction is anchored in historically and culturally constructed sign systems. From this point of view, it is not enough just to translate symbols from one language to another; it is essential to recognize that mathematical meanings are imbricated in social practices, values and conceptions of the world. In multilingual environments, for example, teacher mediation requires attention to the different strategies for interpreting and recreating signs, ensuring that linguistic and cultural plurality is understood as an integral part of the training process.

In the next subsection, when dealing with multiculturalism and cultural influences in international textbooks, we return to this semiotic axis, highlighting how pedagogical materials, when circulating in different contexts, require the negotiation of meanings and the contextualization of content according to the history, languages involved and cultural practices of each school community.

3.2.1.1 Reflections on cultural influences

Studies such as those by Bishop (1988) and D'Ambrósio (2005) draw attention to the cultural dimension of mathematical knowledge, opposing the idea that this science is universal in the sense of being free of values and contexts. In this sense, the textbook is a cultural artifact loaded with meanings, whose production and circulation cannot be dissociated from politics, editorial interests and worldviews (Cassiano, 2004; Munakata, 2016; M. A. da Silva, 2019).

According to Munakata (2016, p. 28), “the curricular selections present in the textbook carry worldviews, values and often silence voices that do not fit the dominant model”.

From a theoretical perspective, the transnational circulation of textbooks transcends the mere translation of words, as it involves dealing with cultural, curricular and even contextual references (Ball, 2012). Critically, this internationalization not only requires linguistic adaptations, but also the consideration of the historical, social and political structures of each location (Valero, 2004). When these adaptations are limited to literal translations, without proper examination of cultural differences or regional educational objectives, there is a risk of intensifying the distance between the proposed content and the students' school reality, undermining the relevance of the teaching.

To deepen the discussion, concepts such as multiculturalism, interculturalism and ethnomathematics become central. According to Banks (2014, p. 9), multiculturalism seeks to “recognize and value the multiple cultural identities present in the classroom, avoiding the imposition of a single perspective”. Candau (2012, p. 75) suggests that interculturality consists of “a pedagogical practice that promotes dialog between cultures, creating new references from the encounter”.

When we take this debate to the teaching of mathematics, D'Ambrósio's (2005) ethnomathematical approach draws attention to the different ways in which social groups deal with notions of quantity, space and shape. In line with this, Bishop (1988) considers that “to speak of a universal mathematics, without considering the experience of each community, can obliterate the richness of different rationalities”.

Addressing cultural influences and the internationalization of textbooks through the lenses of multiculturalism, interculturalism and ethnomathematics shows that school mathematics is not dissociated from social values and practices.

As D'Ambrósio (2005, p. 102) emphasizes, “the recognition of cultural diversity in mathematics teaching allows not only the inclusion of different groups, but also the mutual enrichment of knowledge”. In this sense, understanding the complexity involved in the production, selection and integration of foreign textbooks becomes essential in order to promote teaching that is not limited to the transmission of content, but which makes room for the formation of identities and the appreciation of cultural plurality.

3.2.2 The Linguistic Dimension and Multilingual Education

The understanding that language not only expresses but also structures mathematical thinking is based on Vygotsky (1978). For this author, linguistic signs are instruments that enable abstract reasoning and the formation of scientific concepts (Vygotsky, 1978, 1998). In mathematics, technical terminology (for example, “factoring”, “irreversible fraction”, “combinatorial analysis”) and symbolic representations (such as x, y, f(x) etc.) constitute a semiotic system that students need to internalize (Moschkovich, 2010). However, how this happens depends on social interaction and the cultural context.

When the textbook is in a foreign language, the complexity increases (Planas & Setati, 2009). Students who do not fully master the language may face additional difficulties in understanding specific phrases and vocabulary (Tan & Xun, 2023). When the textbook is in a foreign language, the complexity increases

(Planas & Setati, 2009). Students who do not fully master the language may face additional difficulties in understanding specific phrases and vocabulary (Tan & Xun, 2023). In this scenario, the linguistic transposition of the material requires attention to teaching and learning strategies to ensure that students are not overwhelmed by the language barrier.

In many countries, multilingualism is not optional, but an established fact, either for historical reasons (territories that are home to multiple languages) or due to migratory flows (Planas & Setati, 2009). In Brazil, although Portuguese is the official language, there are private schools that adopt multilingual curricula (often Portuguese-English, Portuguese-French or other combinations) to offer an international education.

In this model, mathematics can be taught wholly or partly in the second language, justifying the integration of foreign textbooks (Erath et al., 2021). Studies by Tan and Xun (2023) on the impact of multilingualism on cognitive development indicate that students exposed to different languages tend to have a greater capacity for abstraction and problem solving. However, this effect depends on instructional practices that integrate the linguistic codes present in the classroom (Planas & Setati, 2009).

If the school manages the foreign language exclusively, failing to recognize the role of the mother tongue in semiotic mediation, there is a risk of creating distancing and reinforcing inequalities (Moschkovich, 2010). Planas and Setati (2009) analyze the practice of code-switching - alternating languages during mathematical activities - as a legitimate pedagogical resource.

The authors show that, when dealing with complex concepts, bilingual students initially resort to the language in which they are most fluent, gradually incorporating the second language (Moschkovich, 2015). Many schools, however, forbid this practice, opting for “total immersion” in the additional language. On the contrary, recognizing and legitimizing code-switching can facilitate conceptual appropriation and the participation of all students (Caligari et al., 2021).

Translation can ease language barriers, but raises questions about quality and fidelity to the original (Erath et al., 2021). Integrating the work without translation, on the other hand, requires greater proficiency on the part of teachers and students, as well as language mediation methods (Planas & Setati, 2009).

Even with translation, certain technical terms generate ambiguity, as some common expressions in mathematical English do not have an exact equivalent in Portuguese or are used differently (Moschkovich, 2010). Mere literal translation can omit nuances and cultural references, resulting in gaps in understanding (Barwell et al., 2008)

D'Amore et al. (2006) highlight the epistemological obstacles in didactic transposition, pointing out that it's not enough just to translate words from one language into another; examples and situations need to be redesigned in the light of the target cultural context. On the other hand, the integration of original books (such as English or French) can stimulate the learning of a second language (Tan & Xun, 2023), provided that the teacher has sufficient fluency and is open to resorting to the mother tongue in the initial phase of comprehension (Moschkovich, 2015).

The adoption of these practices is directly linked to the educational policies that regulate schools or education networks, revealing socio-political dimensions about who defines what should be taught and in what language (Ball, 2012; Valero, 2004). In institutions that are oriented towards offering international education, for example, the belief is often established that the foreign textbook should be used entirely in the target language, prioritizing immersion and, at the same time, silencing the real linguistic diversity of the students (Planas & Setati, 2009). In this way, the policy of using imported materials ends up reflecting ideological choices, which can either enhance or restrict cultural and linguistic plurality in the school environment.

However, when students' different proficiencies are not taken into account, there is a risk of exclusions or inequalities (Tan & Xun, 2023). It is also important to intervene in teacher training: teachers who are not proficient in the second language or are not prepared to deal with linguistic diversity may find it difficult to guide activities based on foreign books (Moschkovich, 2015).

In line with the analysis by Caligari et al. (2021), some schools develop complementary materials to align imported books with local contexts, organizing meetings for teachers to review each chapter, discuss cultural adaptations and create bilingual guides (Ball, 2012).

In certain case studies, interdisciplinary projects are observed which, by comparing agricultural data in different languages, use statistical tools to create connections between geographically distant universes (Moschkovich, 2010). Although digital platforms can offer subtitled videos and activities in multiple languages, infrastructure demands and the need for pedagogical support must be considered, an aspect highlighted by Erath et al. (2021), when addressing the practical implications of such resources in diverse classrooms.

These efforts at adaptation, although they enhance the connection between foreign materials and local realities, do not exhaust the complexities involved in constructing meanings in a different linguistic and cultural context. It is at this point that the semiotic perspective of authors such as Radford (2009, 2012) emerges, who understand mathematical learning as the result of social practices, signs and historically situated interactions, bringing new layers of analysis to the circulation of textbooks in different cultures.

Radford (2009, 2012) presents a semiotic perspective of mathematical knowledge, in which the meanings of a concept emerge from social practices involving signs, gestures and historical references. Teaching materials produced in another language or cultural context mobilize networks of signs that may not correspond directly to different school realities (Presmeg & Radford, 2008). According to Radford (2011), it is in classroom interaction that these signs are revisited and re-signified, enabling the shared construction of mathematical meanings.

Radford's (2009, 2011, 2012) semiotic-cultural perspective highlights that mathematical knowledge is not an autonomous entity to be merely transmitted, but something that is constituted through interactive practices, in which signs, gestures and historical artifacts take on different meanings as people use them in specific situations.

From this perspective, mathematical symbols and concepts have great plasticity, as they are constantly re-signified as teachers and students negotiate interpretations in the classroom. This means that the value or meaning of the same object (for example, an equation or a theorem) is not fixed, but depends on the discursive exchanges and cultural references present in the school environment.

In cases where foreign textbooks are adopted, this process of re-signification is more complex. As the materials are produced in another language or cultural context, they bring with them a set of signs and representations that may not immediately dialogue with the students' linguistic or historical reality (Presmeg & Radford, 2008).

If, on the one hand, the presence of terminology and examples from abroad can broaden the students' repertoire, on the other, there is a risk of generating semiotic mismatches - situations in which the meanings intended by the material diverge from the meanings constructed locally by the students (Planas & Setati, 2009; Moschkovich, 2015). In these multilingual scenarios, the teacher acts as a mediator, as it is up to them to promote cultural interpretation and translation strategies that bring the foreign content closer to school experiences.

The importance of the language of instruction, in this sense, goes beyond learning specialized terms in another language: it involves understanding and assigning meaning to each expression, diagram, illustration or numerical example (Radford, 2011). With each activity, mathematical signs are inserted into existing networks of meanings, relating to both the local culture and the historical dynamics of the foreign material. This clash between different contexts implies reflections on which values and worldviews become dominant in the classroom, and how each discursive practice can reinforce or question beliefs about what quality math teaching is.

However, the transposition of foreign textbooks does not happen in a vacuum. The options for adopting, buying or discarding these materials are crossed by power disputes, market negotiations and public policies aimed at selecting content (Ball, 2012; Valero, 2004, 2023).

Educational institutions, publishing networks, international partnerships and government agreements make up a scenario in which decisions about which textbook to use are related to economic interests, curricular conceptions and school identity formation agendas. Therefore, in addition to being semiotic and cultural, teacher mediation is also political: organizing debates around a foreign textbook, reinterpreting its proposal or adapting it to local realities involves questioning norms, quality standards and educational models that do not always reflect the priorities of the school community (Valero & Knijnik, 2016).

In short, the cultural semiotic approach helps to highlight how the adoption of foreign textbooks requires linguistic and symbolic translation efforts. Each element of the material - be it a problem statement or the way the content is presented - requires negotiation of meanings in the classroom, which is even more intense in multilingual environments. However, the choices surrounding these resources are not just questions of language or method; they are related to a socio-political dynamic in which publishing markets, large-scale assessments and curriculum policies shape what is considered legitimate to teach and learn.

In the next section, which focuses on the Sociopolitical Dimension, we delve into the implications of this negotiation on a political and economic level, explaining the ways in which market disputes, international agreements and public policies intervene in the distribution and legitimacy of math teaching materials, determining which voices and values acquire greater visibility or become hegemonic in the school context.

3.2.3 Sociopolitical dimension

The use of foreign textbooks in multilingual schools is not dissociated from the power relations that cross the educational field, involving public policies for the acquisition of materials, the interests of major publishing houses, international agreements and the ideological values of each school community.

According to Ball (2012, p. 3), “the growing intertwining of educational policies and market networks on a global scale ushers in new formats for the production, circulation and legitimization of pedagogical knowledge”, favoring the import of resources considered advanced or excellent without observing the cultural and linguistic particularities of the destination country. In this way, certain materials acquire worldwide prestige and begin to circulate widely, even though they may generate discrepancies when inserted into other contexts.

From a socio-political perspective, the process of choosing and adopting textbooks is a territory marked by market disputes, institutional interests and ideological values. Although the PNLD establishes parameters for evaluating materials intended for the public school system, private schools often resort to handouts or foreign publications that don't pass this official test (Höfling, 2000). This situation creates a “normative vacuum” (Ball, 2012, p.13), in which the direct acquisition of teaching resources, including those produced in other countries, can be driven by marketing strategies or international prestige, without taking into account the specific conditions of each school context.

The push to adopt these foreign materials is intensifying against a backdrop of large-scale assessments such as PISA, which influences curricular reforms aimed at global competitiveness (Valero & Knijnik, 2016). Textbooks from high-performing regions come to be considered exemplary, under the assumption that they reproduce “successful” methods (Valero, 2018). However, as Valero (2023) warns, the mere transposition of content and approaches from one cultural or linguistic context to another does not guarantee similar results, especially in multilingual schools, where classroom interactions require adaptations that respect local teaching and learning practices.

Along these lines, mathematics teaching is a political arena, as it involves clashes over what is meant by relevant knowledge and which curricular proposals are legitimized (Valero, 2004, 2012, 2023). Foreign materials not only transfer technical content, but also convey models of organization and values that can run counter to established school cultures (Valero, 2018). In many cases, they reinforce neoliberal perspectives that emphasize quantitative results and preparation for performance exams, leaving aside the cultural and linguistic dimensions that are essential for students' comprehensive education. Thus, as Valero (2023, p. 57) points out, by adopting these books without the necessary adaptations, there is a risk of promoting a single vision of math teaching and silencing the plurality of voices and practices that already exist.

The worldwide circulation of textbooks and the development of complete teaching systems, with digital platforms, training and internal assessments, illustrate what Ball (2012) calls the commodification of knowledge. In this context, education takes on the character of a standardized product. Publishers strive to create materials seen as “universal”, reputed for their immediate inclusion in different places, without an effective commitment to sociolinguistic diversity.

Valero & Knijnik (2016) point out that large companies end up imposing standards and influencing judgments of excellence, while Silva (2019) argues that the dissemination of international books tends to consolidate subjectivities based on neoliberal principles, transforming mathematics into a set of market-oriented skills, distancing it from a contextualized education.

In private schools with greater financial resources, the acquisition of international materials is usually seen as a differential, often giving them the label of advanced or superior (Melo, 2016). In this movement, these institutions distance themselves from others that remain linked to the PNLD or restricted to national collections (Ball, 2012). This contrast can widen disparities in educational provision, as certain groups have access to books considered to be more prestigious, while others are limited to domestic options, generating impacts on the perception of quality and deepening pre-existing inequalities.

Pinheiro et al. (2021) and Hodson (2020)highlight the lack of specific rules regarding the acquisition of foreign items, especially in the private sphere. While the PNLD guides the selection of books in the public network, requesting technical and pedagogical evaluation, private schools can import international collections without restrictions. Private schools adopt foreign publications, "without examining the cultural or linguistic suitability of the material" (Pinheiro et al., 2021, p. 2). In this context, financial capacity and editorial marketing become differentials, widening the gap between institutions that have resources and others that depend on official distribution policies.

The socio-political perspective shows that the selection of foreign resources involves commercial agreements and ideological confrontations. "The dissemination of imported works largely reflects the interests of global publishing markets, which are not always attuned to the needs or cultures of the recipient countries" (Ball, 2012, p. 5).

Valero (2023) points out that resorting to such publications does not, in itself, lead to advances in mathematics teaching, as many of these projects convey hegemonic discourses that can silence local characteristics or marginalized groups. (Valero & Zevenbergen, 2004). In contrast, Valero (2012) proposes processes of collective re-signification, inviting teachers, students and administrators to critically examine each piece of content. In this way, each topic in the book would be confronted with the realities and demands of the territory, promoting a more genuine experience.

Teacher mediation from a socio-political perspective plays an important role in reformulating and contextualizing materials (Valero, 2004; Radford, 2009). Without adequate training or freedom of management, teachers tend to apply the textbook in a mechanical way. In this way, the teacher doesn't just translate words from one language into another, but intervenes in the collective construction of meanings, ensuring that external references are reinterpreted in the light of the practices and languages of the school community. Teacher training, then, includes discussions on Interculturality, as well as reflections on the political and economic dimensions of using certain publications.

In this regard, Valero (2023, p. 74) points out: “When the teacher is encouraged to reformulate and contextualize examples, inviting students to relate mathematical concepts to problems in their environment, there is an effective re-signification of books, even if they are foreign.”

There can be benefits in some circumstances. In multilingual schools, the book in another language can stimulate students' linguistic experience, encouraging them to master math terminology in different languages, which favors cognitive flexibility (Valero, 2018). However, this advantage can only be realized if there is support and planning that sees cultural and linguistic diversity as an asset, not an obstacle. When there is no such support, the disparity between what comes from outside and the local reality tends to grow, fueling distancing.

Silva (2019) points out that these books not only convey content, but also act in the formation of identities and behaviors. “The arrival of foreign books in a country like Brazil, which is historically diverse and unequal, calls for reflection on who formulates knowledge and whose interests itresponds to “ (Silva, 2019, p. 6). This argument is echoed by Valero (2018), according to whom the process of signification in mathematics is irremediably political, since the decision to include or omit certain examples, terminologies and methods reveals a way of conceiving society.

Melo (2016) observes that the desire for a so-called “education of excellence”, associated with international textbooks, can mask privatization and centralized management initiatives. In these circumstances, local practices and students' voices end up being sidelined. “It is not uncommon for books from English-speaking contexts to present examples that have little or no connection with Brazilian daily life, such as references to winter sports or currencies and units of measurement that do not correspond to the students' experience” (Melo, 2016, p. 558).

Curricular flexibility has emerged as a way of reducing these dilemmas. When teaching systems allow for adaptations, interdisciplinary activities and the active participation of students, a foreign book can foster cultural exchanges. In this case, teachers can propose analyses comparing Brazilian climatic or socio-economic conditions with those of the country where the book was produced, enriching the students' view of the countless situations in which mathematics is present. “The book itself is not decisive. What counts is how the school uses it to connect local and global knowledge, promoting mathematics that dialogues with the students' experience” (Valero, 2023, p. 66).

On the other hand, rigid curricula, guided by standardized tests, offer little room for adaptation. In this case, the use of foreign textbooks tends to reinforce a technical approach, focused on procedures and ignoring cultural or linguistic dimensions. Ball (2012, p. 9) emphasizes: “When educational success is defined solely by quantitative indicators, there is a risk of importing materials that meet these tests, but not necessarily the cultural and social needs of the local environment.”

These resources can come from commercial agreements or international cooperation commitments in which public administrations contract foreign publishers (Hodson, 2020). If there is no careful analysis, such agreements can introduce practices that are alien to local history. Pinheiro et al. (2021) note that not all managers have the tools to critically evaluate each collection, often relying on the notoriety of the material.

Teacher training appears to be a decisive factor. Even if they receive a foreign textbook, teachers can restructure it according to the students' context, if they are able to act reflexively. Valero & Zevenbergen (2004) argue that the autonomy to change examples or modify exercises prevents mathematics from becoming a tool for cultural alienation. Although the selection of the textbook is the result of different interests, the result in the classroom depends on how it dialogues with the students' languages and cultures.

Thus, the handling of foreign textbooks in multilingual schools is a multifaceted phenomenon. It is not limited to seeking out materials considered to be of high quality; it involves power relations, market dynamics and teaching concepts. The absence of regulations on international resources, combined with the emphasis on global assessments, facilitates the dissemination of products that are not adapted in the country of destination (Ball, 2012). Valero (2012) sees the possibility of creating collective instances of choice, where the school community - teachers, students, families - examine the suitability of each material, suggesting possible reformulations.

If combined with continuing education and openness to interculturality, foreign books can contribute to experiences of cultural exchange, bringing students closer to other universes of mathematics (Valero, 2023). In multilingual schools, this factor encourages the appreciation of different languages and the contrast of terminologies, without restricting oneself to distant standards. “Linguistic and cultural plurality is strengthened when the textbook is treated as a starting point, not as aclosed script” (Valero & Knijnik, 2016, p. 4).

On the other hand, if the automatic handling of imported methods and content prevails, these books reproduce hierarchies of knowledge and affirm a single conception of teaching. Silva (2019, p. 10) points out that “The lack of robust didactic-pedagogical support initiatives means that international textbooks function as cultural reproduction mechanisms, without critical appropriation on the part of those who teach and those who learn”.

In general, the use of foreign textbooks in Brazilian multilingual schools highlights a situation in which power, the market and educational guidelines are intertwined, generating potentialities and challenges in math teaching. These resources can promote contact with other cultures and approaches, but they can also reinforce a process of curricular colonization, depending on whether or not there is a reflective practice adapted to the location (Valero, 2018). The analyses by Ball (2012) and Valero (2023) indicate the need for choice and implementation mechanisms based on criticism, collective participation and respect for linguistic and cultural identities.

“The relevance and impact of a textbook result not only from its provenance or international reputation, but above all from how this material is inserted into the school practices and educational policies of a givenlocation” (Valero, 2012, p. 381). In this way, multilingual schools can be fertile spaces for rethinking the use of foreign publications, testing possibilities for intercultural interaction and encouraging mathematics that makes sense in students' daily lives. After all, the teaching of mathematics is not neutral, as multiple interests can limit or broaden it, affecting the formation of reflective citizens who are aware of the specificities of their environment.

3.3 Critical expansion and interlocutions: the metaphor of the two cities and the research objectives

The relationship between foreign textbooks, especially those produced in Canada, and the Brazilian educational context led us to consider the sociocultural, linguistic and sociopolitical elements that run through this relationship.

Although there are efforts in other contexts to recognize the diversity of different groups and cultures (Banks, 2014), the way in which this recognition is portrayed in teaching materials does not always reflect the complexities of local realities, including historically marginalized groups (Apple, 2019). When transported to Brazil, these materials can reinforce stereotypes or, on the contrary, broaden perspectives, depending on how they are mediated (Candau, 2012; Medrada, 2020).

Considering the objectives of this research, we can see how they relate to the tension between the global and the local. In these situations, the teacher plays a central role in complementing or reinterpreting the content, remembering that the culture conveyed by the material is not necessarily universal (Bishop, 1988). From a linguistic point of view, the presence of technical terms and foreign references requires mediation strategies to ensure student comprehension (Moschkovich, 2010).

In multilingual environments, positive synergies can arise, but also new challenges if the material does not dialog with the languages present in the school community (Planas & Setati, 2009). The socio-political dimension involves reflecting on market pressures, publishing interests and public policies that motivate or restrict the incorporation of these books (Ball, 2012). In some cases, the quest for better performance in international assessments leads schools to import ready-made solutions that are not always suited to local demands.

The song Duas Cidades, by Baiana System (Passapusso & Barreto, 2016), offers a powerful metaphor for understanding this dynamic of approximations and distancing. The verse sabe subir na vida e não sabe subir sambando reveals the contradiction of those who master certain ascension logics, but are unaware of the cultural practices that give meaning to collectivity.

Transporting this image to the context of textbooks, we have schools that opt for foreign materials in search of prestige or quality indexes, without considering the cultures and needs that permeate the classroom (Apple, 2018). In this movement, the upper city symbolizes the discourse of modernity and innovation, while the lower city alludes to concrete realities, local knowledge and cultural expressions that often remain invisible or marginalized (Candau, 2012).

The metaphor of the two cities also evokes the possibility of going up sambando, i.e. valuing traditional practices, regional identities and mother tongues in the very act of ascending to new levels of learning (Passapusso & Barreto, 2016). In this way, the teacher who understands the local context articulates problems, contexts and languages from the foreign book with knowledge from the community, narrowing the gap between the global (what comes from outside) and the local (what is already part of the students' lives).

Thus, the upper city and the lower city cease to be separate compartments and become part of the same territory, where mathematics can be experienced with greater meaning (Candau, 2012). However, in scenarios marked by pressure for results, the absence of continuing education and the lack of openness to autonomous teaching make pedagogical re-signification more difficult (Ball, 2012; Giroux, 2011).

In this context, the foreign book is incorporated as a commodity or status symbol, without any real integration with the local culture (Apple, 2019). The string of garlic and the scapular of music, which represent protection against the adversities of everyday life, can be seen as analogies to the practices of teacher resistance - small strategies and adaptations that teachers make to defend their students from decontextualized materials, even in the midst of a system that privileges market logics (Passapusso & Barreto, 2016).

From this perspective, this theoretical framework - by articulating sociocultural, linguistic and sociopolitical dimensions - points to the need for a reflexive attitude towards foreign textbooks, especially in multilingual schools.

Rather than labeling them as good or bad, it is important to understand how they are received, interpreted and, above all, (re)signified by the school community. The research effort proposed here coincides with the ambition to create bridges between different cities: that of global mathematical knowledge and that of local customs and experiences. If this encounter takes place sensitively, it opens up space for teaching mathematics that, beyond cognitive terms, is plural and aware of the diverse voices that make up the local school universe.

4 TRACING THE HISTORY AND STRUCTURE OF THE MATHEMATICS CURRICULUM IN CANADA: A REGIONAL PERSPECTIVE

Throughout this section, a historical and structural overview of the mathematics curriculum in Canada is presented, highlighting how political, socioeconomic and cultural factors have shaped the teaching of this subject in different regions of the country (CMEC, 2016a; Wallner, 2014). From the colonial period, when mathematics was taught mainly in religious schools, with a focus on basic counting and trade skills (Gaffield, 2015), to the present day, there is a trajectory marked by tensions between traditional practices and more innovative approaches (Ontario Ministry of Education, 2020; Delisle, 2017).

The autonomy of the provinces and territories in relation to education, resulting from the absence of a unified national curriculum, generates diverse educational models, where initiatives such as the Western and Northern Canadian Protocol (WNCP) seek cooperation and regional alignment (WNCP, 2001; 2002; 2008).

In this context, important movements, such as "New Math" in the 1960s and 1970s (Herrera & Owens, 2001), and reforms based on international assessments (such as PISA, conducted by the Organization for Economic Cooperation and Development - OECD) in the 1980s and 1990s (CMEC, 2019; Bennet, 2020), illustrate the confluence between academic theories, market demands, teaching practices and social expectations.

Along this path, the production of mathematics textbooks reflects the tensions and challenges generated by Canadian decentralization, showing different pedagogical emphases over time (Wagner, 2021). At the same time, the recent demands of the 21st century, such as the integration of digital technologies and the promotion of equity and inclusion (Garcia et.al, 2022), reinforce the need to rethink teaching and assessment methodologies to take into account the country's cultural and linguistic diversity (Bruno-Jofré, 1999; Manitoba Education and Training, 2017).

Finally, the current structure of the Canadian education system, marked by variations in the levels and modalities of teaching (Wallner, 2014), as well as the adoption of different assessment strategies (Alberta Education, 2018; CMEC, 2016a), reaffirms the relevance of the WNCP and other cooperation efforts.

In this way, this chapter offers a broad and contextualized view of the paths that mathematics education has taken - albeit not uniformly - throughout Canada, pointing to historical transformations, contemporary challenges and future trends in the teaching of this subject (Ontario Ministry of Education, 2020).

4.1 Historical aspects of the mathematics curriculum in Canada

The history of the mathematics curriculum in Canada is marked by profound transformations that have accompanied the country's socio-economic and political development (Robson, 2019). Unlike some highly centralized national systems, Canada does not have a single, homogeneous national curriculum.

Instead, each province and territory are responsible for its own education, which results in policies, official documents and curriculum guidelines that can vary significantly from one another (CMEC, 2016b). Still, over the centuries, several common trends and influences can be identified in the teaching of mathematics on Canadian soil.

The following is a comprehensive - albeit brief - overview of the history of the mathematics curriculum in Canada, highlighting periods and movements that have contributed to shaping the way the subject has been taught in the country. Although each province has its own particularities, there are educational events and movements of national and even international scope that have had an impact on the formulation of local curricula.

This section will provide an overview from the colonial period to more contemporary influences, highlighting the conceptual and pedagogical evolution that underpinned the development of curriculum documents. It also discusses the main factors that have motivated changes in teaching, such as technological development, the needs of the labor market and global educational reforms, such as the “New Math” movement and the adoption of international performance standards.

4.1.1 The colonial period

Formal education in Canada has roots dating back to the colonial period, when the first schools were established by missionaries and religious communities, mainly Catholics and Protestants (Robson, 2019). In this context, the mathematics taught was often limited to basic arithmetic and practical notions aimed at trade, agriculture and local economic activities. It is worth noting that at the beginning of European colonization, the transmission of mathematical knowledge already existed informally among indigenous peoples, although these knowledge systems were rarely recognized by the colonizers (Wallner, 2014).

In the French colonies, for example, education was strongly influenced by the Jesuits, who introduced elements of algebra and geometry into their schools, aimed mainly at the elite. On the other hand, in the English-speaking colonies, the curriculum was based on British models, emphasizing reading, writing and basic calculation (Gaffield, 2015). During the 18th century, mathematical content was reduced to notions of counting, simple operations and, at most, the introduction of fractions and measures. This approach served the basic demands of an agrarian and mercantile society.

In terms of teaching resources, there was a shortage of material and teachers often lacked specific training in mathematics. In some regions, private schools maintained by merchants emerged, designed to teach only what was necessary to carry out financial activities (Robson, 2019; Charbonneau et al., 2024). The primary concern was to equip future citizens with sufficient calculation skills for local commerce, tax collection and property management. In this scenario, the subject ofmathematicswas restricted to utilitarian functions, and there was little discussion of its theoretical foundations or broader applications.

4.1.2 The Consolidation of the Educational System and the 19th Century

With population growth and the need to organize the infrastructure of a country in formation, education became more important. The creation of public-school systems in the 19th century had an important impact on the definition of curricula. In 1841, the government of the Province of Canada (which was not yet a unified country as it is today) approved measures to organize and finance common schools (or public schools), directly influencing the mathematics curriculum (Robson, 2019; Charbonneau, et.al, 2024).

Although there was still no unified curriculum policy, some provinces began to draw up reference documents to guide teaching. Mathematics teachingin this context retained its practical nature, but began to incorporate slightly more advanced subjects, such as elementary algebra and Euclidean geometry, especially in the final years of elementary school (Wagner, 2012). The European influence, especially British and French, was noticeable in the adoption of textbooks that followed models used in the metropolises. For example, the method of teaching geometry based on Euclid and algebra based on the English tradition were common in secondary schools.

At the same time, there was concern about teacher training, which culminated in the creation of "Normal Schools" and teacher certification programs in several provinces. These programs sought to standardize teaching content and methodologies, so that mathematics began to be approached more systematically. However, the level of depth was still limited, as many teachers lacked solid training in the area (Wallner, 2014).

This period also saw the popularization of competitions and standardized exams, which aimed to verify mastery of essential skills in reading, writing and mathematics. Although these assessments helped drive the inclusion of mathematics in curricula, they often restricted the subject to a series of mechanical calculation procedures (Robson, 2019; Charbonneau, et.al, 2024). The idea of a more exploratory or creative mathematics was not yet widespread, as a utilitarian and practical view prevailed, linked to economic development and the demands of the emerging labor market.

4.1.3 Reforms at the beginning of the 20th century

At the turn of the 19th to the 20th century, Canada began to consolidate its national and industrial identity. Economic growth, coupled with urbanization, put pressure on the education system to train a skilled workforce, which gradually raised expectations regarding the teaching of mathematics (Wallner, 2014).

Debates arose about the need to include topics such as Analytical Geometry, Trigonometry and Statistics. However, the actual inclusion of these subjects was still slow and segmented.

International influences were also beginning to make their presence felt. John Dewey's pedagogical ideas, which were already resonating in the United States, reached Canada, stimulating discussions about active learning methodologies, including in mathematics (Dewey, 1902/2001). The concern with conceptual understanding, rather than simply training algorithms, began to gain ground in academic circles and in some political sectors, although it was not unanimous.

The period between 1910 and 1930 was marked by various educational commissions in the Canadian provinces, seeking to reformulate teaching content and methods. The province of Ontario, for example, drew up new curriculum documents that encouraged the teaching of mathematics as a subject that formed logical reasoning and not just as a tool for practical operations (Ontario Ministry of Education, 2020). In Quebec, on the other hand, the conservative Catholic education system was reluctant to adopt significant pedagogical changes, although from the 1920s onwards it came under pressure to modernize (Delisle, 2017).

In this process, research into learning difficulties in mathematics also emerged, giving rise to more structured teaching methods, such as the use of manipulative materials and contextualized examples. This approach, however, met with resistance from the teaching staff, who were used to “chalkboard teaching” and repetitive exercises. Nevertheless, the beginning of the 20th century planted the first ideas that would later germinate into larger reform movements.

4.1.4 The New Math movement (1960s and 1970s)

One of the most striking chapters in the history of the mathematics curriculum in Canada was the adoption of the so-called “New Math” or “Modern Math” (Herrera & Owens, 2001). This movement originated in international discussions, mainly in the United States and Europe, and arrived in Canada in the early 1960s. The backdrop included the Cold War and the space race, which gave impetus to the idea of strengthening the scientific and technological education of children and young people (Phillips, 2012). New Math proposed a complete reorganization of content, focusing on mathematical structures, set theory and abstract concepts, to the detriment of traditional teaching, based on algorithms and memorization.

In Canada, the adoption of “New Math” varied from province to province, but in general the country followed the international enthusiasm. Textbooks were replaced or rewritten, teachers were called for continuing education courses and topics such as sets, logic, number systems and non-Euclidean geometry were included in the official curricula (Wallner, 2014). There was a belief that introducing these concepts from the early years would stimulate abstract thinking and improve performance in highly complex scientific areas.

However, the implementation encountered difficulties. Many teachers were not sufficiently prepared to teach such abstract content. In addition, some families did not understand the changes, which led to resistance. The main criticism was that children were not learning to do practical everyday math (Phillips, 2012). In some places, such as Quebec, resistance was even greater, given the already tense relationship between conservative sectors of society and pedagogical innovations.

Despite the controversies, the “New Math” movement left a significant legacy: it promoted a broad debate on the meaning of mathematics teaching, opened up space for epistemological discussions and drew attention to teacher training as a crucial element in the implementation of any curriculum reform.

At the end of the 1970s, several provinces revised their curricula to mitigate the exaggerated emphasis on highly abstract concepts, but maintained some of the principles introduced by “New Math”, such as the importance of symbolic language and the understanding of numerical structures (Herrera & Owens, 2001).

4.1.5 The influence of national and international standards (1980s and 1990s)

With the decline of the “New Math” movement from the 1980s onwards, there was a kind of partial return to more traditional methods, but with greater concern for relating mathematics to real-life situations and the development of cognitive skills. In this period, new issues began to shape the curriculum: the expansion of personal computers, the demands of the globalized market and the growing internationalization of academic performance assessment criteria (Wallner, 2014).

The emergence of international assessment bodies such as the International Association for the Evaluation of Educational Achievement (IEA), which promotes studies such as TIMSS (Trends in International Mathematics and Science Study), had a strong impact on Canada. The results obtained in international assessments began to be closely observed by Canadian provinces, influencing the definition of curriculum goals and the emphasis on certain contents (CMEC, 2016).

Faced with these pressures and results, the discussion on accountability arose, based on the idea of holding educational actors responsible for the indices achieved.

Generally speaking, accountability refers to policies and mechanisms that hold different educational actors (governments, education networks, school managers, teachers) responsible for the results obtained by students (Bennett, 2020). In practical terms, this means that performance in tests such as PISA (Programme for International Student Assessment) and TIMSS (Trends in International Mathematics and Science Study) provides public data that can guide political and budgetary decisions.

When math scores fail to meet defined targets, provinces and territories can respond politically, for example by revising curricula, creating continuing education programs for teachers or adopting improvement strategies inspired by other regions in Canada and abroad (Wallner, 2014). From a financial point of view, resources are often reallocated to support schools and districts where performance is below expectations, either by hiring math teaching specialists or by purchasing support materials (Bennett, 2020).

This transparency in results also generates pressure from communities and the media, who demand quick solutions to the problems indicated by the proficiency indices. At the same time, there are those who criticize an excessive emphasis on quantitative results, claiming that it can limit more creative or inclusive pedagogical approaches (Gallagher, 2013; Ontario Ministry of Education, 2020).

Still, the reality is that the focus on accountability has strongly influenced education policies in Canada, especially in recent decades, where both international assessments (such as PISA and TIMSS) and provincial standardized assessments serve as a parameter for setting priorities, targets and, in some cases, even for comparing interprovincial performance (CMEC, 2016).

In addition, the document “Curriculum and Evaluation Standards for School Mathematics”, published by the National Council of Teachers of Mathematics (NCTM) in 1989 in the United States, had repercussions in Canada. Although the country did not adopt the same national system of standards, many Canadian educators saw the NCTM document as an important reference for curriculum reforms. The focus became the development of broader competencies, such as problem solving, mathematical communication and logical reasoning, alongside procedural skills (NCTM, 2000).

During the 1980s and 1990s, most Canadian provinces undertook significant curriculum revisions, resulting in more detailed documents on learning objectives, teaching strategies and formative assessment (Ontario Ministry of Education, 2020). In the province of Ontario, for example, the curriculum began to emphasize the connection between mathematics and other areas of knowledge, and the importance of developing a positive attitude towards the subject. In Alberta, the curriculum incorporated constructivist principles, valuing guided discovery and a deep understanding of key concepts.

In Quebec, after the Quiet Revolution, the education system was reformulated, gradually moving closer to North American trends. However, a certain independence was maintained, reflected in the development of its own teaching materials and guidelines in French (Delisle, 2017). Across Canada, the inclusion of subjects such as Statistics and Probability has become more evident, in line with the growing importance of these subjects in the scientific context and in the job market.

4.1.6 The 21st century and new curriculum demands

With the turn of the millennium, the mathematics curriculum in Canada was faced with even more complex challenges. Technological advances and the ubiquity of digital devices have highlighted the need for mathematical skills linked to data analysis, programming and modeling (CMEC, 2016). International assessments have continued to influence the formulation of educational policies, with particular attention to the results of the Programme for International Student Assessment (PISA), conducted by the Organization for Economic Cooperation and Development (OECD).

In this scenario, the provinces have intensified their search for approaches that stimulate critical thinking and problem-solving in authentic situations. The use of educational technologies, such as dynamic geometry software and computer algebra tools, has gained ground in the curriculum, with the aim of both improving student motivation and deepening conceptual understanding (Ontario Ministry of Education, 2020). The idea of a more interactive mathematics classroom, with investigative tasks and group work, has established itself as a trend.

Another relevant point was the growing attention to equity and inclusion. Research (Bennett, 2020; Wallner, 2014) showed significant discrepancies in performance and access to learning opportunities between different socioeconomic groups and communities, including indigenous, Afro-descendant and immigrant populations (Wallner, 2014). In response, curricula began to include specific guidelines for creating learning environments that value cultural diversity, including examples of mathematical applications in different cultures. Some curriculum documents, for example, began to explicitly mention the contributions of indigenous peoples to mathematics, as a way of legitimizing and valuing traditional knowledge (Battiste, 2013).

In addition, the importance of formative assessment practices and assessment tools that went beyond standardized tests was increasingly recognized. The use of portfolios, projects, self-assessments and collaborative activities to measure student progress was expanded, reflecting a more comprehensive vision of what it means to be mathematically proficient (Ontario Ministry of Education, 2020).

In terms of policies, there has been an effort at cooperation between the provinces through the Council of Ministers of Education, Canada (CMEC). The body has promoted dialogues and benchmarks to compare and continuously improve the curricula of each region. Although this has not resulted in a unified national curriculum, there is now greater convergence of guiding principles, especially in relation to the emphasis on 21st century skills (CMEC, 2016).

4.2 Mathematics textbooks in Canada

The history of mathematics textbooks in Canada, when analyzed from academic references, makes up a field of research rich in nuances that integrate political, social and epistemological dimensions. This specific panorama emerges largely from the country's federal characteristics - each province and territory develops its own curriculum - and from international influences which, at different times, have dictated substantial educational reforms.

Together, these factors give rise to textbooks whose contents, methodologies and examples reflect the tensions between homogenization and plurality, tradition and innovation, as well as the pressures exerted by debates on inclusion, cultural diversity, accountability and teacher training.

The autonomy of the Canadian provinces in drawing up curricula and defining textbook adoption policies has historical roots linked to the Constitutional Act of 1867 and its subsequent amendments (Young & Levin, 2002). This legal framework explains why, unlike countries with unified guidelines at the federal level, Canada has a mosaic of educational systems, each with its own nuances in terms of the math content that should or should not be included in curricula.

Research by Waddington (2018) andFriesenand Jacobsen (2020) show that this decentralized nature has an impact on the production of teaching materials, as publishers have to meet the specific requirements of different provinces, resulting in textbooks with significant methodological variations. Despite initiatives such as the WNCP, which sought to harmonize elements of the curricula of the western provinces, local sovereignty remains a defining feature of Canadian education policy.

In Canada, this methodological shift permeated the production of textbooks, which began to incorporate examples from everyday life, discussions of practical cases and scripts for investigative activities, allowing students to understand mathematics in contexts closer to their experiences (Robson, 2019). At the same time, there were suggestions that authors should adopt a more accessible language, while maintaining conceptual rigor, in order to overcome the barrier of excessive formalism characteristic of the previous period.

This line of materials focused on problem-solving was also influenced by the demands of educational accountability , especially since the 1990s, when large-scale assessments began to be used more heavily as a parameter of performance and quality of teaching (Bennett, 2020).

The participation of Canadian provinces in tests such as the Programme for International Student Assessment (PISA) reinforced the value placed on mathematical reasoning skills over the mere memorization of algorithms. Thus, textbooks gradually began to include sections dedicated to developing the skills assessed by these tests, such as interpreting graphs, reading statements critically and solving interdisciplinary problems (Boaler, 2002; Bennett, 2020). This curricular restructuring is not homogeneous across Canada, but it has directed editorial production towards formats that balance theory, exercises and practical application activities.

An even more recent aspect concerns the inclusion of themes related to cultural diversity and the inclusion of historically marginalized groups (Hildebrand, 2019). In a country recognized for its multiculturalism and the significant presence of different ethnicities, not only in large metropolises but also in rural areas, there is a growing concern to make mathematics textbooks reflect these contexts.

In addition, the issue of First Nations, Métis and Inuit (native peoples of Canada) has gained relevance in educational debates, especially since the work of the Truth and Reconciliation Commission (2015), which recommended the “decolonization” of the curriculum (Garcia et.al, 2022). As a result, several textbooks began to include references to traditional mathematical practices, such as counting systems and concepts of space and shape found in indigenous cultures, as well as examples linked to festivals, commemorative dates and everyday situations in immigrant communities.

This search for diversity in authorship and content stems not only from a political impulse, but also from theoretical reflections on the sociocultural role of mathematics. While the more formalist approaches of the past considered mathematics to be a universal and decontextualized knowledge, current research, such as that by Garcia et.al (2022) and Charbonneau, et.al, (2024), highlights that the meaning attributed to mathematical concepts is, in part, shaped by linguistic and cultural factors. Thus, including different references in the textbook does not simply "enrich" the activities, but rather recognizes that diverse knowledge and practices contribute to the formation of a more global and integrative mathematical knowledge.

From a technological point of view, the turn of the 21st century brought a number of significant transformations, as textbooks began to dialog with the use of digital tools, dynamic geometry software and graphing calculators(Chorlay et.al, 2022). Studies published in the Canadian Journal of Science, Mathematics and Technology Education (for example,Chorlay et.al, 2022) indicate that this integration of digital resources is linked to the growing acceptance of constructivist approaches and the perspective that the student must actively investigate and construct meaning.

In this context, the most recent textbooks often guide the use of applications, simulators and online platforms so that students can make discoveries autonomously and collaborate with peers in synchronous or asynchronous activities. This change in the profile of teaching materials broadens the scope of the teacher's work, requiring technological skills and the ability to manage less linear teaching processes, where the book becomes a curriculum guide and not just a compendium of exercises.

The evolution of mathematics textbooks in Canada, therefore, is not merely a process of replacing one material with another, but a complex phenomenon in which epistemological disputes, pedagogical debates and political interests come into play (McLean, 2007).

In recent decades, academic production has emphasized that these textbooks should not just be seen as neutral containers of information, but as cultural and historical artifacts that reflect specific visions of how mathematics should be taught and learned (Bruno-Jofré, 1999; Lerman, 2014). By analyzing the layout, the selection of examples, the language used and the organization of the chapters, researchers can

In analyzing the layout, selection of examples, language used and organization of the chapters, researchers have identified teaching concepts that range from an emphasis on mechanical exercise to proposals that encourage debate and metacognitive reflection.

Canada's linguistic diversity also plays an important role in this scenario, with the maintenance of a strong tradition of textbooks in French, especially aimed at the province of Quebec and the French-speaking communities that spread to other regions of the country (Robson, 2019). This production, while dialoguing with European references from French-speaking countries, is concerned with maintaining links with local curricula and Canadian cultural specificities, reinforcing the multiple and often segmented nature of the educational publishing market in the country.

Francophone textbooks can bring different examples, terms and cultural contexts to those found in Anglophone textbooks, even though the mathematical object of study is the same. This linguistic duality fosters a constant exchange of experiences and enhances the debate on the universality and contextualization of mathematics teaching.

Furthermore, the role of the teacher in mediating between the textbook and the student should not be overlooked. Research (Fan & Kaeley, 2020; Yu et.al, 2025; Scott & Husain, 2021) suggests that even if textbooks are carefully designed to include active strategies, practical applications and different learning styles, the success of this approach depends to a large extent on formative and institutional conditions. Yu et.al (2025) explored the dual influence of textbooks on teaching methodologies and student learning outcomes and Scott & Husain (2021) highlight the challenges posed by reliance on traditional textbooks in underfunded schools, particularly in adopting student-centered learning approaches.

When there is no consistent ongoing training program, or when the teaching staff is overloaded, the pedagogical intentions embedded in the textbook may not materialize in the classroom. This mismatch, already observed in the days of the “New Mathematics”, remains a constant warning: the introduction of new materials or methodologies requires adequate preparation to avoid the superficiality of innovations.

In summary, the analysis of the trajectory of mathematics textbooks in Canada shows a path marked by cycles of reform, sometimes driven by international trends, sometimes shaped by internal demands for inclusion, diversity and meeting regional specificities (Bruno-Jofré, 1999; Tomkins, 2008; Yan, 2019; Young & Levin, 2002). These books cannot be seen as mere objects for automatic distribution in schools, but as social constructions that emerge from the dialog - or confrontation - between political values, pedagogical approaches and conceptions of mathematical knowledge. In this way, they are both a reflection of the historical transformations of the Canadian education system and an active force in directing classroom practices.

By situating these publications within a broader framework of historical and sociological studies, it becomes clear how the content of a simple exercise or the narrative of a chapter can carry profound implications for what is meant by quality teaching, for teacher training and, ultimately, for the expectations that Canadian society projects onto the mathematical education of its new generations.

4.3 Structure of the Canadian education system and the WNCP

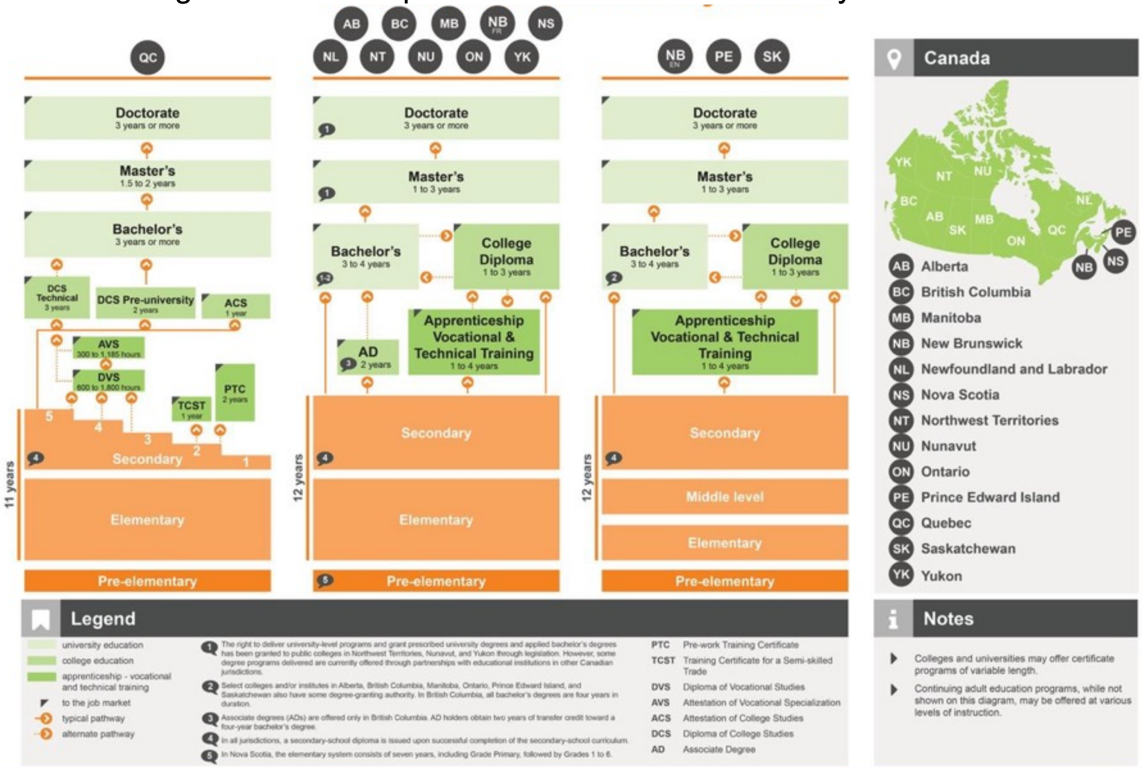
The structure of the mathematics curriculum in Canada varies according to the different provinces and territories, but converges on some fundamental principles and areas of knowledge.

In general, the educational pathway comprises Kindergarten, followed by Elementary School and High School, although there is the possibility of subdivision into Junior High and Senior High in certain regions (Wallner, 2014).

Each jurisdiction draws up official guidelines, but these documents share objectives that emphasize conceptual understanding, the practical application of concepts and problem solving as the central axes of mathematics teaching.

Figure 3 seeks to illustrate, in general terms, the Canadian educational structure, highlighting the fact that, although there are cooperation bodies, each province and territory operates autonomously in relation to the curriculum and assessments.

Figure 3 - The Map of the Canadian Education System



Source: CIC (2024, website[29])

In Early Childhood Education, children between the ages of four and six have their first formal contact with mathematical notions through activities that seek to develop an interest in and initial understanding of quantities, shapes and patterns. This period is not limited to memorizing numbers or algorithms; on the contrary, it aims to establish a conceptual basis for future progression, especially through playful activities and interactive explorations (Ontario Ministry of Education, 2020).

As students progress through elementary school, there is a gradual expansion of concepts. From the first years (equivalent to the first three years of elementary school) to the final years of this stage, there is a growing

[29] Recuperado em CIC - Canadian Information Centre for International Credentials: *An Overview of Education in Canada*. Academic Credentials." Accessed January 28, 2025. https://www.cicic.ca/1149/academic_credentials.canada

systematization of topics such as arithmetic, elementary algebra, geometry, measurements, statistics and probability, often organized into “strands” or “domains”. The general proposal involves not only mastering procedures, but also developing the ability to reason about problems of different kinds, involving everyday situations or open questions that require creativity in solving (Alberta Education, 2018).

In secondary education, which usually covers the 9th to 12th grades, different paths are established according to the students' interests and the academic or professional demands they wish to meet. The subjects become segmented and can include anything from in-depth algebra, trigonometry, pre-calculus and differential and integral calculus, to more practical applications, depending on the path chosen (Delisle, 2017).

In some provinces, the credit structure allows for greater flexibility, as students complete a specific number of hours for each subject. In places like Quebec, there are differentiated models that provide for an intermediate cycle (CEGEP) between secondary school and university entrance, which requires a curricular organization adjusted to these transition requirements.

Although the Canadian provinces are different in their curricular guidelines, several official documents attach importance to the use of digital technologies in the teaching of mathematics. In Ontario, for example, the Ministry of Education (Ontario Ministry of Education, 2020) recommends dynamic geometry software, computer algebra tools and online teaching platforms as part of the strategies that enable exploration, analysis and communication of results in the classroom. In this context, the use of technological supports has been understood as a way of expanding teaching practices, although the way and degree of adoption may vary according to each location.

This technological integration seeks to respond to contemporary demands, encouraging computational thinking and the ability to solve problems collaboratively. With regard to assessment processes, the provinces also have different structures, often combining standardized exams at the end of each cycle with formative assessments such as projects, presentations and the creation of portfolios (Wallner, 2014).

4.3.1 The Western and Northern Canadian Protocol

The Western and Northern Canadian Protocol (WNCP) is an educational cooperation agreement that brings together provinces and territories in western and northern Canada to jointly develop curricula and teaching materials (WNCP, 2001).

Created in 1993, the WNCP initially involved the provinces of British Columbia, Alberta, Saskatchewan and Manitoba, as well as the territories of Yukon, the Northwest Territories and Nunavut. Its central objective was to develop a common curriculum framework in order to promote greater educational consistency and pedagogical alignment in these regions.

This protocol seeks to align pedagogical objectives, assessment standards and learning content, ensuring that there is coherence in the educational programs of the participating regions.

In the context of our research, the textbooks analyzed were developed precisely under this WNCP guideline. This means that they reflect specific guidelines regarding the didactic approach, the development of competences and skills, as well as the proposed content. Therefore, when analyzing these books, it is essential to understand the larger context in which they were created - in other words, the WNCP framework - in order to fully understand their characteristics and the principles that guide the development of the materials.

Delving deeper into the WNCP allows us to see how the content, teaching methodologies and assessment strategies present in the books are connected to the learning objectives set by the educational authorities involved. It also makes it possible to see how the topics covered relate to regional needs and specificities, as well as broader trends in contemporary education.

The WNCP emerged as a response to these inconsistencies, offering a framework designed to standardize key learning outcomes across provinces, while leaving room for regional adaptations. For example, Alberta and Manitoba adopted similar math competencies, but maintained distinct approaches to instructional methods, reflecting their unique socioeconomic conditions (WNCP, 2001; 2002; 2008).

In 2006, the WNCP released the "Common Curriculum Framework for K-9 Mathematics," followed by the "Common Curriculum Framework for 10th-12th Grade Mathematics" in 2008. These frameworks aimed to standardize mathematics education, emphasizing problem solving, reasoning and the integration of technology into learning (WNCP, 2001; 2002; 2008).

In the context of educational initiatives such as Science, Technology, Engineering and Mathematics (STEM) and competency-based assessments, WNCP regions have demonstrated varying degrees of engagement. While some WNCP provinces have integrated STEM principles into their curricula, others have adopted more traditional approaches.

For example, Alberta has incorporated STEM-focused programs into its education strategy, while Saskatchewan has maintained a conventional curriculum structure (Alberta Education, 2016; Saskatchewan Ministry of Education, 2017). Regarding competency-based assessments, the WNCP does not implement a unified standardized testing system similar to Ontario's Education Quality and Accountability Office (EQAO). Instead, assessment practices differ between member regions, leading to diverse methods for assessing student competencies (Council of Ministers of Education, Canada, 2018).

The era of accountability[30] has influenced education policy across Canada, with notable differences between WNCP regions and provinces like Ontario.

Ontario's EQAO assessments provide standardized data on student performance, informing policy decisions and educational strategies (EQAO, 2019). In contrast, the WNCP's decentralized approach results in varied assessment practices that reflect regional priorities and educational philosophies. For example, Alberta administers provincial achievement tests, while British Columbia uses the Basic Skills Assessment to evaluate student performance (Alberta Education, 2016; British Columbia Ministry of Education, 2018). These

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[30] A era da responsabilidade refere-se a um período, iniciado nos anos 1990, caracterizado pelo aumento do uso de testes padronizados, responsabilização de professores e escolas pelo desempenho dos alunos, e uma maior ênfase em resultados mensuráveis nas políticas educacionais (Volante & Ben Jaafar, 2008). Volante, L., & Ben Jaafar, S. (2008). *Educational assessment in Canada: Critical issues and future directions*. Springer

different approaches highlight the decentralized nature of Canada's education system and the autonomy of the provinces and territories in educational governance.

The WNCP curriculum was analyzed in terms of its impact on equity and achievement, particularly in relation to Indigenous and minority students. Efforts to incorporate Indigenous perspectives into the curriculum have been made with varying degrees of implementation in the WNCP regions. For example, Manitoba has integrated Indigenous knowledge and perspectives into its math curriculum to create a more inclusive educational environment (Manitoba Education and Training, 2017).

However, challenges remain to ensure equitable results for all groups of students. The use of standardized assessments, when implemented, has been criticized for its potential limitation in addressing existing disparities between student populations, rather than directly causing them (Wallner, 2014).

In the broader context of Canada's decentralized education system, the WNCP represents a collaborative effort to harmonize curricula across multiple jurisdictions. This collaboration seeks to balance regional autonomy with the benefits of a standardized educational framework.

The decentralized nature of the system allows provinces and territories to adapt education policies to their specific contexts, but also presents challenges to achieving uniformity in curriculum and assessment practices (Council of Ministers of Education, Canada, 2018). The WNCP's approach reflects an attempt to navigate these complexities by promoting cooperation between its member regions, while respecting their individual educational priorities.

5 METHODOLOGICAL PATH

The aim of this section is to present the investigative path that guided this study, highlighting the theoretical-methodological choices and the stages of data collection, organization and analysis. Initially, the qualitative approach adopted is discussed, based on the reflections of authors such as Bicudo (2005), Bogdan & Biklen (1994) and Deslandes et al. (2025).

This approach underscore the descriptive and interpretive nature of educational phenomena, as well as the relevance of the meanings attributed by the subjects or by the materials analyzed themselves (Bicudo, 2005; Deslandes et al., 2025). In this way, it is emphasized that the results emerge from an inductive process, in which the researcher maintains a continuous interaction with the reality observed (Bogdan & Biklen, 1994; Denzin & Lincoln, 2006).

Next, the procedures and scenarios that guide the investigation are discussed, describing the corpus selected and the ways of collecting and producing the data, as recommended by authors such as Goldenberg (2004) and Laville and Dione (1999). The structural characteristics of the MMS collection are presented, as well as the techniques used to identify, organize and quantify the sections and tasks in each volume.

Finally, the categories of analysis resulting from the full reading of the books and alignment with the theoretical framework are presented, indicating how these categories will be interpreted in subsequent chapters. In this way, the methodology outlined here enables an in-depth understanding of the phenomenon under investigation, reinforcing the multidimensional nature of textbooks and the relevance of examining cognitive, cultural and pedagogical aspects in the field of Mathematics Education.

5.1 The research's methodological approach

This investigation adopts the perspective of qualitative research because it considers that “this type of research emphasizes description” (Bicudo, 2005, p. 122) and focuses on understanding the meanings attributed to educational phenomena. According to Bogdan and Biklen (1994), although qualitative research can take on different configurations, it is characterized, among other things, by being essentially descriptive and by emphasizing “the process rather than simply the results or products” (Bogdan & Biklen, 1994, p. 47-50). Furthermore, these authors point out that “abstractions are built up as the particular data that have been collected are grouped together” (Bogdan & Biklen, 1994, p. 50), highlighting the inductive nature of the reasoning employed.

The qualitative approach, in addition to descriptions, involves interpreting the events observed, as well as critically reflecting on the meanings attributed by the participants or the materials investigated (Goldenberg, 2004). This is in line with the observation that qualitative research “works with the universe of meanings, motivations, aspirations, beliefs, values and attitudes, which corresponds to a deeper space of relationships [...] that cannot be reduced to the operationalization of variables” (Deslandes et al., 2025, pp. 21-22).

From this perspective, the researcher assumes that experiences or empirical evidence can provide support for understanding and interpreting the phenomenon, without the intention of exhausting or statistically generalizing the conclusions reached (Bogdan & Biklen, 1994; Denzin & Lincoln, 2005).

As far as the occasional use of quantitative data is concerned, there is no incompatibility with the qualitative framework. According to Goldenberg (2004, p. 53) “Most researchers in the social sciences now admit that there is no single technique, no single valid means of collecting data in all research” and, in this sense, “qualitative and quantitative methods, from this perspective, are no longer seen as opposites but as complementary”. In this way, numerical information, such as the number of tasks, sections or occurrences in textbooks, can be explored alongside interpretative and reflective analysis, broadening the view of the object of study (Deslandes et al., 2025).

By adopting this approach, it is recognized that the results emerge from a continuous interaction between the researcher and the investigated reality, in which the “perspective of the participants (individuals or group of people to be researched) on phenomena that surround them” is observed. (Sampieri et al., 2013, p. 376).

These voices are considered in their context because, in the view of Creswell and Creswell (2022) research questions can change as new data is gathered, since “questions emerge during a qualitative study” (Creswell & Creswell, 2022, p. 187). Furthermore, the interpretation proposed by the researcher is not neutral, because “it means that the researcher filters the data through a personal lens situated in a specific socio-political and historical moment” (Creswell & Creswell, 2022, p. 187).

In this scenario, qualitative research in Mathematics Education is relevant for examining aspects such as the use and selection of textbooks. The subjectivity of teachers and students, as well as the possible educational implications, demand an analysis that goes beyond the mere collection of statistical or standardized data (Bicudo, 1993).

These analyses, in turn, can unfold into theoretical and practical reflections, contributing to the understanding of educational phenomena in all their complexity, especially when these phenomena concern teaching and learning processes in a social, cultural and historically situated environment. (Laville & Dione, 1999).

Thus, the qualitative approach adopted in this study directs attention to the “universe of meanings” (Deslandes et al., 2025, p. 21) and for the interpretations that emerge from contact with the sources analyzed (Bogdan & Biklen, 1994)b. The descriptions presented here, combined with critical reflection and theoretical analysis, aim to ensure a more in-depth look at the object of investigation.

In this way, far from excluding any quantitative data, qualitative research promotes a broader and more coherent understanding of educational phenomena, preserving their complexity and ensuring room for interpretations that value the process, subjectivity and social interactions (Goldenberg, 2004; Lüdke & André, 2013).

5.2 Procedures and scenarios

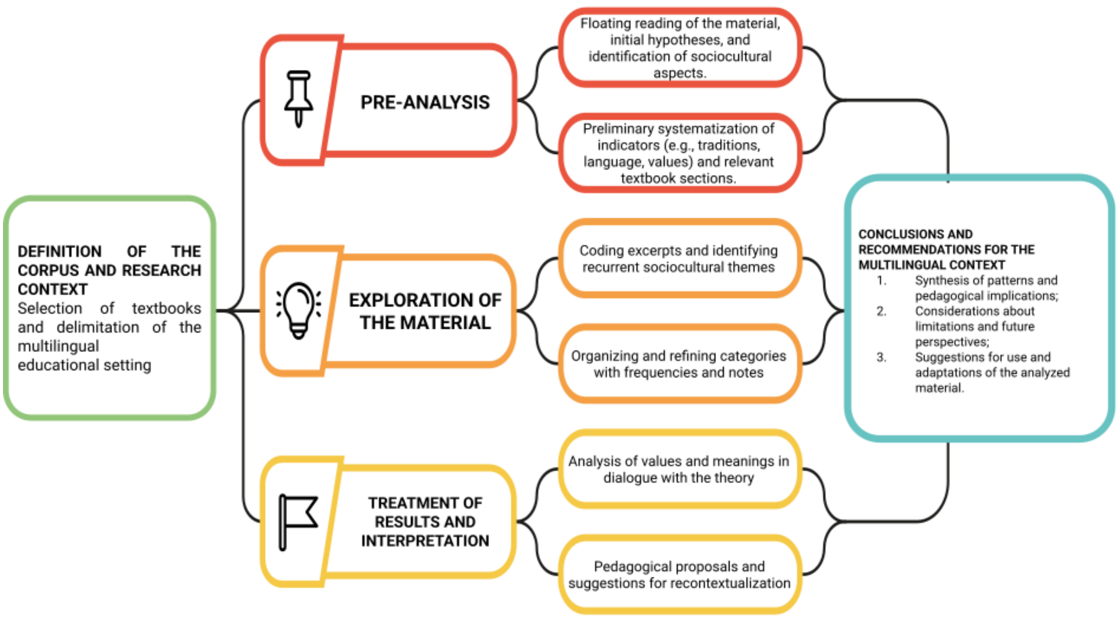
This research studies the MMS collection, focusing on how Canadian sociocultural elements are incorporated into these textbooks and how they can be interpreted in multilingual Brazilian schools.

The content analysis proposed by Bardin (2016), referenced by Dalla Valle and Ferreira (2023), is adopted in this research to systematically interpret textual and visual data from Mathematics Makes Sense textbooks, making it possible to identify how Canadian sociocultural elements are incorporated and can be contextualized in Brazilian multilingual schools.

This method is organized in three interconnected phases. Pre-analysis focuses on defining the corpus, reading through it and choosing the relevant materials, as well as formulating hypotheses, objectives and indicators. In the material exploration phase, coding takes place, which includes grouping recording units, categorizing and comparing passages that show similarities or differences. Finally, in the treatment of results and interpretation stage, the findings are linked to the theoretical framework and inferences are made, giving consistency to the discussion on cultural, linguistic and political values associated with the collection analyzed.

This chain, described by Bardin (2016) and corroborated by Dalla Valle and Ferreira (2023), guides the construction of inferences that are not limited to the manifest content, facilitating the apprehension of implicit and latent meanings that emerge in the discourse. Figure 4, inspired by Dalla Valle and Ferreira (2023) and adapted from Bardin (2016), illustrates the process of intersection between the phases.

Figure 4 - Methodological journey



Source: The author (2025) based on Bardin (2016) and Dalla Valle and Ferreira (2023)

In order to adopt this methodology, some preliminary procedures were established, as shown in figure 4. In the pre-analysis stage, it was planned to conduct a floating reading of each volume to point out possible indicators related to Canadian daily life, such as sports and festivities, which could appear in the representation of mathematical problems or contextual examples. This initial reading was considered relevant in order to make operational the first hypotheses about how these elements could manifest themselves and what adaptations would be necessary to bring them closer to the reality of Brazilian students in multilingual environments.

In the material exploration phase, the intention was to apply a coding system based on categorizing passages that alluded to Canadian values and traditions, as well as those that suggested ways of comparing them with the local context. It was hoped to group together fragments that discussed, for example, extreme temperatures, the practice of specific sports or references to Canadian indigenous populations, in an attempt to determine the degree of proximity or distance from the experience of Brazilian students. This coding process would be based on the indicators established in the pre-analysis, resulting in categories that would support comparison and analysis.

Finally, when processing the results and interpreting them, we tried to relate the categories formed to the theoretical framework underpinning the research, checking to what extent the Canadian references could be re-signified. Above all, the plan was to assess whether the adoption of MMS books would foster cultural plurality or whether, on the contrary, it would create distancing that would require greater teacher mediation for the content to make sense in Brazilian multilingual schools. These inferences would be discussed taking into account the cultural, linguistic and political values present in this exchange of teaching materials, as established by the research objectives.

5.2.1 Getting to know Mathematics Makes Sense WNCP Edition

In this phase of the material exploration, the study examines the four volumes of the MMS collection (MMS6, MMS7, MMS8, and MMS9), which are designed for students in grades 6 to 9. This selection proves especially relevant in view of the notable expansion of Canada-based bilingual school franchises in Brazil, where one such network – introduced in the mid-2000s – already operates over 150 units across multiple states (Silva & Martins, 2022).

As these institutions increasingly adopt textbooks from countries that perform well on international education indexes, the MMS series has become part of everyday practice in multilingual settings. This trend also reflects a broader perception that Canadian-produced content may be "superior" (Brito, 2020), further reinforcing its appeal. Against this backdrop, the content analysis guides the focus on structural and conceptual dimensions, directing attention to how each textbook is organized and presented, and highlighting the sociocultural features embedded in these materials.

The initial reading of each volume entailed mapping its overall structure, with a particular focus on the thematic units. According to Bardin (2016), this step corresponds to a "floating reading," in which the researcher becomes acquainted with the content without adhering to rigid categories. We observed that each unit begins with a "Launch Activity," where a practical or contextual scenario introduces the main topic, establishing links between mathematical concepts and everyday situations. Immediately following this, a "Learning Objectives" section outlines the intended skills and knowledge, ensuring consistency with current curriculum guidelines.

In line with Bardin's (2016) phases, this first reading (pre-analysis) was used to pinpoint potential emergent categories, such as approaches to contextualization, real-world connections, and the explicit mention of pedagogical aims. Relevant elements (e.g., scenarios of practical application, key topics, and expected learning outcomes) were then identified and coded, subsequently organized into categories that reflect the textbooks' didactic structure. These categories underpinned the examination of how Canadian sociocultural aspects are conveyed and how they might address the needs of Brazilian multilingual schools, in alignment with the research objectives.

The MMS Collection is structured into thematic units, each beginning with a Launch activity (see Figure 5) that presents a practical or contextualized scenario to introduce the main topic of the unit.

Figure 5 - MMS Pearson WNCP Edition (K-9)

Source: Pearson Canada (2024)

This opening segment suggests an application of mathematical concepts in everyday contexts, aiming to establish connections between the topics discussed and real-world problems. Following this, the Learning Goals section (see Figure 5, highlighted in yellow) delineates, in a clear and objective manner, the skills and knowledge that students are expected to develop throughout each unit.

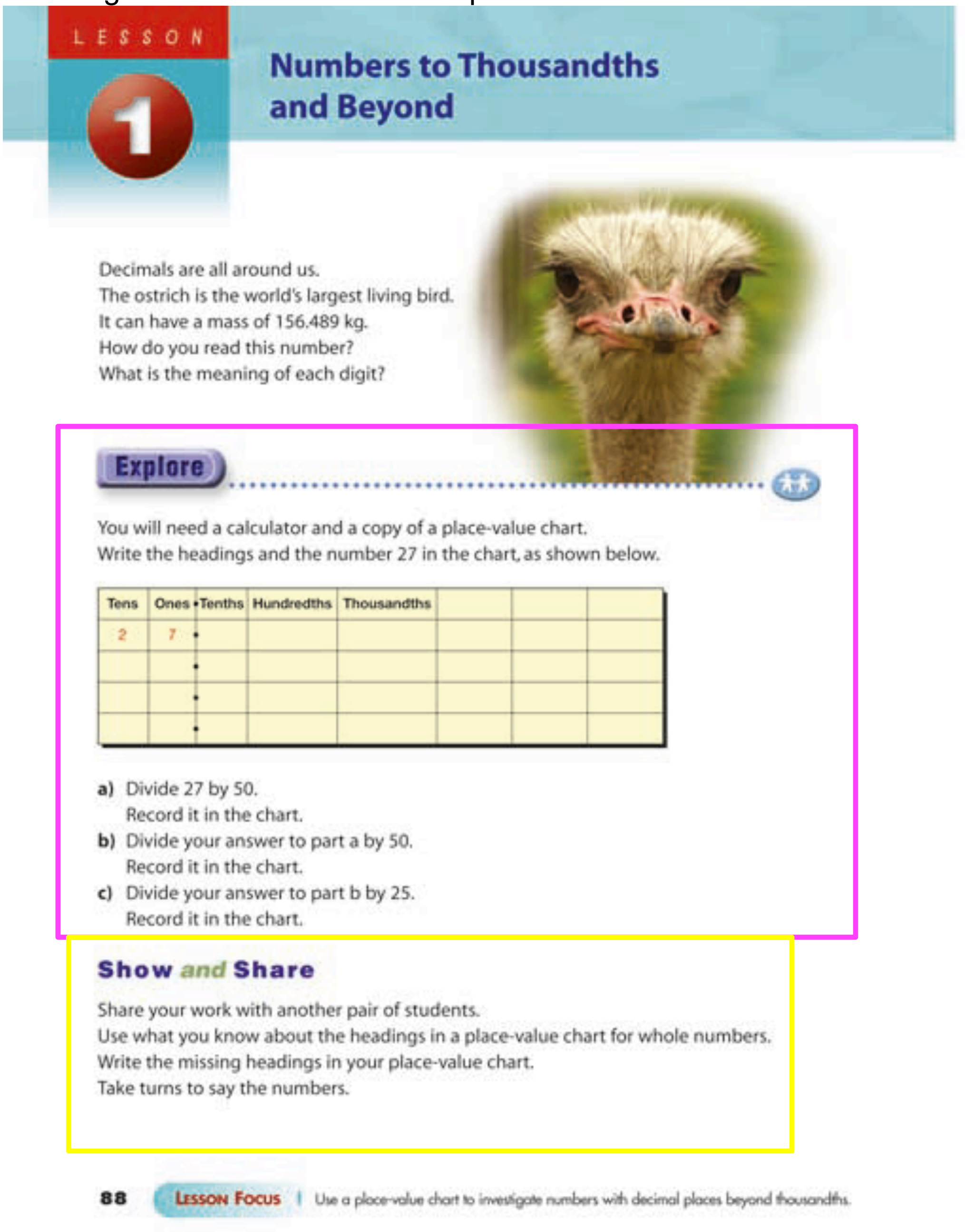
Figure 6 - Example of a Launch Activity and Definition of Learning Goals

Source: Pearson Canada - MMS 6 (2024, p 288 - 289)

Each lesson is segmented into several sections designed to facilitate investigative exploration and the exchange of ideas among students. The Explore section (see Figure 7, highlighted in pink) organizes practical activities that may support collaborative work and experimentation in the study of mathematical fundamentals.

Following this, the Show and Share section (see Figure 7, highlighted in yellow) is intended to prompt the communication of strategies and the discussion of resolution methods, offering an opportunity for critical reflection on the proposed solutions.

Figure 7 - Structure of the Explore and Show and Share Sections



LESSON 1

Numbers to Thousandths and Beyond

Decimals are all around us.
The ostrich is the world's largest living bird.
It can have a mass of 156.489 kg.
How do you read this number?
What is the meaning of each digit?

Explore

You will need a calculator and a copy of a place-value chart.
Write the headings and the number 27 in the chart, as shown below.

Tens	Ones	•Tenths	Hundredths	Thousandths			
2	7	•					
		•					
		•					
		•					

a) Divide 27 by 50.
Record it in the chart.
b) Divide your answer to part a by 50.
Record it in the chart.
c) Divide your answer to part b by 25.
Record it in the chart.

Show and Share

Share your work with another pair of students.
Use what you know about the headings in a place-value chart for whole numbers.
Write the missing headings in your place-value chart.
Take turns to say the numbers.

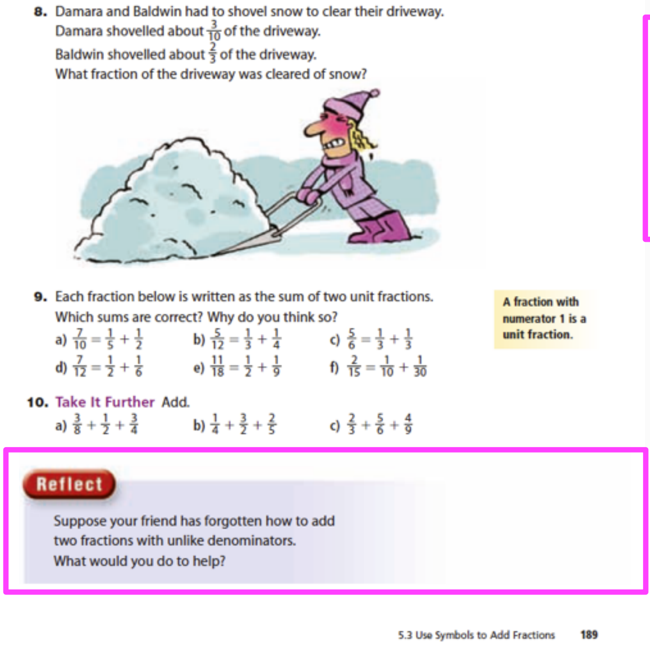
88 LESSON FOCUS | Use a place-value chart to investigate numbers with decimal places beyond thousandths.

Source: Pearson Canada - MMS 6 (2024, p. 88)

To consolidate learning, the Practice Questions section (see Figure 8) includes exercises designed to review the concepts addressed, while the Reflect section (see Figure 7, highlighted in pink) prompts students to consider what they have learned and identify their own learning strategies.

Several lessons also include a section titled Strategies Toolkit (see Figure 9), where various problem-solving techniques are outlined. Additionally, certain units incorporate games and interactive activities (see Figure 9) to introduce alternative approaches to engaging with the material.

Figure 8 - Practice Questions and Reflect Space exercises



8. Damara and Baldwin had to shovel snow to clear their driveway.
Damara shovelled about $\frac{3}{10}$ of the driveway.
Baldwin shovelled about $\frac{2}{3}$ of the driveway.
What fraction of the driveway was cleared of snow?

9. Each fraction below is written as the sum of two unit fractions.
Which sums are correct? Why do you think so?

a) $\frac{7}{10} = \frac{1}{5} + \frac{1}{2}$ b) $\frac{5}{12} = \frac{1}{3} + \frac{1}{4}$ c) $\frac{5}{6} = \frac{1}{3} + \frac{1}{3}$

d) $\frac{7}{12} = \frac{1}{2} + \frac{1}{6}$ e) $\frac{11}{18} = \frac{1}{2} + \frac{1}{9}$ f) $\frac{2}{15} = \frac{1}{10} + \frac{1}{30}$

A fraction with numerator 1 is a unit fraction.

10. Take It Further Add.

a) $\frac{3}{8} + \frac{1}{2} + \frac{3}{4}$ b) $\frac{1}{4} + \frac{3}{2} + \frac{2}{5}$ c) $\frac{2}{3} + \frac{5}{6} + \frac{4}{9}$

Reflect

Suppose your friend has forgotten how to add two fractions with unlike denominators. What would you do to help?

5.3 Use Symbols to Add Fractions 189

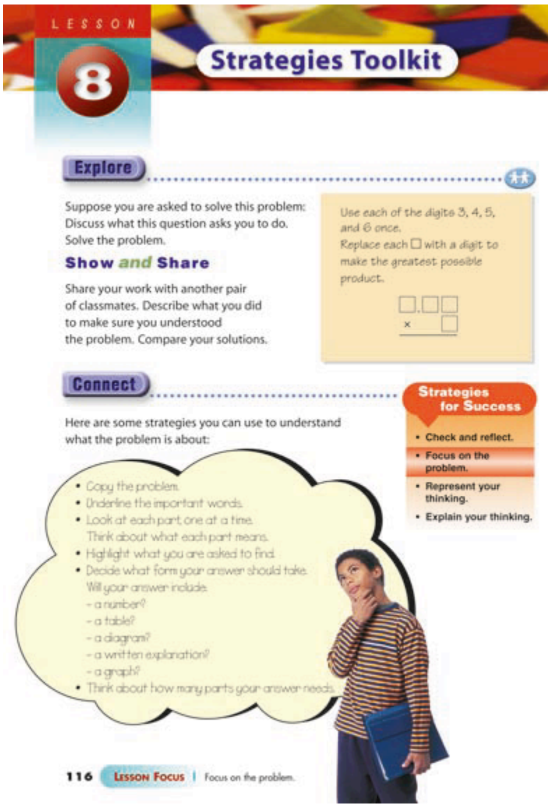
Reflect & Share

- Read over what you have written.
  Will someone else be able to follow your thinking?
- Share your Thinking Log with a classmate.
  Was your classmate able to follow your thinking and understand your solution? Explain.
- Describe any changes you would make to improve your Thinking Log.

Source: MMS 7 (2007, p.189) Source: MMS 8 (2007, p.14)

In several lessons, a Strategies Toolkit section is included, outlining various problem-solving approaches. Additionally, games and interactive activities (see Figure 9) are incorporated to provide alternative contexts for engaging with the material.

Figure 9 - Demonstration of the Strategies Toolkit Section and Interactive Games



LESSON 8

Strategies Toolkit

Explore

Suppose you are asked to solve this problem:
Discuss what this question asks you to do.
Solve the problem.

Use each of the digits 3, 4, 5, and 6 once.
Replace each ☐ with a digit to make the greatest possible product.

Show and Share

Share your work with another pair of classmates. Describe what you did to make sure you understood the problem. Compare your solutions.

Connect

Here are some strategies you can use to understand what the problem is about:

- Copy the problem.
- Underline the important words.
- Look at each part one at a time. Think about what each part means.
- Highlight what you are asked to find.
- Decide what form your answer should take. Will your answer include
  - a number?
  - a table?
  - a diagram?
  - a written explanation?
  - a graph?
- Think about how many parts your answer needs.

Strategies for Success

- Check and reflect.
- Focus on the problem.
- Represent your thinking.
- Explain your thinking.

116 Lesson Focus | Focus on the problem.

Source: MMS 6 (2007, p.116)

Game

Spinning Fractions

HOW TO PLAY

Your teacher will give you a copy of the spinner.
Use an open paper clip as the pointer.
Use a sharp pencil to keep the pointer in place.
Record the scores in a chart.

1. Player A spins the pointer twice.
Player A adds the fractions.
Player B multiplies the fractions.
The player with the greater result gets one point.

2. Player B spins the pointer twice.
Player A adds the fractions.
Player B multiplies the fractions.
The player with the greater result gets one point.

3. Players continue to take turns spinning the pointer.
The first person to get 12 points wins.

YOU WILL NEED
A copy of the spinner; an open paper clip; a sharp pencil

NUMBER OF PLAYERS
2

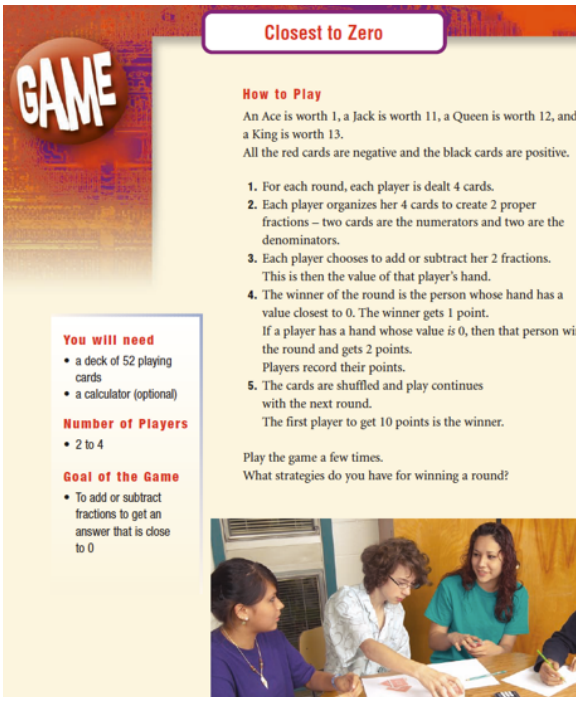
GOAL OF THE GAME
To be the first to get 12 points

REFLECT

- Do you think this game is fair?
How many games do you need to play to find out?
a) If this game is fair, explain how you know.
b) If this game is not fair, how could you make the game a fair game?
- Without playing the game many times, how else could you find out if the game is fair?

Game: Spinning Fractions 127

Source: MMS 8 (2008, p.127)



GAME

Closest to Zero

How to Play

An Ace is worth 1, a Jack is worth 11, a Queen is worth 12, and a King is worth 13.
All the red cards are negative and the black cards are positive.

1. For each round, each player is dealt 4 cards.
2. Each player organizes her 4 cards to create 2 proper fractions – two cards are the numerators and two are the denominators.
3. Each player chooses to add or subtract her 2 fractions. This is then the value of that player's hand.
4. The winner of the round is the person whose hand has a value closest to 0. The winner gets 1 point.
If a player has a hand whose value *is* 0, then that person wi[ns] the round and gets 2 points.
Players record their points.
5. The cards are shuffled and play continues with the next round.
The first player to get 10 points is the winner.

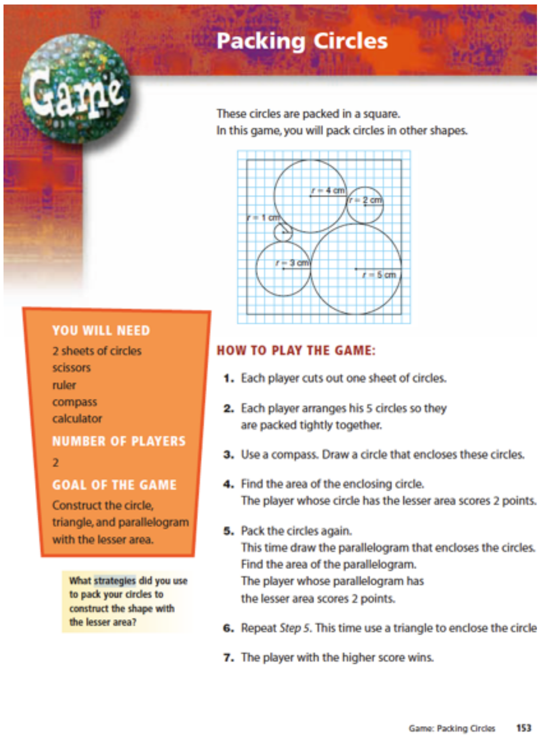
Play the game a few times.
What strategies do you have for winning a round?

You will need
- a deck of 52 playing cards
- a calculator (optional)

Number of Players
- 2 to 4

Goal of the Game
- To add or subtract fractions to get an answer that is close to 0

Source: MMS 9 (2009, p.122)



Game

Packing Circles

These circles are packed in a square.
In this game, you will pack circles in other shapes.

r = 4 cm
r = 2 cm
r = 1 cm
r = 3 cm
r = 5 cm

YOU WILL NEED
2 sheets of circles
scissors
ruler
compass
calculator

NUMBER OF PLAYERS
2

GOAL OF THE GAME
Construct the circle, triangle, and parallelogram with the lesser area.

HOW TO PLAY THE GAME:

1. Each player cuts out one sheet of circles.
2. Each player arranges his 5 circles so they are packed tightly together.
3. Use a compass. Draw a circle that encloses these circles.
4. Find the area of the enclosing circle.
The player whose circle has the lesser area scores 2 points.
5. Pack the circles again.
This time draw the parallelogram that encloses the circles.
Find the area of the parallelogram.
The player whose parallelogram has the lesser area scores 2 points.
6. Repeat *Step 5*. This time use a triangle to enclose the circle[s]
7. The player with the higher score wins.

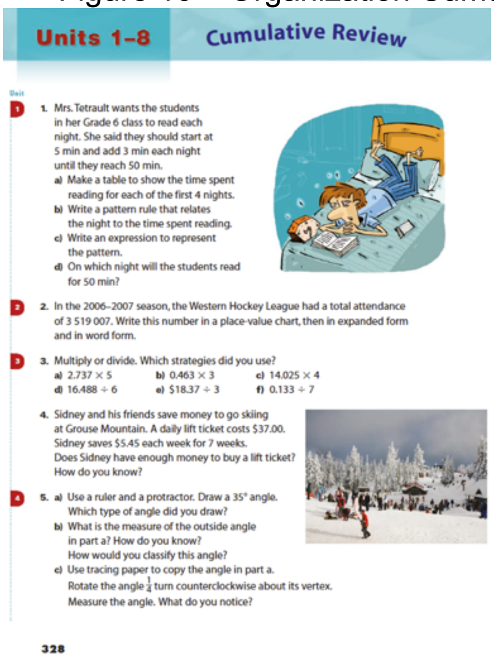
What strategies did you use to pack your circles to construct the shape with the lesser area?

Game: Packing Circles 153

Source: MMS 7 (2007, p.153)

At the end of each unit, a Unit Review section offers revision exercises (see Figure 10), followed by a Unit Problem that integrates various concepts presented within the unit into a single task. In some instances, a Cumulative Review is provided to revisit content from multiple units.

Figure 10 – Organization Cumulative Review and Unit Review


Units 1–8 Cumulative Review

1. Mrs. Tetrault wants the students in her Grade 6 class to read each night. She said they should start at 5 min and add 3 min each night until they reach 50 min.
a) Make a table to show the time spent reading for each of the first 4 nights.
b) Write a pattern rule that relates the night to the time spent reading.
c) Write an expression to represent the pattern.
d) On which night will the students read for 50 min?

2. In the 2006–2007 season, the Western Hockey League had a total attendance of 3 519 007. Write this number in a place-value chart, then in expanded form and in word form.

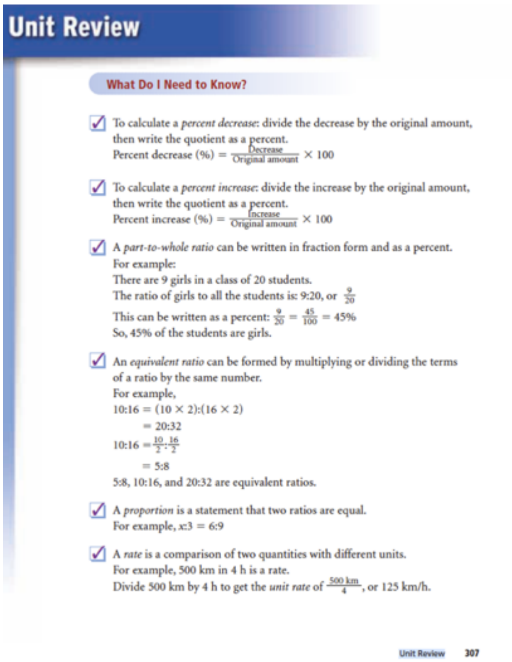
3. Multiply or divide. Which strategies did you use?
a) 2.737 × 5 b) 0.463 × 3 c) 14.025 × 4
d) 16.488 ÷ 6 e) $18.37 ÷ 3 f) 0.133 ÷ 7

4. Sidney and his friends save money to go skiing at Grouse Mountain. A daily lift ticket costs $37.00. Sidney saves $5.45 each week for 7 weeks. Does Sidney have enough money to buy a lift ticket? How do you know?

5. a) Use a ruler and a protractor. Draw a 35° angle. Which type of angle did you draw?
b) What is the measure of the outside angle in part a? How do you know? How would you classify this angle?
c) Use tracing paper to copy the angle in part a. Rotate the angle $\frac{1}{4}$ turn counterclockwise about its vertex. Measure the angle. What do you notice?

328

Source: MMS 6 (2007, p.328)


Unit Review

What Do I Need to Know?

To calculate a *percent decrease*: divide the decrease by the original amount, then write the quotient as a percent.
Percent decrease (%) = $\frac{\text{Decrease}}{\text{Original amount}} \times 100$

To calculate a *percent increase*: divide the increase by the original amount, then write the quotient as a percent.
Percent increase (%) = $\frac{\text{Increase}}{\text{Original amount}} \times 100$

A *part-to-whole ratio* can be written in fraction form and as a percent.
For example:
There are 9 girls in a class of 20 students.
The ratio of girls to all the students is: 9:20, or $\frac{9}{20}$
This can be written as a percent: $\frac{9}{20} = \frac{45}{100} = 45\%$
So, 45% of the students are girls.

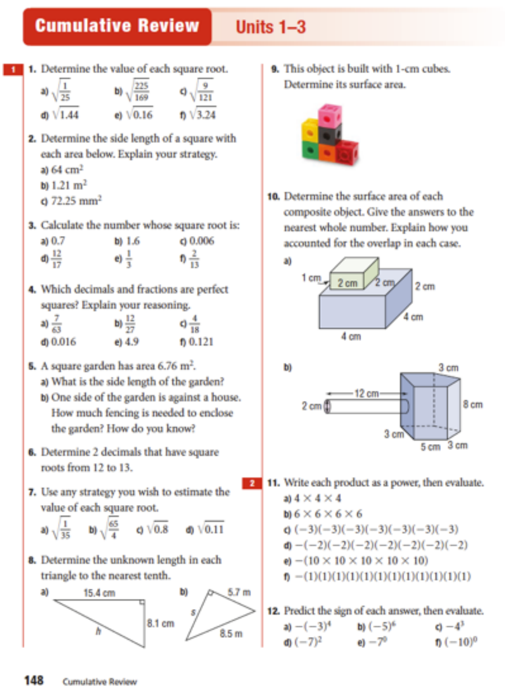
An *equivalent ratio* can be formed by multiplying or dividing the terms of a ratio by the same number.
For example,
10:16 = (10 × 2):(16 × 2)
= 20:32
10:16 = $\frac{10}{2}$:$\frac{16}{2}$
= 5:8
5:8, 10:16, and 20:32 are equivalent ratios.

A *proportion* is a statement that two ratios are equal.
For example, x:3 = 6:9

A *rate* is a comparison of two quantities with different units.
For example, 500 km in 4 h is a rate.
Divide 500 km by 4 h to get the *unit rate* of $\frac{500\text{ km}}{4}$, or 125 km/h.

Unit Review 307

Source: MMS 8 (2008, p.307)


Cumulative Review Units 1–3

1. Determine the value of each square root.
a) $\sqrt{\frac{1}{25}}$ b) $\sqrt{\frac{225}{169}}$ c) $\sqrt{\frac{9}{121}}$
d) $\sqrt{1.44}$ e) $\sqrt{0.16}$ f) $\sqrt{3.24}$

2. Determine the side length of a square with each area below. Explain your strategy.
a) 64 cm²
b) 1.21 m²
c) 72.25 mm²

3. Calculate the number whose square root is:
a) 0.7 b) 1.6 c) 0.006
d) $\frac{12}{17}$ e) $\frac{1}{3}$ f) $\frac{2}{13}$

4. Which decimals and fractions are perfect squares? Explain your reasoning.
a) $\frac{7}{63}$ b) $\frac{12}{27}$ c) $\frac{4}{18}$
d) 0.016 e) 4.9 f) 0.121

5. A square garden has area 6.76 m².
a) What is the side length of the garden?
b) One side of the garden is against a house. How much fencing is needed to enclose the garden? How do you know?

6. Determine 2 decimals that have square roots from 12 to 13.

7. Use any strategy you wish to estimate the value of each square root.
a) $\sqrt{\frac{1}{35}}$ b) $\sqrt{\frac{65}{4}}$ c) $\sqrt{0.8}$ d) $\sqrt{0.11}$

8. Determine the unknown length in each triangle to the nearest tenth.

9. This object is built with 1-cm cubes. Determine its surface area.

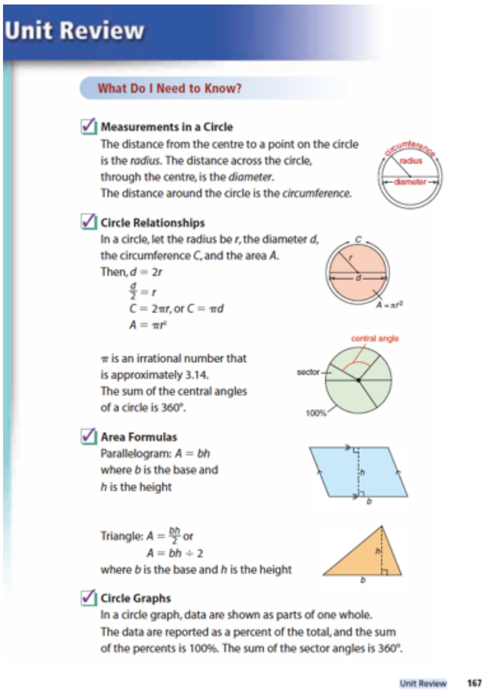
10. Determine the surface area of each composite object. Give the answers to the nearest whole number. Explain how you accounted for the overlap in each case.

11. Write each product as a power, then evaluate.
a) 4 × 4 × 4
b) 6 × 6 × 6 × 6
c) (−3)(−3)(−3)(−3)(−3)(−3)
d) −(−2)(−2)(−2)(−2)(−2)(−2)
e) −(10 × 10 × 10 × 10 × 10)
f) −(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)

12. Predict the sign of each answer, then evaluate.
a) $-(-3)^4$ b) $(-5)^6$ c) -4^3
d) $(-7)^2$ e) -7^0 f) $(-10)^0$

148 Cumulative Review

Source: MMS 9 (2009, p.148)


Unit Review

What Do I Need to Know?

Measurements in a Circle
The distance from the centre to a point on the circle is the *radius*. The distance across the circle, through the centre, is the *diameter*.
The distance around the circle is the *circumference*.

Circle Relationships
In a circle, let the radius be *r*, the diameter *d*, the circumference *C*, and the area *A*.
Then, $d = 2r$
$\frac{d}{2} = r$
$C = 2\pi r$, or $C = \pi d$
$A = \pi r^2$

π is an irrational number that is approximately 3.14.
The sum of the central angles of a circle is 360°.

Area Formulas
Parallelogram: $A = bh$
where *b* is the base and *h* is the height

Triangle: $A = \frac{bh}{2}$ or
$A = bh \div 2$
where *b* is the base and *h* is the height

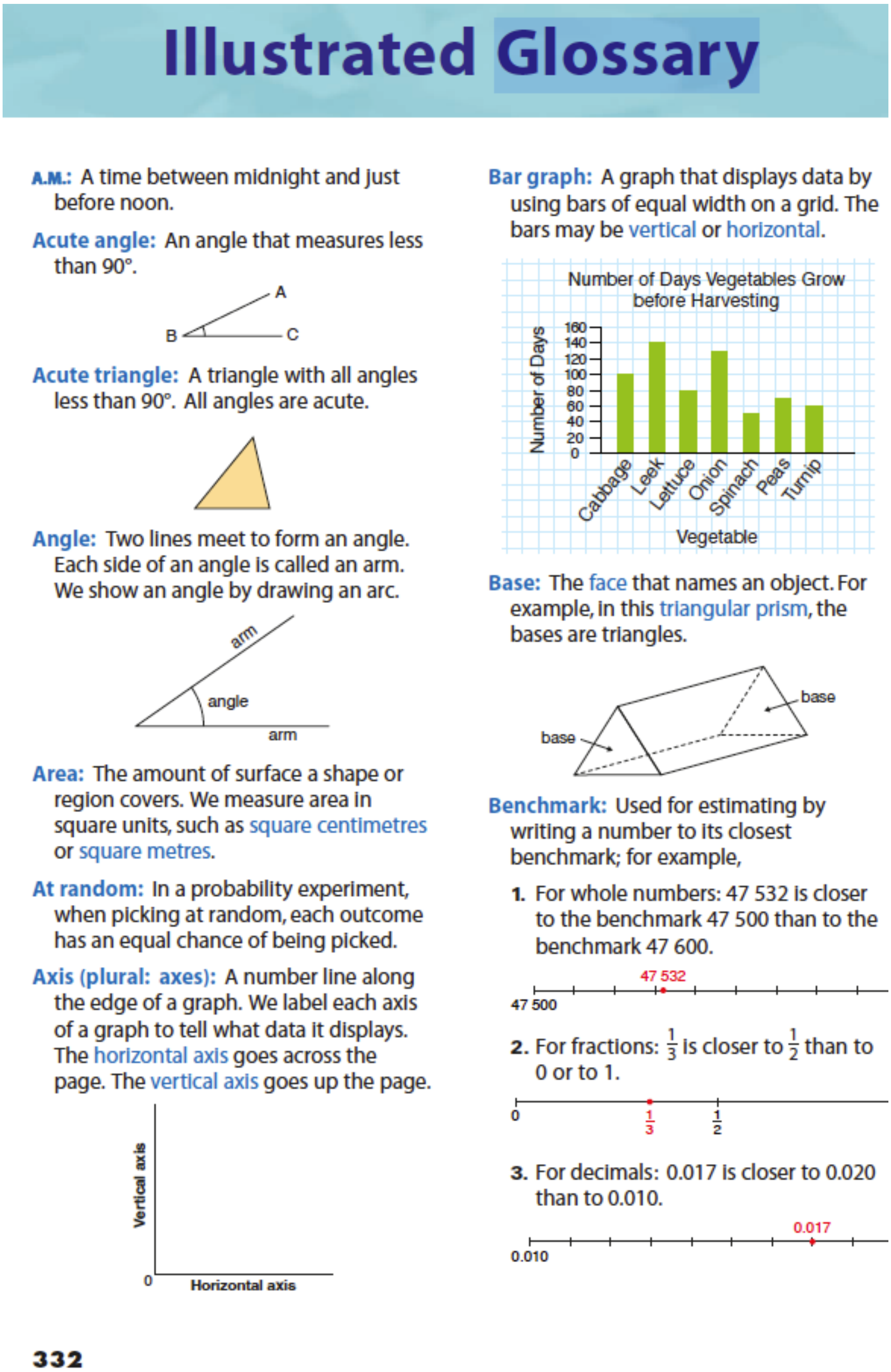
Circle Graphs
In a circle graph, data are shown as parts of one whole.
The data are reported as a percent of the total, and the sum of the percents is 100%. The sum of the sector angles is 360°.

Unit Review 167

Source: MMS 7 (2007, p.167)

At the end of the book, an illustrated glossary is provided to define specific terms, along with an index that organizes topics for content location (see Figure 11). Each level of the series exhibits distinct features: MMS6 includes an “At Home” section (see Figure 15); MMS7 contains a section on Reading and Writing in Math (see Figure 14); MMS8 features a section titled Strategies for Success (see Figure 13); and MMS9 incorporates “Start Where You Are” and Study Guide sections at the beginning of each unit (see Figure 12). These differences reflect the incorporation of complementary resources that correspond to the varying demands of each age group or educational stage.

Figure 11 - Example of Illustrated Glossary



Illustrated Glossary

A.M.: A time between midnight and just before noon.

Acute angle: An angle that measures less than 90°.

A
B C

Acute triangle: A triangle with all angles less than 90°. All angles are acute.

Angle: Two lines meet to form an angle. Each side of an angle is called an arm. We show an angle by drawing an arc.

arm
angle
arm

Area: The amount of surface a shape or region covers. We measure area in square units, such as square centimetres or square metres.

At random: In a probability experiment, when picking at random, each outcome has an equal chance of being picked.

Axis (plural: axes): A number line along the edge of a graph. We label each axis of a graph to tell what data it displays. The horizontal axis goes across the page. The vertical axis goes up the page.

Vertical axis
0
Horizontal axis

Bar graph: A graph that displays data by using bars of equal width on a grid. The bars may be vertical or horizontal.

Number of Days Vegetables Grow before Harvesting
Number of Days
160 140 120 100 80 60 40 20 0
Cabbage Leek Lettuce Onion Spinach Peas Turnip
Vegetable

Base: The face that names an object. For example, in this triangular prism, the bases are triangles.

base
base

Benchmark: Used for estimating by writing a number to its closest benchmark; for example,

1. For whole numbers: 47 532 is closer to the benchmark 47 500 than to the benchmark 47 600.

47 532
47 500

2. For fractions: $\frac{1}{3}$ is closer to $\frac{1}{2}$ than to 0 or to 1.

0 $\frac{1}{3}$ $\frac{1}{2}$

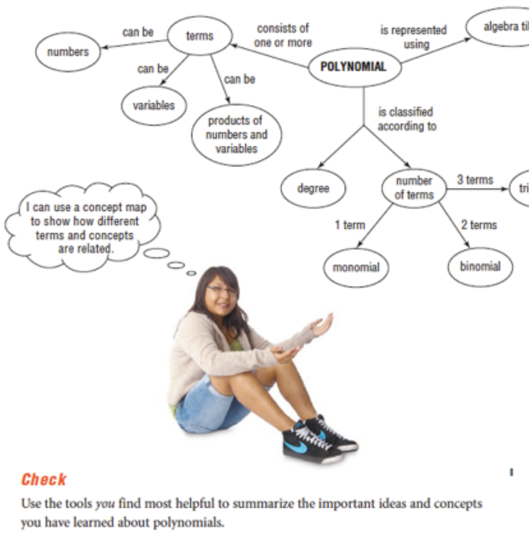
3. For decimals: 0.017 is closer to 0.020 than to 0.010.

0.017
0.010

332

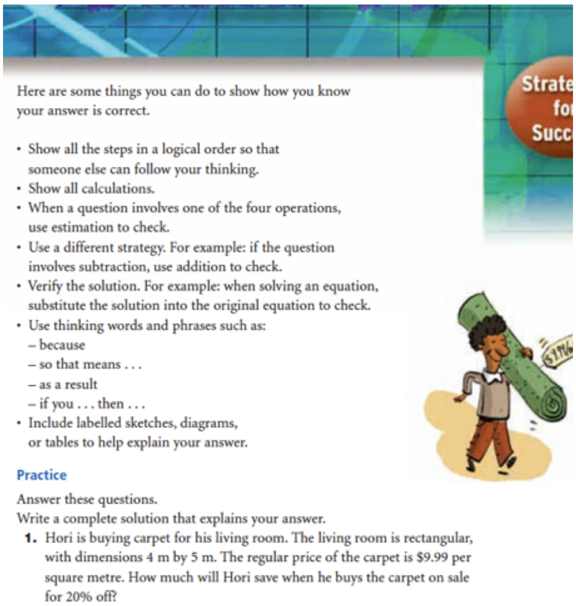
Source: MMS 6 (2007, p.332)

Figure 12 - Example of activities Start Where you Are



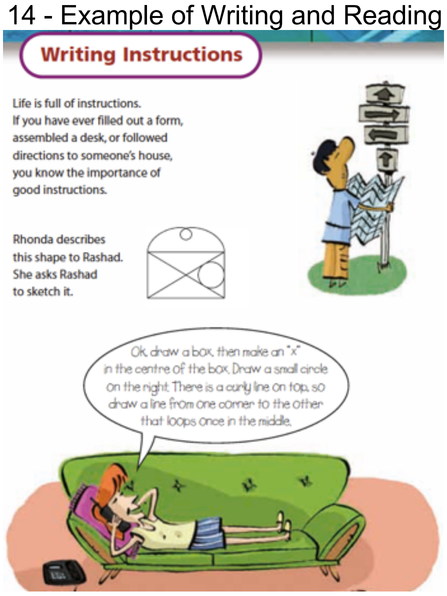
Source: MMS 9 (2009, p.148)

Figure 13 - Example Strategies for Sucess



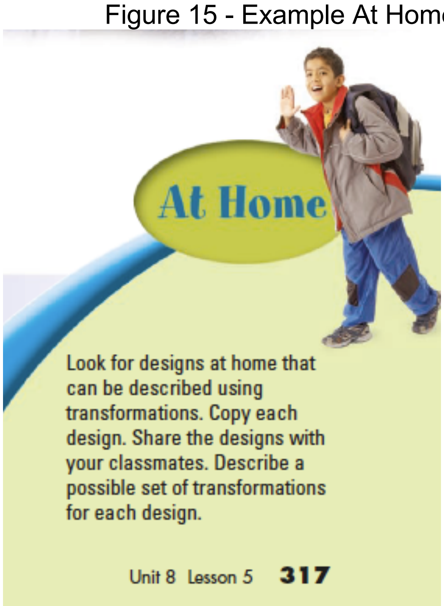
Source: MMS 8 (2008, p.307)

Figure 14 - Example of Writing and Reading



Source: MMS 7 (2007, p.118)

Figure 15 - Example At Home



Source: MMS 6 (2007, p.317)

From the presentation of the four volumes of the MMS collection, we can see a didactic organization that includes Launch activities, Explore and Show and Share sections, Practice Questions exercises and Reflect moments.

In addition, each level of the series features additional resources, such as the Strategies Toolkit, interactive games, unit and cumulative reviews, an illustrated glossary and specific sections (for example, At Home, Reading and Writing in Math, Strategies for Success or Start Where You Are) distributed throughout the units, provide subsidies for studying how mathematical content is introduced, explored and returned to in multiple learning moments.

In view of the variety and internal structure of the thematic units in the analyzed series, the subsequent phase provided a detailed account of the procedures for data collection and production. The following subsection outlines the methods used to select and organize the materials (the four volumes of the MMS collection), as well as the recording and documentation strategies that support the analysis of how Canadian sociocultural elements could be identified and potentially recontextualized in Brazilian multilingual schools.

Consistent with Bardin (2016), determining the corpus and defining analytical categories at this stage facilitates the identification of fundamental themes and patterns for the interpretive investigation conducted in the next sections, thus shedding light on the implications of transferring and using foreign textbooks in the local context.

5.2.2 Data processing, coding and interpretation

Based on the initial mapping of the MMS collection, a systematic, iterative approach was applied to analyze the textbooks. Each volume was first examined structurally, with sections categorized and recorded in spreadsheets (Excel, Microsoft Office 365, 2024) to facilitate the identification of content organization patterns and variations. This step included comparing recurring features such as the frequency of “Unit Problems,” “Games,” and “Technology” sections, as well as classifying tasks according to Litoldo and Amaral-Schio (2021).

Data coding and analysis highlighted not only each book’s internal organization and conceptual progression but also served as the basis for a comparative assessment of MMS6, MMS7, MMS8, and MMS9. Following Bardin’s (2016) guidelines for a systematic understanding of messages, the categories identified were linked to the research questions, particularly to examine how Canadian sociocultural references appear in the books and can be adapted for multilingual Brazilian schools.

The results of this analysis are organized into distinct categories that address both the structural organization and the content presentation of the textbooks. These findings provide a basis for discussing the extent to which the MMS collection reflects the intended pedagogical and curricular orientations, as well as how cultural aspects are embedded within the materials. The forthcoming discussion will relate these findings to the broader research objectives, examining potential implications for educational practices in Brazilian multilingual settings.

This methodological process made it possible to identify the sections that make up each volume, as well as to analyze similarities and differences between them, as discussed in subsection 4.2.2. These analyses made it possible to understand the progressive development of the contents within the MMS collection and to observe the extent to which its editorial choices dialogue with the curriculum guidelines and with theoretical references in mathematics education.

It was observed that MMS6, MMS7 and MMS8 are organized into eight units each, while MMS9 has nine units. These units are made up of various sections, which fulfill different pedagogical functions throughout the material. Table 2 shows some of these sections and their distribution in the books.

Table 2 - Distribution of units within the MMS collection

Section name	Quantification
Game	28
Investigation	7
Mid-Unit Review	25
Practice Test	25
Project	3
Reading and Writing in Math	8
Review	9
Start Where You Are	9
Strategies for Success	8
Study Guide	9
Technology	23
The World of Work	2
Unit Problem	33

Section name	Quantification
Unit Review	24

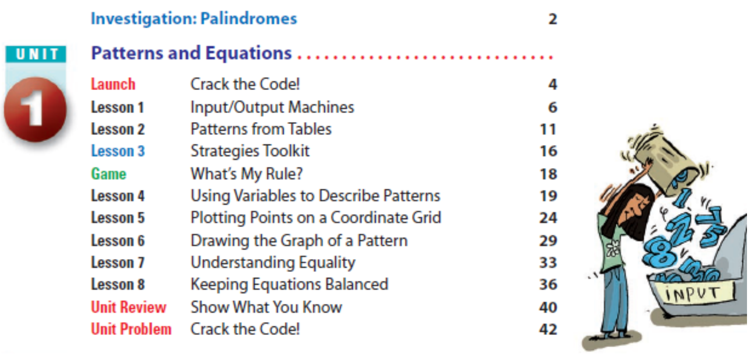
Source: The author (2025)

Table 2 shows some reflections on the structure of the textbooks analyzed. Some sections, such as Unit Problem (33 occurrences), Game (28 occurrences) and Technology (23 occurrences), are more frequent, while others, such as Project (3 occurrences) and The World of Work (2 occurrences), are more occasional.

The sections Mid-Unit Review (25 occurrences), Practice Test (25 occurrences) and Unit Review (24 occurrences) occur with considerable frequency, suggesting an emphasis on continuous revision of content. Sections such as Reading and Writing in Math (8 occurrences) and Strategies for Success (8 occurrences) appear in smaller numbers, but indicate an effort to integrate complementary skills such as reading, writing and study strategies.

The similarities and differences between the books in the MMS collection were also analyzed. It was found that they all start their units with the Investigation section (in the case of MMS6, MMS7 and MMS8) or Project (in the case of MMS9). After this initial section, the organization differs: the Launch section is present in all volumes, except for MMS6, where the units are structured in Lessons. On the other hand, in books MMS7, MMS8 and MMS9, each unit is subdivided into sub-units, indicating a different approach to presenting the content. Figure 16 illustrates these structural specificities.

Figure 16 - Summary of the content


Investigation: Palindromes 2

UNIT 1

Patterns and Equations

Launch	Crack the Code!	4
Lesson 1	Input/Output Machines	6
Lesson 2	Patterns from Tables	11
Lesson 3	Strategies Toolkit	16
Game	What's My Rule?	18
Lesson 4	Using Variables to Describe Patterns	19
Lesson 5	Plotting Points on a Coordinate Grid	24
Lesson 6	Drawing the Graph of a Pattern	29
Lesson 7	Understanding Equality	33
Lesson 8	Keeping Equations Balanced	36
Unit Review	Show What You Know	40
Unit Problem	Crack the Code!	42

Source: MMS6 (2009, p.vii)

Source: MMS (2009, p. v)

Another feature identified in all four MMS books, as shown in Figure 16, was the recurrence of the Game and Technology sections throughout the units. These proposals focused on playful exploration (in the case of Game) and the use of digital or technological resources (in the case of Technology), promoting both logical reasoning and student interaction with mathematical concepts in practical or virtual scenarios. In addition, it was noted that in MMS6, the units concluded with a Unit Review and then a Unit Problem, which suggested a systematic review of the content before a more extensive exercise, while MMS7, MMS8 and MMS9 also included sections such as Practice Test and Study Guide, expanding the possibilities for review and further study.

Also noteworthy was the Cumulative Review, present at the end of each set of three units in all volumes, creating moments of synthesis and consolidation that favored returning to previous content and connecting different themes. Each book also had an Illustrated Glossary, Index and Acknowledgements at the end, which made it easier to locate the topics studied and formally acknowledged the contributions made in preparing the materials.

After this preliminary check of the structural elements, we proceeded with the Content Analysis process, combining the phases proposed by Bardin (2016) - pre-analysis, exploration of the material and treatment of the results - to examine how the sections identified related to the research objectives and the framework adopted. During the full reading of the books (pre-analysis), both aspects aligned with the previously defined categories of analysis and peripheral elements that did not immediately fit the proposed questions were observed. This initial reading guided coding, and the subsequent categorization (exploration of the material) culminated in the definition of three categories of analysis, in line with gaps identified in the literature (Category 1) and with the theoretical framework adopted (Categories 2 and 3). These categories formed the basis of the reflections presented in the following chapters.

Additionally, it is important to note that the subcategory names draw inspiration from unit titles and sections featured in the MMS textbooks themselves, thus preserving the material's original thematic structure:

a) Category 1: Constructing Mathematical Meanings for Integers, Fractions, and Language: Bridging the Literature Gap and Recontextualizing Canadian Textbooks, which is broken down into:
    i. Discovering Integers: Models, Properties and Practices
    ii. Multiplying and Dividing without Mystery: The Magic of Models
    iii. Language and Visual Paths: Lines, Colored Pieces and Signs

In this category, we underscore the lack of specialized research on the use of foreign textbooks – particularly Canadian ones – in multilingual Brazilian contexts, highlighting the need to investigate how integers, fractions, and linguistic elements are presented and potentially recontextualized within these materials.

b) Category 2: The Sociocultural and Sociopolitical Dimensions in Mathematics: Sports, Art and Festivities as Learning Scenarios, subdivided into:
    i. From Hockey to Football: The Sporting Universe as a Learning Scenario
    ii. Art, Monuments and Architecture: When Geometry Comes to Life
    iii. Cultural Festivals and Regionalisms: When Mathematics Dresses Up as a Party

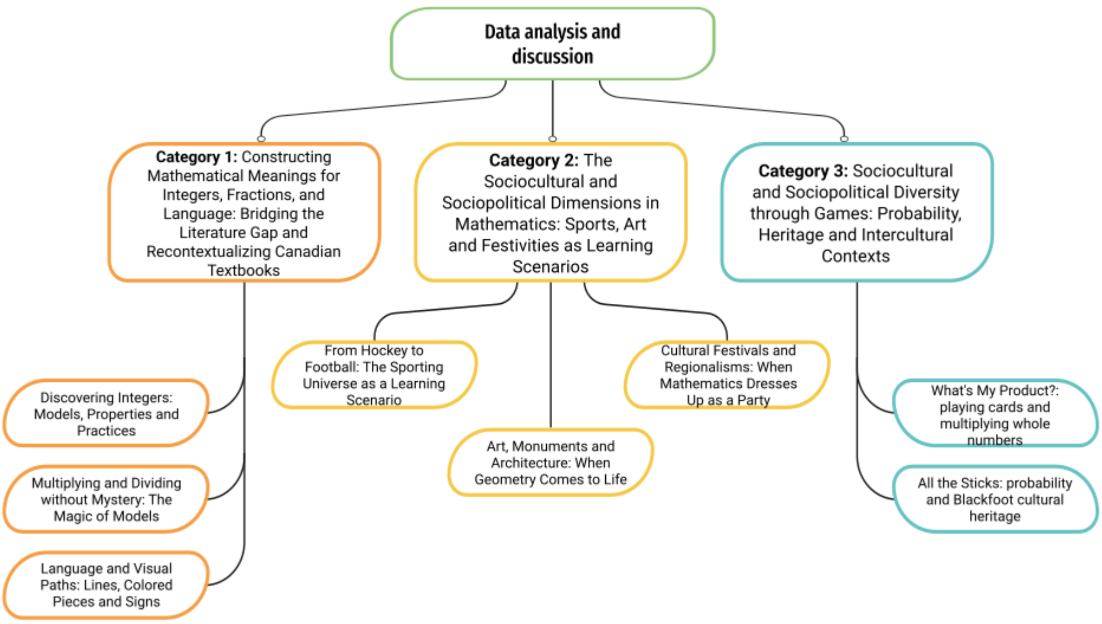
This category draws on sociocultural theories, exploring how everyday references – such as sports, art, and local festivities – can shape students' engagement with mathematical concepts in different socio-educational realities.

c) Category 3: Sociocultural and Sociopolitical Diversity through Games: Probability, Heritage and Intercultural Contexts, organized from:
   i. What's My Product?: playing cards and multiplying whole numbers
   ii. All the Sticks: probability and Blackfoot cultural heritage

By examining games and their sociocultural roots both local and indigenous, this category builds on the same theoretical framework and expands it, investigating how playful activities intersect with broader social and political dimensions of textbook adoption.

Based on these three categories, the analyses were developed on the basis of the theoretical framework pertinent to each one, with the next chapters detailing how these elements manifest themselves in the MMS collection. Figure 17 summarizes this structure of categories and subcategories of analysis, showing how the data was organized throughout the research process. The subdivision of Category 2 reflects a significant broadening of its scope, integrating different sociocultural contexts into the study.

Figure 17 - The Categories of Analysis



Source: The author (2025)

In the stage of structuring the analysis, methodological strategies were defined to organize and interpret the data in line with Bardin (2016), which provides for the selection of recording units or appropriate contexts, we chose to consider the tasks as central units of observation. The term tasks was defined on the basis of Litoldo and Amaral-Schio (2021), who conceptualize them as "activities to be solved by the student," describing such activities as "any type of proposal in the textbook that involves a solution process" (Litoldo & Amaral-Schio, 2021, p. 512, our translation). In the MMS collection, these tasks can be found in specific sections, such as Practice, Reflect, and Reflect & Share.

Thus, in line with the material exploration stage in Content Analysis (Bardin, 2016), a criterion was established for quantifying the tasks based on each item or question present in specific sections. Thus, if the "Practice" section contained three items, each would be treated as an independent task. The aim of this practice was to make the examination more detailed, so as to allow a thorough observation of the variations and strategies proposed in the books. At the same time, recording the occurrences and systematically organizing the data in tables or spreadsheets facilitated subsequent coding and categorization, in line with the procedures suggested by Bardin, to ensure greater clarity in identifying patterns and meanings.

Once the categorization phase was complete, we moved on to a qualitative analysis anchored in the theoretical framework defined for each of the three study categories. Although all the information was produced together throughout the process, the analytical presentation of each category was planned in specific sections, respecting the conceptual peculiarities of each one. By linking these results to the aims of the research, the aim was to investigate how foreign textbooks could be examined, appropriated or reinterpreted in Brazilian multilingual contexts, thus meeting the central problem of this thesis.

The definition of categories (integers, fractions and language; sociocultural and sociopolitical dimensions; and cultural diversity and games) emerged both from points of attention indicated in the literature and from systematic contact with the tasks in the exploration phase. Thus, the process ranged from the initial mapping (pre-analysis) and coding of excerpts and proposals (exploration) to the interpretative interweaving with the theoretical framework (treatment of results).

Finally, the methodological approach adopted aimed to point out ways of answering the question about the implications and potential of using Canadian textbooks in Brazil, particularly in the multilingual context. The application of the aforementioned steps addressed points that had been little explored in previous research, especially with regard to the treatment of whole numbers, fractions and the use of language, and at the same time deepened the debate on socio-cultural and socio-political dimensions and issues related to cultural diversity and games in the mathematics curriculum, reinforcing the importance of investigating foreign teaching materials in local educational contexts. It also responds to a notable gap in the literature (Category 1) by incorporating objective (d) – examining how these materials approach integers, fractions, and language – while highlighting sociocultural and sociopolitical dimensions (Category 2) and broadening the discussion to cultural diversity and games (Category 3).

6 ANALYSIS AND DISCUSSION OF THE DATA: A SOCIOCULTURAL AND SOCIOPOLITICAL PERSPECTIVE

The aim of this section is to present a summary of the data produced and the main reflections that guide the research. The aim is to show how the textbooks analyzed use visual models, games and everyday references to address topics such as whole numbers, fractions, geometry, and probability, linking these contents to cultural elements.

The aim is to show that these strategies can promote conceptual understanding and stimulate student engagement, although they require adaptation to the specific context of each school. In particular, the aim is to show that tasks linked to climatic and sporting characteristics typical of Canada may not connect with the reality of students in other settings. In addition, the aim is to point out that the presence of games and references to festivities, monuments and indigenous traditions indicates an attempt to diversify learning, even if this approach sometimes lacks the cultural and historical depth to generate greater identification.

Thus, although the books present consistent methodological proposals, their full potential depends on teacher mediation that contextualizes the content, promotes critical reflection, and values sociocultural plurality, making mathematics more alive and connected to the students' experience.

6.1 Constructing Mathematical Meanings for Integers, Fractions, and Language: Bridging the Literature Gap and Recontextualizing Canadian Textbooks

In this first category, we engage with the mathematical approaches in the MMS collection while addressing the literature gap on how integers, fractions, and language use are represented in foreign textbooks, incorporating objective (d) to examine how Canadian textbooks approach these topics in light of existing research. This focus connects directly to the final guiding question of this dissertation – what cultural aspects of the Canadian context are embedded and made explicit in mathematics textbooks, and how can these elements be interpreted in the context of multilingual schools in Brazil? By examining how the MMS materials employ models for conceptual development, multiple visual representations, and specific lexical choices in a multilingual environment, we also reflect the general objective of exploring how sociocultural differences in foreign textbooks might influence Brazilian educational settings.

Moreover, this category aligns also with specific objectives (a) and (b), which aim to analyze sociocultural and linguistic aspects present in Canadian textbooks and their potential interplay with Brazilian contexts. As we delve into the subcategories on operations with integers, fractions, and the relationship between mathematics and language, we show how the collection frames these concepts in ways that may invite recontextualization or adaptation for local realities.

6.1.1 Discovering integers: models, properties, and practices

The teaching of whole numbers in mathematics education has been the focus of various investigations, often highlighting how the study of numerical sets and their operations lays groundwork for subsequent mathematical concepts. Several studies address these topics by examining, for instance, bibliographic surveys on the teaching of integers in Brazil (Rubin et al., 2014), the influence of textbooks (Besen, 2006), games as an instructional resource (Oliveira, 2022), models for developing operations with integers (Menon, 2015; Rubin et al., 2014), and future teachers' knowledge in this domain (Reeder & Bateiha, 2016). From a historical perspective, Moretti (2012) notes that the sign rule for multiplication was introduced by Diophantus of Alexandria without proof and later rigorously demonstrated by Hankel in 1867, underscoring its mathematical relevance and the ongoing challenges it presents in teaching.

In the textbook MMS, the second unit, titled "Integers," addresses operations with integers, specifically multiplication and division. This unit clarifies integer concepts and outlines multiplication and division procedures through models and rules (Figure 18). It also discusses order of operations and includes classroom activities such as "What Is My Product?" along with strategies for problem solving. Although these activities diversify approaches to integer operations, there is limited critical reflection on whether they accommodate different learning contexts.

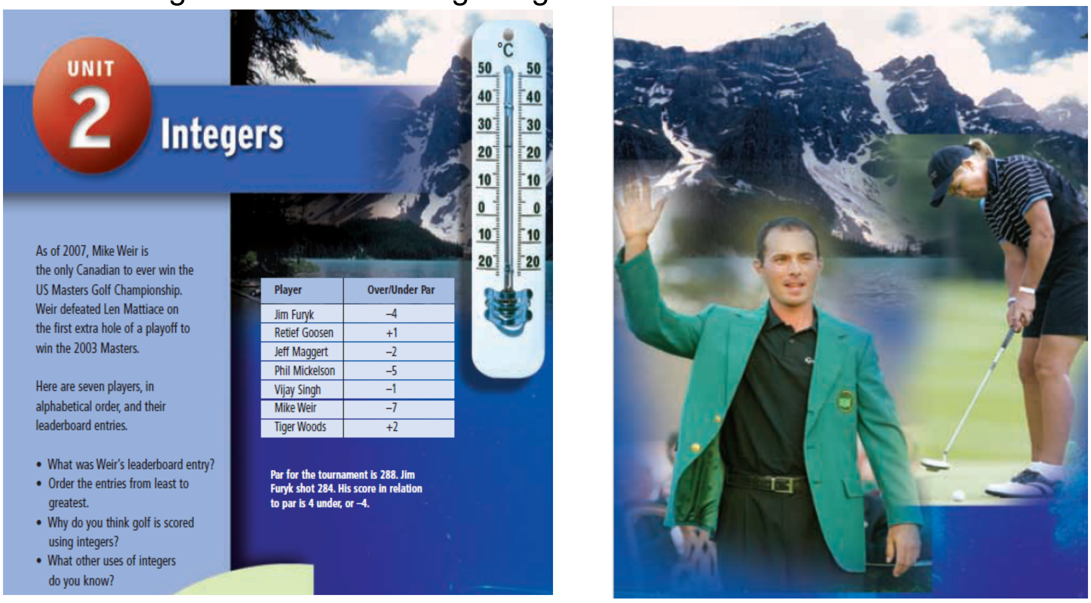
Figura 18 - Unit Overview Integers no MMS8

UNIT 2 **Integers**

Source: MMS8 (2009, p. v)

In addition to numerical exercises, the unit uses contexts involving sports, people, and everyday situations (Figure 19). Golf is one of the featured themes, referencing well-known figures in the sport and events such as the US Masters Golf Championship. While these examples highlight rankings and scores to illustrate integer applications, Moretti (2012) points out that traditional models – like "gain/loss" or "past/future" – do not necessarily address all conceptual difficulties, especially the formal properties of negative numbers and their multiplication rules.

Figura 19 - Associating Integers with the World of Golf

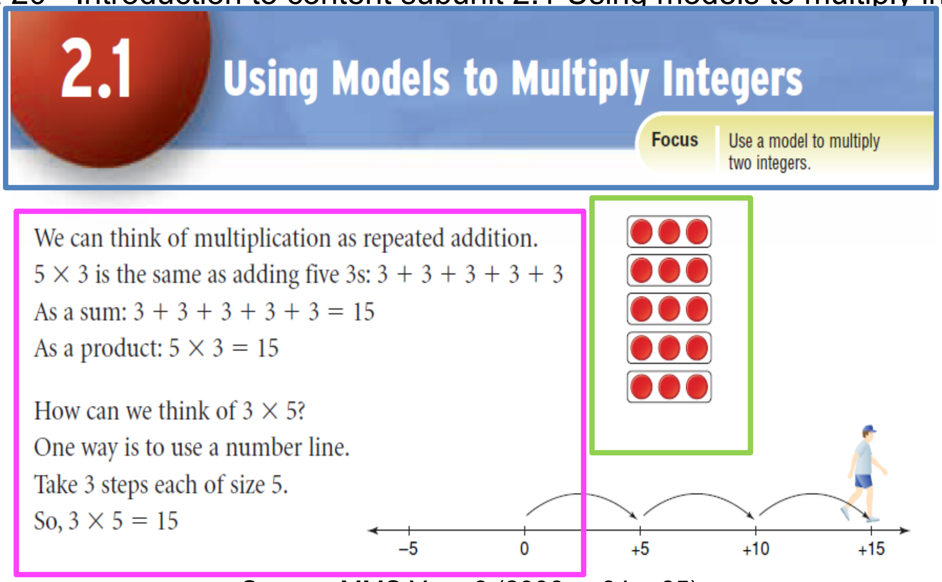


Source: MMS8 (2009, p. 62-62)

The unit's primary objective is to develop students' competence in working with positive and negative integers through various operations, particularly multiplication and division. Concepts progress from foundational ideas to more applied tasks, drawing on models, visual resources, and pattern recognition. However, a stronger emphasis on the extension principle (Moretti, 2012) – the idea of preserving distributive properties from positive to negative numbers – might clarify why the sign rule for multiplication takes its usual form.

Initially, the unit focuses on models for multiplying integers (Figure 20). Subunit 2.1, "Using Models to Multiply Whole Numbers," illustrates multiplication as repeated addition, which aligns with certain historical perspectives. Moretti (2012) cautions, however, that while repeated addition can be a starting point, it may not capture the full complexity of multiplication when extended to negative factors. The material offers examples and exercises that can reinforce procedural understanding, but the text does not address potential conceptual gaps that could arise when students move beyond concrete representations to more formal algebraic contexts.

Figura 20 - Introduction to content subunit 2.1 Using models to multiply integers



Source: MMS Year 8 (2006, p.64 – 65).

At the beginning, the students are shown the representation of 5 × 3 by the sum of five parts equal to 3, which favors an intuitive understanding of multiplication based on the concept of repeated addition (highlighted in blue in Figure 20).

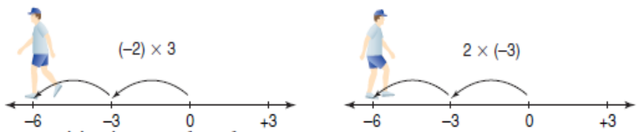
The representation of 5 × 3, through the rectangular layout (highlighted in green). In addition to these, the number line appears as a third representation (highlighted in pink), and the latter is explored in the sequence indicating work with pairs (Figure 21).

Figure 21 - Exploring the Number Line with the Investigate Proposal

Investigate

Work with a partner.
You will need masking tape and a metre stick.
Make a large number line across the floor.
Divide the line into intervals of 15 cm.
Label the line from −15 to +15.

- Walk the line to multiply integers.
  – Start at 0.
  – For negative numbers of steps, face the negative end of the line before walking.
  – For negative step sizes, walk backward.
  For example:



- Choose 2 different positive integers less than +5.
- Find all possible products of the integers and their opposites.
  Take turns. One partner walks the line to find each product.
  The other partner records on a number line and writes the multiplication equation each time.

Reflect & Share

Share your results with the class. What patterns do you notice?
How can you predict the product of two integers?

Source: MMS8 (2009, p. 64)

The Investigate proposal, followed by the Reflect and Share section (Figure 21), uses the number line as a central representation to explore the multiplication of whole numbers, taking into account positive and negative signs. This practice, designed to be done in pairs, encourages students to work collectively to understand fundamental concepts of arithmetic (Vygotsky, 1998), while using the number line to visualize mathematical operations, namely multiplication, and their results.

The proposal begins with the construction of a number line, marked out in regular 15 cm intervals and labeled -15 to +15. Students are instructed to start at ground zero and move to the right or left, depending on the sign and number factor. Moving to the right represents positive steps, while moving to the left represents negative steps. This practical and kinesthetic approach not only illustrates the abstract concepts of multiplication, but also reinforces the relationship between numbers and direction on the number line, facilitating the construction of meaning.

Multiplication of integers is further explored by varying the signs of the factors. For example, in the case of $(-2) \times 3$ the student walks backwards two units of 15 cm each, repeating the movement three times, ending up at position -6. Similarly, the multiplication $(-2) \times 3$ demonstrates the reversal of direction on the number line.

This methodology helps students understand how the sign of the product of two integers is determined, promoting the recognition of mathematical patterns, such as: a) the multiplication of two numbers with equal signs results in a positive product and b) the multiplication of numbers with opposite signs results in a negative product.

In addition, by asking students to share their observations and conclusions about the patterns, the proposal fosters the development of critical thinking, mathematical argumentation and collaboration in the classroom. The playful and interactive practice contributes to the internalization of concepts in a meaningful way, going beyond the simple memorization of rules.

Thus, it is considered that this proposal is a relevant example of how mathematics can be taught through interactive and concrete practices. highlight that the use of practices based on activities and interactions in the classroom promotes the development of critical thinking, mathematical argumentation and collaboration, facilitating the meaningful understanding of mathematical concepts, such as whole numbers, rather than relying on the memorization of rules.

The work on multiplication between whole numbers continues with the section entitled Connect, which now explores the representation of multiplication via a rectangular layout, using colored pieces to model whole numbers and operations (Figure 22).

The colored pieces, yellow +1 and red -1, refer to the property of the opposite or symmetrical element. This property exists in whole numbers, but not in natural numbers. The combination of these two pieces forms the neutral element, represented by the numeral one (Figure 22, highlighted in red). The 'commands' for modeling are mentioned below (Figure 22, highlighted in green).

Figure 22 - Colored Pieces to Represent Operations with Integers

Connect

Recall that we can use coloured tiles to model integers.
One yellow tile models +1, and one red tile models –1.

They combine to form a zero pair: (+1) + (–1) = 0

We can extend our use of coloured tiles to model the multiplication of two integers
Let a circle represent the "bank." Start with the bank having zero value.
The first integer tells us to deposit (put in) or to withdraw (take out).
When the first integer is positive, put tiles in.
When the first integer is negative, take tiles out.
The second integer tells us what to put in or take out.

Source: MMS8 (2009, p. 65)

Figure 23 shows the four possible ways of multiplying whole numbers relative to the signs of their numerals presented in the book: 1) $(+4) \times (+3)$; 2) $(+4) \times (-3)$; 3) $(-4) \times (-3)$ 4) $(-4) \times (+3)$. The Possibilities provided not only illustrate integer multiplication using a visual approach (colored tiles), but also implicitly address several fundamental mathematical properties.

These properties are important for building algebraic reasoning and for a deep understanding of multiplication, especially in the context of whole numbers. The mathematical properties underlying multiplication present in the examples in Figure 23 are: commutative, distributive over addition, inverse element and neutral element (Stewart et al., 2015).

Figure 23 - Four Multiplication Scenarios: Visualizing Signals and Properties

Possibilidade 1

➤ Multiply: (+4) × (+3)
+4 is a positive integer.
+3 is modelled with 3 yellow tiles.
So, *put* 4 sets of 3 yellow tiles *into* the circle.

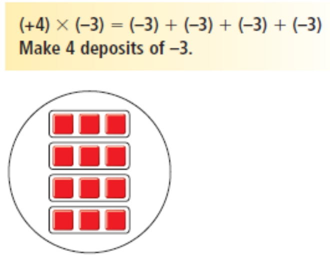
There are 12 yellow tiles in the circle.
They represent +12.
So, (+4) × (+3) = +12

(+4) × (+3) = (+3) + (+3) + (+3) + (+3)
Make 4 deposits of +3.

Possibilidade 2

➤ Multiply: (+4) × (–3)
+4 is a positive integer.
–3 is modelled with 3 red tiles.
So, *put* 4 sets of 3 red tiles *into* the circle.

There are 12 red tiles in the circle.
They represent –12.
So, (+4) × (–3) = –12

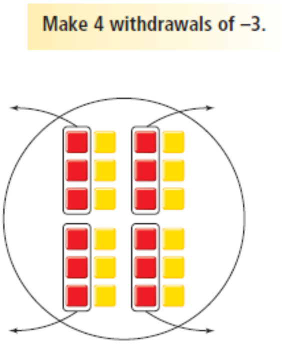


Possibilidade 3

➤ Multiply: (–4) × (–3)
–4 is a negative integer.
–3 is modelled with 3 red tiles.
So, *take* 4 sets of 3 red tiles *out* of the circle.
There are no red tiles to take out.

So, add zero pairs until there are enough red tiles to remove.
Add 12 zero pairs.
Take out 4 sets of 3 red tiles.

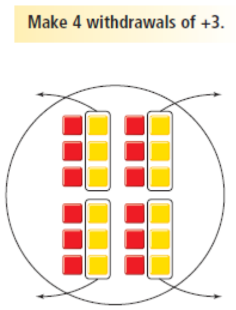
There are 12 yellow tiles left in the circle.
They represent +12.
So, (–4) × (–3) = +12



Possibilidade 4

➤ Multiply: (–4) × (+3)
–4 is a negative integer.
+3 is modelled with 3 yellow tiles.
So, take 4 sets of 3 yellow tiles out of the circle.
There are no yellow tiles to take out.
So, add zero pairs until there are enough yellow tiles to remove.
Add 12 zero pairs.
Take out 4 sets of 3 yellow tiles.

There are 12 red tiles left in the circle.
They represent –12.
So, (–4) × (+3) = –12



Source: MMS8 (2009, p. 65-66)

The Commutative Property of Multiplication refers to the fact that the order of the factors does not alter the product. For example, $(+4) \times (+3) = (+3) \times (+4) = +12$. Although the possibilities provided in the book (Figure 23) do not directly explore this commutation, the use of sets (tokens) suggests that the number of elements grouped does not depend on the order of the factors.

It is possible to think of Possibility 1 as exploiting this property by investigating that it can be physically demonstrated by reversing the roles of the number of groups and the number of elements per group, showing that the result remains the same.

The Distributive Property of Multiplication over Addition states that, given a number a and a sum b + c, the multiplication of a by the sum can be distributed, i.e: $a \times (b + c) = (a \times b) + (a \times c)$. In the context of Possibilities 2, 3 and 4 (Figure 23), this property is worked out implicitly, since, for example, the multiplication $(+4) \times (-3)$ is represented as a repeated sum of equal terms: $(-3) + (-3) + (-3) + (-3) = +12$. The notion of division into repeated sets is explained in the image of the red cards as the sum of four equal sets of -3: $(+4) \times (-3) = (-3) + (-3) + (-3) + (-3)$. Here, the number +4 represents the number of sets, while the number -3 represents the value of each set (or parcel).

This situation deals with the distributive over an implicit sum, which means that this example exploits the same principle of distribution by breaking down multiplication into a sum of equal parts. In this case: $(+4) \times (-3) = (-3) + (-3) + (-3) + (-3)$. The central idea of the distributive property is that a multiplication can be "distributed" into smaller sums, as can be seen with the red (negative) tokens.

In Possibility 2 (Figure 23), the red tokens are organized into four groups of three elements each, reinforcing the idea that multiplication is equivalent to the repeated sum of equal values. The circle containing the sets helps the student to see that the multiplication operation is not something abstract, but can be broken down into concrete steps, such as, put 4 sets of 3 red pieces in the circle.

Thus, by observing that the four sets add up to -12, it is possible for students to develop an understanding that the number +4 acts as a multiplier (or repeater) of -3, which is directly aligned with the distributive concept: $a \times b = b + b + b + ..$ (repeated several times).

Possibilities 2, 3 and 4, which make it possible to work with this property, albeit implicitly, promote the student's development of an initial intuition for the distributive property by allowing them to: (a) visualize multiplication as a sum of repeated portions, (b) see that the product can be constructed incrementally, distributing +4 over the sums of -3 and, (c) understand that the negative sign in the factor -3 is preserved and accumulates, resulting in a negative product -12.

In addition, the representation with tokens contributes to the development of logical reasoning and can prepare students to work with more complex distributions, such as: $(+4) \times [(-2) + (-1)] = [(+4) \times (-2)] + [(+4) \times (-1)]$.

Although Possibility 2 (Figure 23) is restricted to a single multiplied factor (-3), it can easily be extended to scenarios where explicit sums are worked on, exploiting the distributive directly. As: $(+4) \times (-3) = (+4) \times [(-2) + (-1)] = [(+4) \times (-2)] + [(+4) \times (-1)]$.

In this case, each partial sum could be represented by separate cards, further reinforcing the concept.

It is therefore considered that Possibilities 2, 3 and 4 presented by the book provide a practical basis for developing understanding of the Distributive Property of Multiplication over Addition, connecting it to a concrete visual representation and helping students to build solid intuitions before moving on to abstract representations.

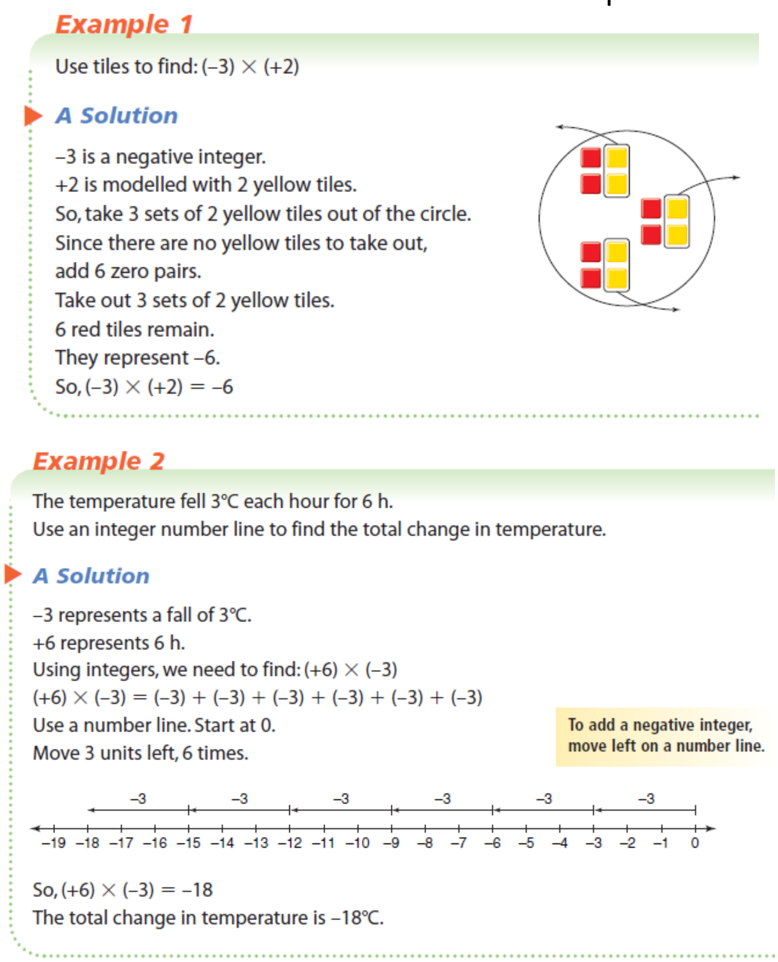
Regarding the Inverse Element Property of Multiplication, multiplying any number by 1 result in the number itself. Although this property is not explicit in the possibilities presented, it is implicit in the idea of sets containing a single element. For example, $(+4) \times (+1)$ would represent four sets of a yellow token, resulting in +4.

Finally, the Neutral Element of Multiplication Property refers to any number multiplied by 0 resulting in 0. Likewise, this property is not presented directly in the examples, however, it can be inferred from the absence of tokens. If no set is created, the product will be zero regardless of the number of tokens per set.

Following the presentation of these possibilities, the book shows two examples (Figure 24). Example 1 refers to the process of solving a multiplication between integers via modeling with colored chips. The second example addresses the process of solving multiplication by modeling the number line.

It's important to note that while in the first example the multiplication context doesn't involve everyday situations, but is limited to the use of yellow and red chips (as in the four possibilities illustrated in Figure 24), the second example refers to a context related to temperature, associating negative values with cold and positive values with heat, thus anchoring itself to the number line.

Figure 24 - From Concrete to Abstract: Colored Chips and Number Line



Example 1

Use tiles to find: (–3) × (+2)

A Solution

–3 is a negative integer.
+2 is modelled with 2 yellow tiles.
So, take 3 sets of 2 yellow tiles out of the circle.
Since there are no yellow tiles to take out, add 6 zero pairs.
Take out 3 sets of 2 yellow tiles.
6 red tiles remain.
They represent –6.
So, (–3) × (+2) = –6

Example 2

The temperature fell 3°C each hour for 6 h.
Use an integer number line to find the total change in temperature.

A Solution

–3 represents a fall of 3°C.
+6 represents 6 h.
Using integers, we need to find: (+6) × (–3)
(+6) × (–3) = (–3) + (–3) + (–3) + (–3) + (–3) + (–3)
Use a number line. Start at 0.
Move 3 units left, 6 times.

To add a negative integer, move left on a number line.

So, (+6) × (–3) = –18
The total change in temperature is –18°C.

Source: MMS8 (2009, p. 66-67)

The purpose of these two examples is to encourage students to think about different possible models for solving multiplications between whole numbers. The space provided by the book to stimulate this reflection consists of Discuss the ideas, in which the students' preference for certain models is questioned and the possibility of other models is explored.

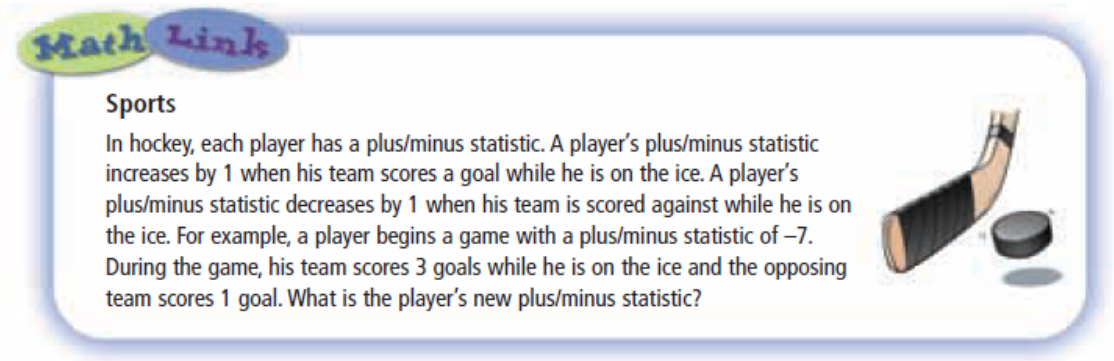
1. Which model do you find easiest to use to multiply integers?
2. What other models can you think of?

3. What do you notice about the effect of the order of the integers on the product?
4. Use a model to explain multiplication of an integer by 0 (MMS8, 2009, p. 67)

At this point, the book directly introduces the discussion of important properties, such as commutativity and the neutral element of multiplication. This is evident in the questions posed to the student, for example: 3. What do you notice about the effect of the order of the integers on the product?? and 4. Use a model to explain multiplication of an integer by 0. These questions aim to deepen conceptual understanding and promote reflection on these properties.

To conclude subunit 2.1, Using models to multiply whole numbers, the book presents a box entitled Math Links (Figure 25). The proposal presented uses a sporting context, hockey, to introduce and explore the addition/subtraction of whole numbers in an everyday situation. The task contextualizes a player's plus/minus statistic, which varies according to the goals scored or conceded by his team while he is on the pitch.

Figure 25 - Connecting Math to Hockey: Math Links and Integers


Math Link

Sports

In hockey, each player has a plus/minus statistic. A player's plus/minus statistic increases by 1 when his team scores a goal while he is on the ice. A player's plus/minus statistic decreases by 1 when his team is scored against while he is on the ice. For example, a player begins a game with a plus/minus statistic of −7. During the game, his team scores 3 goals while he is on the ice and the opposing team scores 1 goal. What is the player's new plus/minus statistic?

Source: MMS8 (2009, p. 67)

The mathematics behind this task is related to the addition and subtraction of whole numbers. The plus/minus system establishes two rules: 1) for each goal scored by the team while the player is on the pitch, +1 is added to his statistic and 2) for each goal conceded while he is on the pitch, -1 is subtracted from the statistic.

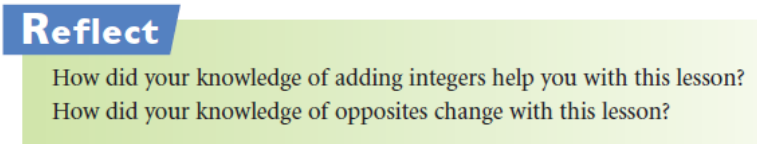
From the situation presented in the task, the player starts with a statistic of -7. His team scores 3 goals (+1 for each) and concedes 1 goal (-1 for each). Thus, the total change in the statistic is calculated as: $-7 + (+3) + (-1) = -7 + 2$resulting in the player ending up with a statistic of -5.

This task requires students to carry out sequential operations, understanding that the order of events (goals scored and conceded) does not alter the result, reinforcing the concept of associativity of the addition of whole numbers: $(-7 + 3) + (-1) = -7 + (3 - 1) = -7 + 2 = -5.$

It also allows them to explore how statistics vary depending on the team's performance and the impact this has on a player's individual assessment. By repeating the task with different scenarios (more goals conceded or scored), students can observe patterns such as: i) the predominance of goals scored leads to positive statistics, and ii) the predominance of goals conceded results in negative statistics.

Subunit 2.1, Using Models to Multiply Integers, concludes with a section entitled Practice, which includes various tasks coupled in sections such as Check, Apply, 15. Assessment Focus and Take It Further, followed by a box entitled Reflect (Figure 26), which offers students a moment to reflect on their learning, connecting previous knowledge to the experience developed throughout the tasks.

Figure 26 - Reflecting on Learning: Space for Metacognition


Reflect

How did your knowledge of adding integers help you with this lesson?
How did your knowledge of opposites change with this lesson?

Source: MMS8 (2009, p. 69)

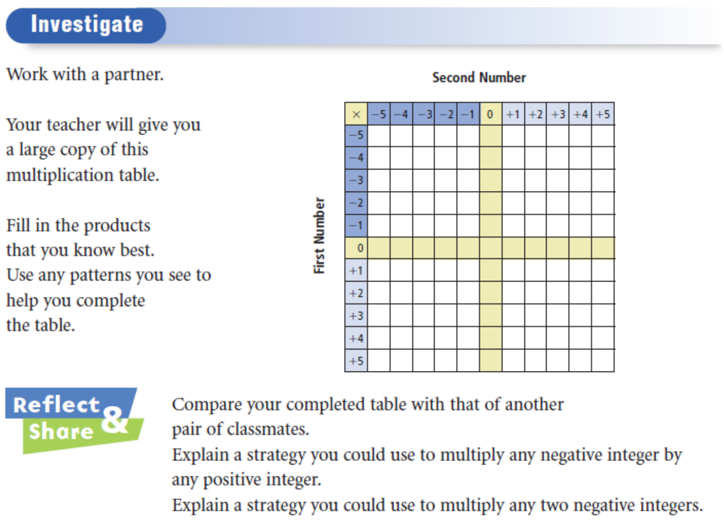
This structure not only consolidates the subject of integer multiplication, but also values the role of critical thinking and metacognition in the learning process. Menon (2015) points out that the structure for teaching integer multiplication, when supported by models and reflective practices, promotes the connection between previous knowledge and new learning experiences, highlighting the importance of critical thinking and metacognition for the consolidation of mathematical concepts.

Subunit 2.2, entitled Developing Rules to Multiply Integers, focuses on identifying patterns and constructing the sign rule for the multiplication operation between integers (MMS8, 2009, p. 70). For example, the subunit presents the fact that multiplying two integers of the same sign results in a positive product, while integers with opposite signs generate a negative product.

With a proposal entitled Investigate (Figure 27), the book offers the opportunity to work in pairs to explore and reinforce the rules for multiplying whole numbers by means of patterns identified in a multiplication table. This approach encourages the construction of knowledge based on observation and deduction, using a visual model to facilitate learning.

The mathematical objectives underlying this investigation lie in identifying patterns by exploring that the structure of the multiplication table allows students to observe and recognize regularities in the multiplication of positive and negative integers. The expected patterns include: multiplying two positive numbers results in a positive product; multiplying a positive number by a negative number generates a negative product, and vice versa; multiplying two negative numbers results in a positive product.

Figure 27 - Investigate Proposal: Discovering Patterns in Integer Multiplication



Source MMS8 (2009, p. 70)

These patterns provide a logical and visual basis for generalizing the rules of signs in multiplication. The proposal is that students formulate the rules for multiplying integers, as in Table 3:

Table 3 - Overview of the Sign Rules in Multiplication of Integers

1.	(+) × (+) = (+)	'Equal signs result in positive'
2.	(-) × (-) = (+)	
3.	(+) × (-) = (-)	'Different signs result in negative'
4.	(-) × (+) = (-)	

Source: The author (2025)

Identifying and validating these rules promotes conceptual understanding rather than mechanical memorization.

Pedagogically, the Investigate task provides an opportunity to work in pairs or groups, promoting discussion and the sharing of ideas, allowing students to compare their observations and reasoning. This approach helps develop mathematical communication skills.

This type of task gives students the responsibility of filling in the table and identifying patterns, rather than providing ready-made answers. This process activates autonomy, promotes logical thinking and encourages exploration.

According to Vygotsky (1998), learning is essentially a social process, where interaction between peers plays a central role in cognitive development.

The Zone of Proximal Development (ZDP), a key concept in his theory, refers to the space between what the student can achieve independently and what they can achieve with the collaboration of others. Pedagogical tasks such as Investigate, which involve working in pairs or groups, align directly with this perspective, as they allow students to learn from each other, discuss ideas and negotiate meanings.

This type of interaction not only reinforces mathematical communication skills, but also facilitates the internalization of concepts, fostering broader and deeper learning.

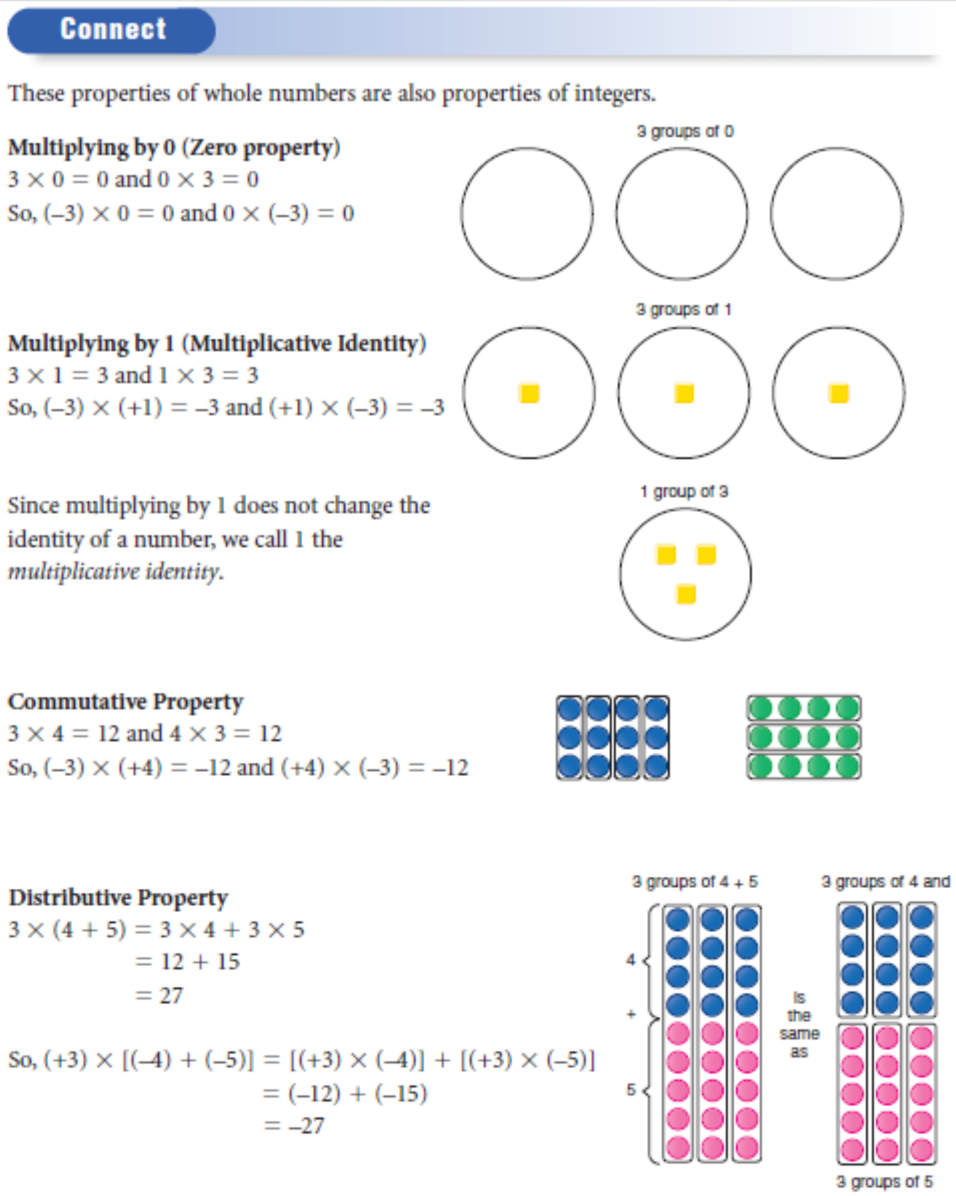
The use of a table to be filled in can be considered a visual tool that makes it possible to understand how integers are related in multiplication. This representation organizes the information clearly and gives a global view of the patterns. This contributes to the exploration of multiplication strategies, which is highlighted by the Reflect & Share box, which guides students to explain specific strategies for multiplying whole numbers.

Next, the book proposes the Connect section, which now directly provides work with the properties of multiplication. This sub-unit integrates and formalizes the properties of whole numbers (Figure 28).

The book's proposal, which links patterns and properties, seeks to provide a solid basis for students to assimilate the rules of integer multiplication in a cohesive, coherent way, based on comprehensive mathematical principles.

After this presentation, the book gives three examples in which the student is asked to Find each product. In each example, the book proposes a solution. Each one is considered.

Figure 28 - Connecting Properties: The Formalization of Integer Multiplication



Connect

These properties of whole numbers are also properties of integers.

Multiplying by 0 (Zero property)
$3 \times 0 = 0$ and $0 \times 3 = 0$
So, $(-3) \times 0 = 0$ and $0 \times (-3) = 0$

Multiplying by 1 (Multiplicative Identity)
$3 \times 1 = 3$ and $1 \times 3 = 3$
So, $(-3) \times (+1) = -3$ and $(+1) \times (-3) = -3$

Since multiplying by 1 does not change the identity of a number, we call 1 the *multiplicative identity*.

Commutative Property
$3 \times 4 = 12$ and $4 \times 3 = 12$
So, $(-3) \times (+4) = -12$ and $(+4) \times (-3) = -12$

Distributive Property

$$3 \times (4 + 5) = 3 \times 4 + 3 \times 5 = 12 + 15 = 27$$

So, $(+3) \times [(-4) + (-5)] = [(+3) \times (-4)] + [(+3) \times (-5)] = (-12) + (-15) = -27$

Source: MMS8 (2009, p. 71)

In Example 1 (Figure 29), the procedure involves two steps: i) considering the multiplication of the two positive integers; and ii) evaluating the signs of the multiplied integers and 'applying' the sign rule. In Example 2 (Figure 30), the proposal is to use the distributive property after rewriting the number -36 (in expanded form as $(-30) + (-6)$).

The highlight of these examples is the third (Figure 31), where for the first time, the book explicitly presents the rectangular layout as one of the possible models for considering the multiplication of integers. Here, the procedure is to start multiplication as a rectangular model of positive integers, and then evaluate the signs of the numbers and 'apply' the sign rule.

Figure 29 - Example 1: Steps to Multiply Two Positive Integers

Example 1

Find each product.
a) (–9) × (+4) b) (–4) × (–9) c) (+4) × (+9)

A Solution

a) Multiply the numbers as if they were positive.
9 × 4 = 36
The integers have opposite signs, so the product is negative.
So, (–9) × (+4) = –36

b) The integers have the same sign, so the product is positive.
So, (–4) × (–9) = +36

c) The integers have the same sign, so the product is positive.
So, (+4) × (+9) = +36

Figure 30 - Example 2: When the Distributive Enters the Scene in Integer Multiplication

Example 2

Find the product: (+20) × (–36)

A Solution

(+20) × (–36) = (+20) × [(–30) + (–6)]
= [(+20) × (–30)] + [(+20) × (–6)]
= (–600) + (–120)
= –720
So, (+20) × (–36) = –720

Write –36 in expanded form.
Use the distributive property.

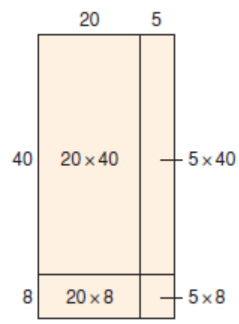
Figure 31 - Example 3: Modeling Integer Multiplication in Rectangular Format

Example 3

Find the product: (–25) × (–48)

A Solution

Multiply the numbers as if they were positive: 25 × 48
Use a rectangle model.
(25) × (48) = (20 × 40) + (5 × 40) + (20 × 8) + (5 × 8)
= 800 + 200 + 160 + 40
= 1200
The integers have the same sign, so the product is positive.
So, (–25) × (–48) = +1200



Source: MMS8 (2009, p. 72)

The Discuss the Ideas box again acts as a moment to consolidate learning, encouraging students to reflect on and systematize the sign rules in integer multiplication and to explain their strategies for solving these operations.

1. Think about the work of this lesson and the previous lesson.
What is the sign of the product when you multiply 2 integers:

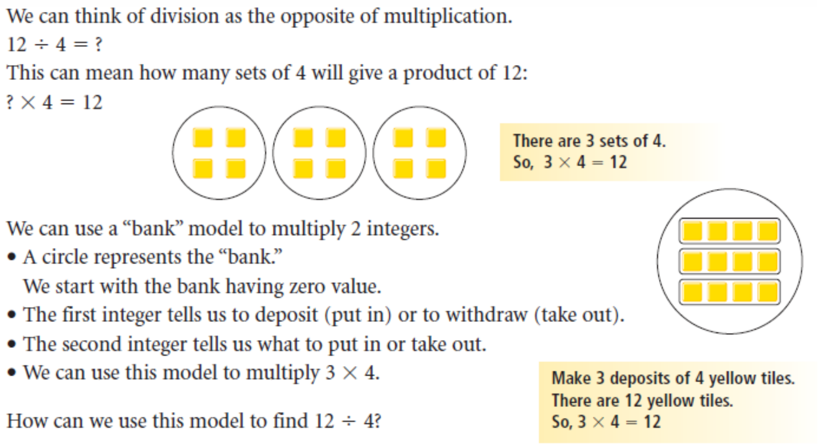
• if both integers are positive?
• if one integer is positive and the other integer is negative?
• if both integers are negative?
2. Explain your strategy to multiply 2 integers.
(MMS8, 2009, p. 73)

This approach encourages critical thinking, metacognition and the construction of clear explanations, allowing students to internalize the concepts explored throughout the subunit (Menon, 2015).

When analyzing the subsequent sections of the book, there is a gradual transition from concrete models and exploratory proposals to the establishment of formal rules, applicable to multi-step operations, including the introduction of integer division (MMS, 2009, p. 77).

Through visual models and structured tasks, the text explores division as the inverse of multiplication (Figure 32), returning to concepts already presented and deepening understanding in progressively more complex mathematical contexts, following the structure of the subunits: 2.4 Developing Rules to Divide Integers, whose focus is: Use patterns to develop the rules for dividing integers and, 2.5 Order of Operations with Integers, which aims to Apply the order of operations with integers.

Figure 32 - Integer Division: Exploring the Inverse of Multiplication



Source: MMS8 (2009, p.77)

In conclusion, Unit 2 addresses the study of multiplication with whole numbers using visual models and social learning strategies, based on fundamental principles of mathematical reasoning (National Governors

Association Center for Best Practices & Council of Chief State School Officers, 2010).

The introduction of materials such as playing cards, however, requires careful analysis in school environments, in order to respect different cultural norms and ensure the relevance of the resources adopted. In this sense, integrating peer interactions into collective learning requires careful teacher management, ensuring not only the equal participation of all students, but also the consolidation of a solid and meaningful understanding of the mathematical concepts involved.

6.1.2 Multiplying and dividing without mystery: the magic of models

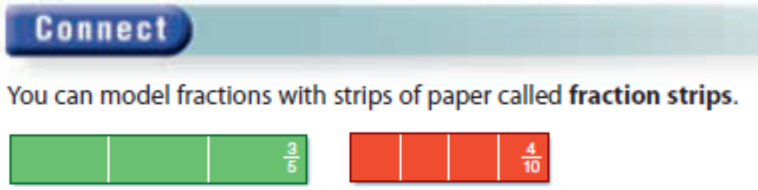
The concept of fraction remains one of the most complex in mathematics, posing challenges for students and teachers alike Behr et al. (1983); Getenet & Callingham (2017); Lamon, (2020) and Margolinas, (2014). This complexity tends to make these contents highly problematic, but also fundamental for learning during basic education.

The centrality of these notions in the development of mathematical thinking highlights their relevance in textbooks, as they are essential for understanding broad concepts such as proportion, ratio and numerical operations. In addition, visual representations of fractions play a fundamental role in mediating knowledge, allowing students to establish concrete connections between symbolic representations and everyday situations.

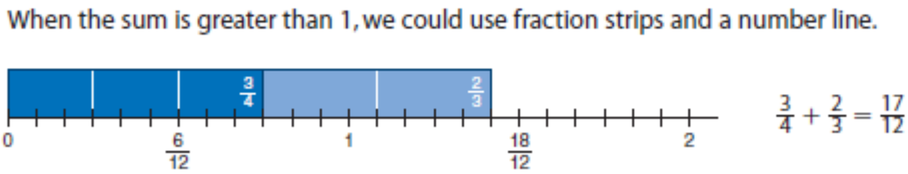
This approach is supported by studies in Mathematics Education, which emphasize the importance of different types of semiotic representation registers (Presmeg et al., 2016).

The textbooks analyzed highlight the potential of visual representations by integrating multiple models when working with operations, such as adding fractions. Examples include fraction bars, fraction strips, number lines and circle representations (Figure 16). These resources are used to explore concepts in an integrated way, highlighting the importance of diverse representations in developing mathematical understanding.

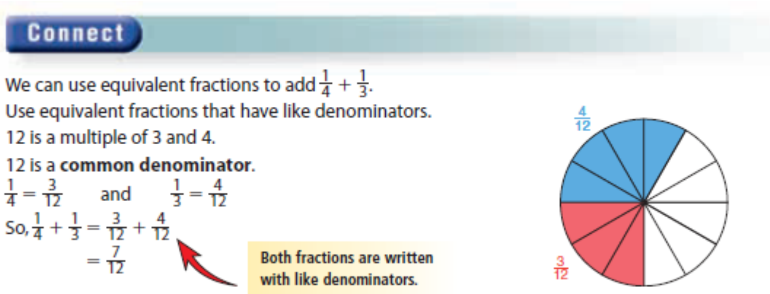
Figure 33 - Three Ways of Seeing Fractions: Bars, Circles and Number Lines



Source: MMS7 (2009, p. 181)



Source: MMS7 (2009, p. 182)



Source: MMS7 (2009, p. 186)

Studies such as Fan et al. (2013)as well as Rezat et al. (2021)point out that combining graphic elements, such as number lines, with textual explanations promotes a more robust integration between algebraic reasoning and geometric intuition. This approach allows students not only to perform numerical calculations, but also to visualize the results in a concrete way, such as aligning or overlapping segments on the number line. The inclusion of operations with fractions in textbooks, combined with the use of visual resources, proves to be pertinent not only for conceptual construction, but also to strengthen didactic clarity. The integration of multiple visual models contributes to reducing ambiguity and helps to form a “visual ballast” that supports mathematical understanding (Rezat et al., 2021).

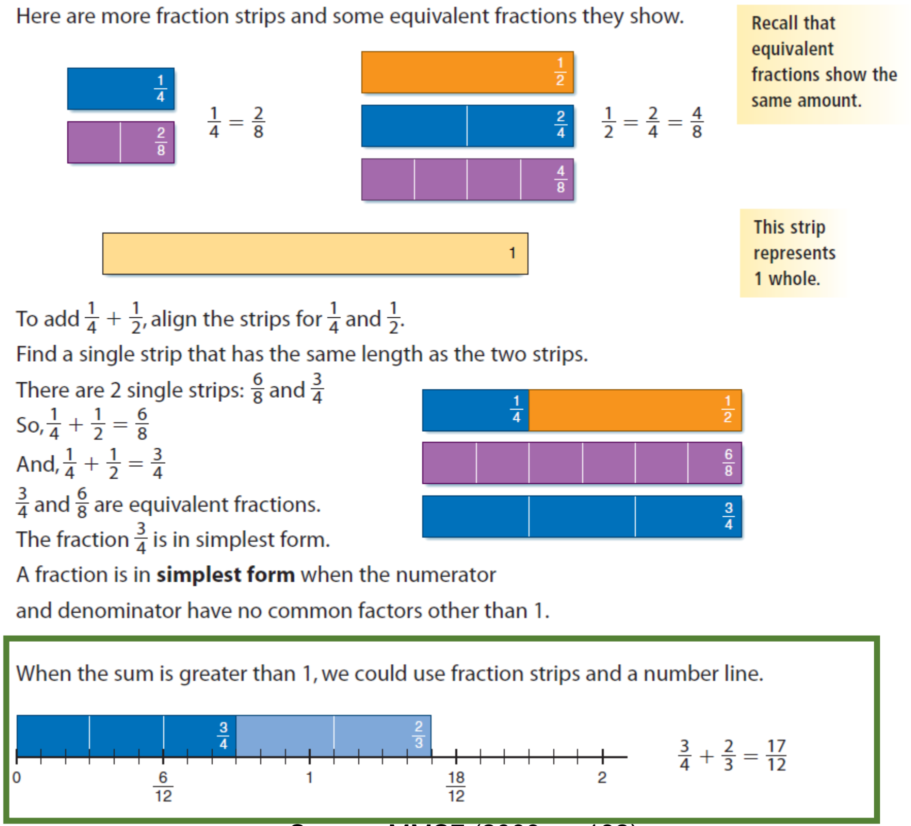
The use of these visual representations (Figure 33) plays a central role in the teaching of fractions. These representations act as a “bridge” between practical manipulation and symbolic formalization (Moschkovich, 2015), allowing students to see the addition of fractions as a concrete action of “putting parts together” on the bar or number line, for example, before resorting to algebraic expressions.

This pedagogical move is in line with the studies of (Duval & Moretti, 2012; Niemann, 2013), who highlight the importance of semiotic representation registers in the cognitive functioning of mathematical learning. According to the author, the transition between different registers, such as visual, numerical and algebraic, is essential for the construction of mathematical meanings.

Figure 34 shows a mathematical approach that makes use of fraction strips to introduce concepts such as equivalence between fractions (for example, $\frac{1}{2}$ e $\frac{2}{4}$ e $\frac{4}{8}$) and adding fractions with different denominators. Aligning the strips on the same base allows students to visualize the relationships between fractions, reinforcing the idea of equivalence in a concrete way.

Next (highlighted in green in Figure 34), the transition to the number lines allows for greater abstraction, connecting the visual concepts to the positional value of the fractions. This process reflects what Duval & Moretti (2012) call articulation between registers, in which coordination between different representations contributes to understanding complex mathematical concepts.

Figure 34 - Fraction Strips: Visualizing Equivalences and Additions



Source: MMS7 (2009, p. 182)

In addition, the inclusion of alternative strategies, such as the use of fractional circles, expands the possibilities for exploring the concepts. This diversity of representations is particularly important in heterogeneous educational contexts, as it caters for different learning styles and makes the content more accessible. Niemann (2013) argues that the multiplicity of semiotic registers is fundamental to prevent students from getting 'stuck' in a single form of representation, which could limit their ability to generalize mathematical concepts.

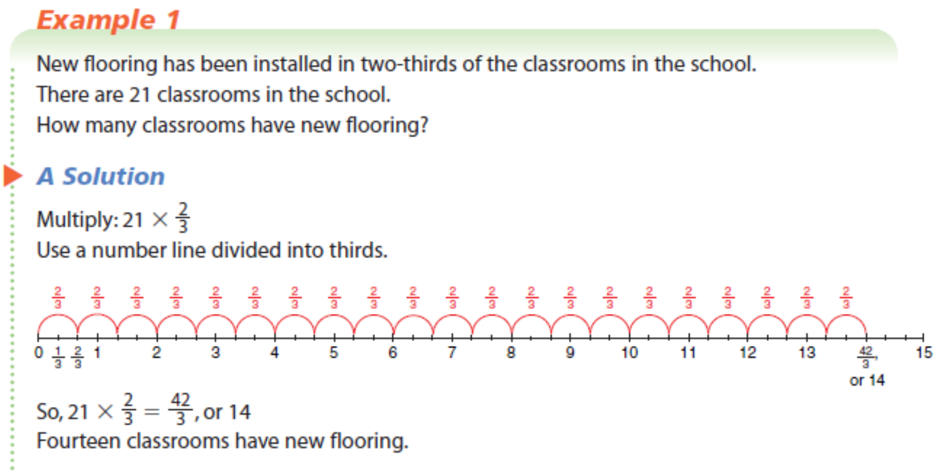
Another relevant point is the evidence of a pedagogical movement that the literature describes as a transition from the exploratory to the conceptual (Fan et al., 2013). Initially, the text directs students to explore fraction strips, discussing in groups how $\frac{1}{2}$ is related to $\frac{2}{4}$. Soon after, terminology and algebraic modeling are introduced, consolidating the idea that “adding fractions with equal denominators corresponds to adding numerators” (D'Amore & Sbaragli, 2017, p. 141).

As highlighted in research in the field of Mathematics Education (Caligari et al., 2021; Kirsch & Duarte, 2020; Moschkovich, 2010; Stathopoulou & Kalabasis, 2007; Tan & Xun, 2023) in multilingual contexts, the simultaneous manifestation of various representations - numerical, visual and verbal - contributes to the formation of robust meanings, especially when mathematical vocabulary can be an additional challenge (Barwell et al., 2008; Moschkovich, 2015b).

This approach is consistent with the principles advocated by D'Amore et al. (2006); Radford (2013) which emphasizes that meaningful learning depends on the ability to articulate representations and not just to manipulate symbols mechanically.

Working with the use of models to multiply a fraction by a whole number addresses different visual representations, such as number lines (Figure 35), counters (Figure 36), fraction circles (Figure 37) and subdivided rectangles (Figure 38).

Figure 35 - Number Lines in Multiplying Fractions by Integers

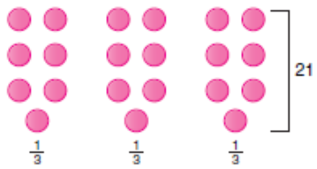


Source: MMS8 (2009, p. 106)

Figure 36 - Using Counters: A Model for Multiplying Fractions by Integers

Example 1
Another Solution

Find: $\frac{2}{3}$ of 21
Use counters.
Model 21 with counters.



Find thirds by dividing the counters into 3 equal groups.
Each group contains 7 counters.
$\frac{1}{3}$ of $21 = 7$
So, $\frac{2}{3}$ of $21 = 14$
Fourteen classrooms have new flooring.

Source: MMS8 (2009, p. 106)

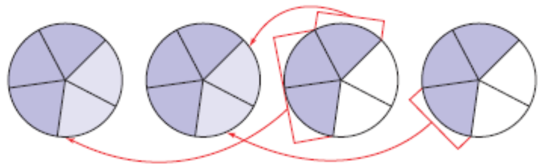
Figure 37 - Fractions in Circles: Another Approach to Multiplication

Example 2

An office building with four floors has rented out $\frac{3}{5}$ of each floor.
How many floors of the building have been rented?

A Solution

Multiply: $4 \times \frac{3}{5}$
$4 \times \frac{3}{5} = \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5}$
Model the expression $\frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5}$ with fraction circles.



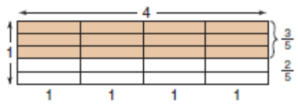
Put the fifths together to make wholes.
2 wholes and two fifths equal $2\frac{2}{5}$.
So, $4 \times \frac{3}{5} = 2\frac{2}{5}$
$2\frac{2}{5}$ floors of the office building have been rented.

Source: MMS8 (2009, p. 106-107)

Figure 38 - Subdivided Rectangles: Exploring Fraction Multiplication

Example 2
Another Solution

Multiply: $4 \times \frac{3}{5}$
Sketch a rectangle with base 4 units and height 1 unit.
Divide the height into fifths.
Shade the rectangle with base 4 and height $\frac{3}{5}$.
The area of the shaded rectangle is:
base × height = $4 \times \frac{3}{5}$
Each small rectangle has area: $1 \times \frac{1}{5} = \frac{1}{5}$
So, the shaded area is: $12 \times \frac{1}{5} = \frac{12}{5}$, or $2\frac{2}{5}$
So, $4 \times \frac{3}{5} = 2\frac{2}{5}$
$2\frac{2}{5}$ floors of the office building have been rented.



Source: MMS8 (2009, p. 107)

At the end, in Discuss the ideas, the book presents pertinent questions about these representations linked to their mathematical purpose.

1. Why can a product be written as repeated addition?
2. When might you not want to use repeated addition to find a product?
3. How could you use a rectangle model to solve the problem in Example 1?
4. How could you use a number line to solve the problem in Example 2? (MMS8, 2009, p. 107)

Based on the questions posed above, it is possible to develop a discussion that explores, in an integrated way, the mathematical approach in the context of visual representations. Multiplication can be understood as a form of repeated addition, especially in the case of fractional numbers. This idea can be visually demonstrated through representations such as number lines and counters (Getenet & Callingham, 2017).

For example, multiplying $21 \times \frac{2}{3}$ the number line (Figure 21) allows the student to visualize the sum of $\frac{2}{3}$ repeated 21 times, reinforcing the concept of multiplication as an iterative process. This visual representation enables the transition and connection between the numerical and graphical registers, promoting meaningful articulation, as pointed out by (Presmeg et al., 2016).

However, repeated addition may not be suitable for solving all multiplication situations, especially when dealing with non-unit fractions or larger numbers, as in the case of $\frac{4}{7} \times \frac{2}{5}$. For these cases, visual models such as subdivided rectangles or fraction circles are more appropriate, as they allow the student to understand the concept of a fraction of another fraction, without the need for manual repetition.

The idea of a fraction of another fraction implies that when you multiply two fractions, you are not just applying an algorithm (multiplying numerators and denominators), but actually calculating a part of an existing part (Menon, 2015). This methodological choice emphasizes that multiplying fractions goes beyond the accumulation of parts and involves understanding operations with fractional parts, as well as expanding the student's mathematical repertoire with rational numbers.

Rectangle models, for example, are particularly useful for solving problems such as the one presented here. $\frac{4}{7} \times \frac{2}{5}$. In this case, $\frac{4}{7} \times \frac{2}{5}$ can be represented by dividing a rectangle into five columns for fifths and subdividing each column into seven rows for sevenths. By shading the intersection corresponding to $\frac{4}{7}$ of $\frac{2}{5}$ the student sees the resulting fraction ($\frac{8}{35}$).

This approach highlights the articulation between graphical and numerical registers, as highlighted by Duval & Moretti (2012), and contributes to the conceptual understanding of the process. The case mentioned above is covered in the book (Figure 39).

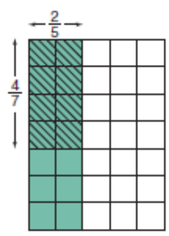
Figure 39 - Modeling Fraction with Rectangles

Connect

Here is an area model to show: $\frac{4}{7} \times \frac{2}{5} = \frac{8}{35}$

The product of the numerators is:
$4 \times 2 = 8$
The product of the denominators is:
$7 \times 5 = 35$
That is, $\frac{4}{7} \times \frac{2}{5} = \frac{4 \times 2}{7 \times 5}$
$= \frac{8}{35}$

Check if there are common factors in the numerator and denominator.



So, to multiply two fractions, multiply the numerators and multiply the denominators.

We can use this method to multiply proper fractions and improper fractions.

Source: MMS8 (2009, p. 116)

This reasoning is important because it shifts the focus from simply doing the math (multiplying numerators and denominators) to understanding what is happening in the process: you are calculating the part (fraction) of another part (fraction), that is, a portion of something that was no longer whole (Lamon, 2020).

Studies such as D'Amore & Sbaragli (2017), Fan et al. (2013) and Rezat et al. (2021) point out that this interpretation can help students understand why multiplying fractions usually results in values smaller than each initial portion (when both fractions are smaller than 1), and not just mechanically apply the rule of multiplying numerators and denominators. Thus, the use of these different visual representations - number lines, counters, fraction circles and subdivided rectangles - enables students to articulate complex mathematical concepts in complementary registers. As argued by Duval & Moretti (2012) this articulation between registers is important for the development of robust meanings, preventing students from limiting themselves to the mechanical memorization of algorithms.

In addition, the exploration of multiple representations promotes cognitive flexibility, allowing students to choose the most appropriate strategy for different mathematical contexts. In this way, the integrated approach of the materials analyzed broadens the understanding of concepts and favors meaningful learning. Another interesting aspect of the MMS8 book is the inclusion of multiple solutions to the same question, accompanied by captions such as Another Solution (Figures 36 and 38). One of these solutions uses rectangular blocks with subdivisions, allowing the student to visualize the fraction as part of a rectangle and then as a part of that part (Figure 39).

This type of variation in strategies reinforces what Fan et al. (2013) classify as a comprehensive instructional perspective. The textbook does not limit itself to a single method but invites the learner to recognize correspondences between different models of representation. This practice is advocated by research that analyzes learning in multilingual or multicultural contexts (Moschkovich, 2015; Planas & Setati, 2009), as it allows each student to find a representation with which they identify best, overcoming possible difficulties related to vocabulary or algebraic abstraction. By including varied approaches, such as the use of number lines or rectangular subdivisions, the examples presented in the MMS collection go beyond a mere algorithmic demonstration.

According to Duval & Moretti (2012), the articulation between different semiotic registers - visual, symbolic and algebraic – is essential for the development of broad and deep meanings in mathematics. When compared to traditional Brazilian practices, these illustrations confirm the observations of D'Amore & Sbaragli (2017), according to whom international teaching materials tend to be more explicit in the different stages of calculation and in the use of visual resources to anchor each mathematical process.

While Brazil generally favors a synthetic presentation of the rule - how numerators and denominators are multiplied - the examples in the book indicate a deliberate effort to show the why and how of the operations (Nacarato et al., 2009). This clarity, presented step by step and combined with the use of different representations, enhances students' learning, especially when they work collaboratively or in pairs (Fi & Degner, 2012).

In the Brazilian context, the teaching of fractions has historically prioritized algebraic treatment, with a focus on operations and simplifications (Nacarato et al., 2009). Although the BNCC (Brazil, 2017) encourages the use of varied representations, it is common for Brazilian textbooks to present fractions mainly through verbal explanations, numerical tasks and, occasionally, static figures of fraction circles or bars. However, these representations lack a systematic use of number lines or visual sequences that demonstrate equivalences and operations in a dynamic and interactive way.

The inclusion of multiple solutions and approaches reflects an effort to integrate different learning styles and promote cognitive flexibility among students. As highlighted by Radford (2013) the coordination of different semiotic registers not only facilitates the understanding of complex concepts, but also prepares the student to move between representations in a fluid and meaningful way. In this way, the use of visual representations and the exploration of varied strategies, such as those analyzed in the books in the MMS collection, can inspire improvements in Brazilian textbooks, expanding the possibilities for teaching and learning operations with fractions.

Under these conditions, the role of the teacher is essential to mediate and exploit the potential of these multiple representations in the teaching of fractions. The teacher acts as a facilitator by helping students to identify connections between different semiotic registers and to understand how visual representations are linked to mathematical concepts. This mediation requires the teacher to master the various forms of representation – counters, fractional circles and subdivided rectangles - and the conversions between them, promoting integrated learning.

According to Radford (2000) and Radford and Puig (2007)The teacher must help students overcome the natural difficulty of converting information from one register to another, ensuring that understanding goes beyond the mechanical application of algorithms. In addition, the teacher should create a learning environment that values experimentation and dialog, encouraging students to explore different strategies and discuss their choices with their classmates. This practice not only promotes collaboration, but also allows students to realize the multiplicity of possible ways to solve the same problem, developing autonomy and confidence in the learning process.

In this context, the notion of the didactic tetrahedron, as presented by (Lellis, 2022) highlights the interaction between four fundamental elements: the textbook, the teacher, the students and mathematical knowledge. The teacher plays a central role in this model, adapting the textbook's proposals to the specific needs and characteristics of their class, linking mathematical content with pedagogical practices and different visual representations.

Ongoing teacher training is essential to enable them to balance these dimensions, using the available resources to mediate learning in a broad and deep way (Perovano, 2022).

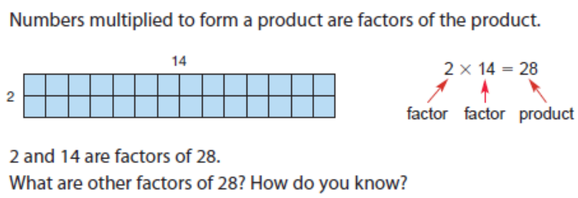
6.1.3 Language and visual paths: straight lines, colored pieces and signs

In environments that opt for foreign textbooks, the linguistic dimension requires a conscious effort to avoid reproducing barriers and, at the same time, take advantage of broader learning opportunities (Moschkovich, 2015). Learning specific mathematical terms in second language contexts - for example, prime factor or composite number - requires attention not only to the literal translation, but also to the conceptual background and cultural practices (Radford et al., 2011).

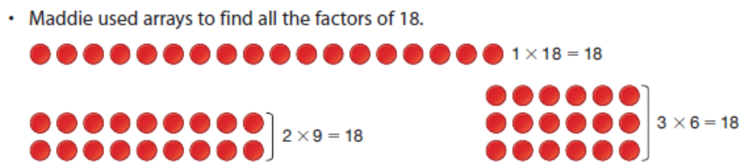
Expressions such as hypotenuse or perfect number have specific meanings that are rarely present in everyday interactions, which requires students to have a conceptual mastery that goes beyond simple linguistic decoding (Radford, 2011). Thus, they need to connect the mathematical meaning of the term to the sociocultural references of their linguistic repertoire (Barwell et al., 2008; Moschkovich, 2015).

In bilingual education, the use of visual representations, such as triangle diagrams to illustrate the concept of hypotenuse or the decomposition of numbers to prime factor, for example, can support comprehension beyond lexical barriers (Craig & Morgan, 2015). (Craig & Morgan, 2015). In addition, mathematical narratives or tasks contextualized in the students' daily lives can facilitate the understanding of mathematical language. The books in the MMS collection usually present/explore the mathematical concept accompanied by, for example, small captions, the use of arrows, geometric figures, Euler-Venn diagrams, factor trees and repeated divisions (Figure 40).

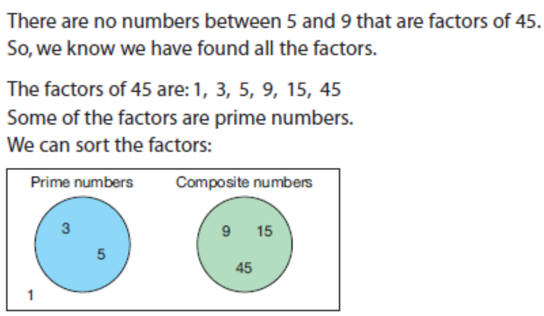
Figure 40 - Semiotic Resources for Factor and Divisor Concepts



Source: MMS6 (2009, p. 59)



Source: MMS6 (2009, p. 60)



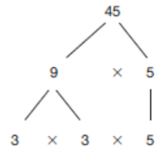
Source: MMS6 (2009, p. 64)

Here are two ways to find the factors of 45 that are prime.

- Draw a factor tree.

  Write 45 as the product of 2 factors.
  9 and 5 are factors of 45.
  9 is a composite number, so we can factor again.

  So, 3 and 5 are the factors of 45 that are prime numbers.



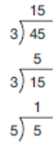
- Use repeated division by prime numbers.

  Begin by dividing 45 by the least prime number that is a factor: 3

  Divide by this prime number until it is no longer a factor.

  Continue to divide each quotient by a prime number until the quotient is 1.

  The factors of 45 that are prime numbers are 3 and 5.

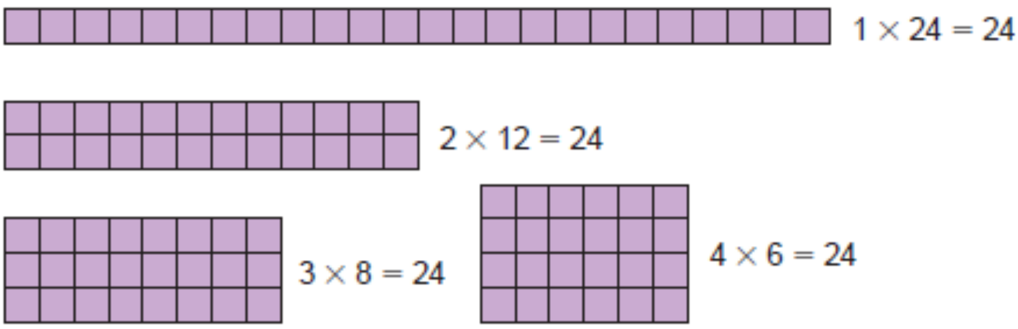


Source: MMS6 (2009, p. 64)

This visual arrangement, according to Getenet & Callingham (2017) and Radford et al. (2011), tends to facilitate understanding, as it graphically demonstrates the reasons why a given number is classified as prime or composite. For example, in the representation of the factors of the number 24 with rectangular arrangements (Figure 41), students see how numbers can be organized into different products. This approach reinforces the idea that factoring is not just an abstract exercise, but also a practice that can be explored visually.

Figure 41 - Unraveling 24: Factors in Rectangular Layouts

➤ Suppose you have 24 Colour Tiles.
You can make 4 different rectangles with 24 tiles.



24 has 8 factors: 1, 2, 3, 4, 6, 8, 12, and 24
The factors that are prime numbers are 2 and 3.

Source: MMS6 (2009, p. 60)

The exploration of this strategy is presented at the beginning of Lesson 4, where work on factoring using rectangular arrangements is covered (Figure 42).

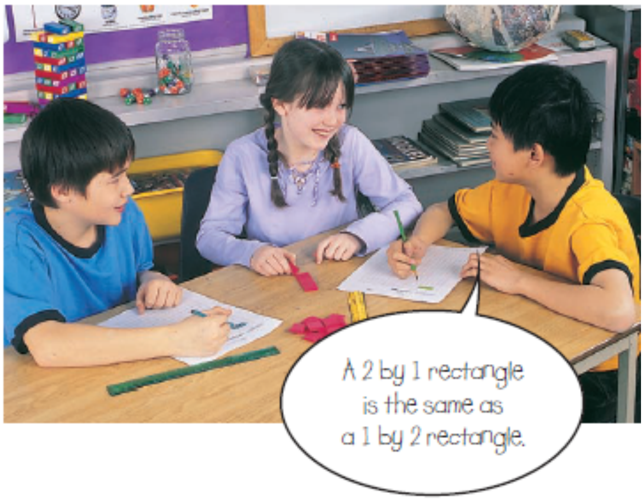
In this scenario, students have the opportunity to create their own representations, promoting the construction of individual meanings and exploring different forms of expression.

Figure 42 - Task with factoring using rectangular arrangements

Explore

You will need Colour Tiles or congruent squares and grid paper.

- Find all the different rectangles you can make using each number of tiles from 2 to 20.
  Draw each rectangle on grid paper.
  Write a multiplication sentence that describes the number of tiles in each rectangle.
- For which numbers of tiles could you make only 1 rectangle?
  For which numbers of tiles could you make 2 rectangles?
  3 rectangles?



Source: MMS6 (2009, p. 60)

The use of specific language, such as prime numbers, composite numbers and factors, can present challenges, especially for students in multilingual contexts or with limited linguistic repertoires, as pointed out by (Abtahi & Planas, 2024; Moschkovich, 2010). However, associating mathematical concepts with different representations helps to mitigate this difficulty. In addition, the presentation of objective definitions, such as the one dealing with prime numbers (Figure 26), helps to mitigate these barriers to understanding.

Figure 43 - Definition of prime numbers in Connect section

Connect

- Suppose you have 23 Colour Tiles.
  You can make only 1 rectangle with all 23 tiles.

23 has 2 factors: 1 and 23
A number with exactly 2 factors, 1 and itself, is a **prime number**.
23 is a prime number.

A prime number is a number greater than 1 that is divisible only by 1 and itself.

Source: MMS6 (2009, p. 60)

When analyzing these examples, we see that he assumes that the student already understands English terms such as prime factor and composite. In a multilingual context, this can lead to interpretative gaps, as this is a specific vocabulary of mathematics in English, which does not always have an immediate or intuitive translation into other languages - “especially when these terms are not accompanied by a broader linguistic contextualization” (Planas & Setati, 2009, p. 5). Furthermore, the way in which the caption and text are linked (Figures 24 to 26) presupposes familiarity with certain cultural references, such as the recurrent use of specific prime numbers (2, 3, 5, 7, 11, 13, etc.). In many teaching materials produced in English, for example, there is a tendency to always use this standard set, as it has become commonplace in exercises and examples. In other cultural contexts, however, different numbers may be used or this content may be approached differently.

Another relevant point is the way in which the idea of divisor is introduced. Often, the concept of divisor is anchored in idiomatic expressions in English or in exercises that use common phrases in that language (divide evenly, no remainder, etc.). For students who have not mastered this linguistic base, such terms can pose barriers to comprehension, since the logic of the text presupposes prior knowledge of the grammatical and cultural structures involved. The presence of these references requires special attention when teaching materials are used in multilingual classrooms or by students whose mother tongue is not English.

Without adequate mediation – through glossaries, additional explanations or comparisons between languages – ther is a risk that students will have difficulty understanding the concepts, not necessarily because they don't know the mathematics itself, but because they don't have access to the meanings of the expressions used. As Planas & Setati (2009) emphasize, these gaps tend to get worse if there are no initiatives to adapt or contextualize the content to the students' linguistic and cultural repertoire.

It is therefore essential that teachers (see Andrade, 2024; Perovano, 2022b) who wish to adopt this type of resource consider that not all students are fluent enough in English to interpret terms such as prime factor, remainder or composite.

From this perspective, both the language and the numerical examples should be critically analyzed and, if necessary, adapted to the context of the class. This adaptation prevents linguistic and cultural barriers from hindering the learning of basic mathematical notions - a challenge that emerges when it is generally assumed that all students share the same cultural and linguistic repertoire as the source material (Candau, 2012).

Radford et al. (2011) reinforce the complexity of this scenario by pointing out that understanding the term composite implies more than simply decoding the word: it requires appropriating the history of the concept of factor composition, something that does not always find direct correspondence in other languages. To overcome this difficulty, some teaching materials make use of visual resources, such as rectangular arrangements (on grid paper) and the use of tiles, building a semiotic environment that seeks to make the relationship between the factors more evident. In this proposal, the spatial organization of numbers in rectangular arrangements or the handling of physical pieces can help students understand how the factors are distributed and connected.

However, even materials rich in images and interactive proposals can lack “contextualization for students unfamiliar with the set of symbols and conventions adopted” (Radford et al., 2011, p. 152). For this reason, it is up to the teacher and the authors of teaching materials to offer additional support, explaining the relationships between the representations (tiles, grid paper) and the mathematical meanings at stake. Such care broadens the possibility of including diverse learner profiles, ensuring a more solid understanding of the concept of composite in environments that bring together speakers of different languages or who have little familiarity with these semiotic resources.

Another positive aspect of the book is the diversity of strategies for identifying factors, such as the use of factor trees, successive divisions and Euler-Venn diagrams. These multiple approaches cater for different learning styles, as Getenet & Callingham (2017) argue, allowing students to opt for the method best suited to their cognitive needs.

The factor tree, for example, emphasizes the notion of decomposition, while the Euler-Venn diagram reinforces the connections between numbers. To further enhance this variety, it is recommended to include explanations showing why each strategy is valid, encouraging critical reflection on the choice of procedure.

As for the exploration of square root notions, the book approaches the square root of 16 (Figure 44) in a visual and geometric way, relating factors, forming rectangles and squares, and highlighting the square root as the inverse operation of potentiation.

In this process, the subdivided grid representation demonstrates how the factors of a number can be arranged in pairs that form rectangles. In the case of 16, the pairs $1 \times 16, 2 \times 8 \text{ e } 4 \times 4$ show different possible dimensions, facilitating the assimilation of the concept of factoring and the realization that, when faced with a repeated factor (4×4)you get a perfect square.

This approach strengthens the conceptual understanding of square root by integrating visual aspects and geometric properties, promoting meaningful connections between numerical and spatial representation.

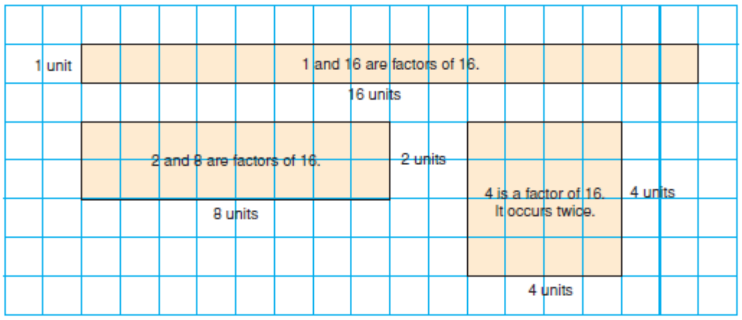
Figure 44 - The Square of 16: Root, Power and Geometric Meanings

Connect

➤ Here are some ways to tell whether a number is a square number.
If we can find a division sentence for a number so that the quotient is equal to the divisor, the number is a square number.
For example, $16 \div 4 = 4$, so 16 is a square number.

dividend **divisor** **quotient**

➤ We can also use factoring.
Factors of a number occur in pairs.
These are the dimensions of a rectangle.



Sixteen has 5 factors: 1, 2, 4, 8, 16

Since there is an odd number of factors, one rectangle is a square.
The square has side length 4 units.
We say that 4 is a **square root** of 16.
We write: $4 = \sqrt{16}$

A factor that occurs twice is only written once in the list of factors.

When a number has an odd number of factors, it is a square number.

When we multiply a number by itself, we square the number.
Squaring and taking the square root are inverse operations. That is, they undo each other.

$4 \times 4 = 16$ $\sqrt{16} = \sqrt{4 \times 4} = \sqrt{4^2}$
so, $4^2 = 16$ $= 4$

Source: MMS6 (2009, p. 12)

This approach proves interesting as it connects the factors to the geometric concept of area, enabling students to visualize the relationship between numbers and shapes.

By showing the transition from a rectangle to the specific case of a square – exemplified in the representation of 4×4 -The book highlights the relationship between the square root and the symmetry and uniformity of the square's dimensions. As can be seen in Unit 1 of the MMS9 (2009, p. 4), this specific work explores and discusses the different squares and the calculation of surfaces, reinforcing the connection between factoring and area.

Despite the benefits, it should be borne in mind that the reliance on visual representations, such as rectangles and squares, can hinder learning in situations where students have little familiarity with geometric concepts or standardized measurement systems. To make this approach more robust, it would be pertinent to add examples that contextualize the calculation of areas in everyday situations (for example, the floor of a room or the plot of land in a house), which would help to broaden the exploration of the concept.

Another relevant point concerns the book's exploration of the inverse operation (highlighted in red in Figure 44). The book emphasizes the inverse relationship between the square root and potentiation, pointing out that $4^2 = 16$ and $\sqrt{16} = 4$. This presentation is fundamental to understanding the concept, as it positions the square root not as an isolated operation, but as part of a system of interconnected mathematical operations (Lellis, 2022).

The visual and symbolic nature of this approach allows students to see how these operations “undo” each other, which favors a deeper understanding of the mathematical properties. In sub-unit 1.1 of MMS9 (2009, p. 4-13), there are additional examples that reinforce the generalization of the concept, including fractions and decimals.

According to Radford et al. (2007) this type of visual representation is not just a simple reminder of how to reverse the power, but relates to how the student conceives of measurement operations and decomposition of areas. In line with this, (Lamon, 2020) draws attention to the fact that if the student does not understand the standard system of measurements or the traditional representation of square areas, the figure may seem abstract and not very meaningful, making it difficult to effectively appropriate the concept. With regard to the properties of square numbers, the book stresses that when a number has an odd number of factors, it is classified as a square number. This explanation is directly related to the concept of geometric symmetry and the repetition of factors - as in the case of 4 being a factor of 16 and, at the same time, its square root.

This approach brings the numerical and geometric dimensions closer together, enriching students' understanding. However, it would be desirable to deepen this concept through activities with non-square numbers, so that students can contrast and understand the lack of symmetry in these cases. The language used in Figure 44, when dealing with terms such as factors, square root and odd number of factors, presents specific lexical challenges, especially in multilingual contexts.

Research by Akkari and Radhouane (2022) and Tan and Xun (2023) shows that there are often no direct equivalents of these terms in different mother tongues, which can hinder semantic transposition and, consequently, conceptual understanding. For example, square root is not just a literal translation, since square in certain cultural contexts does not automatically refer to the idea of the side of a square.

Similarly, expressions like they undo each other can seem ambiguous in languages that don't offer a direct version of root in the mathematical sense. The use of factors to refer to pairs that define the dimensions of rectangles is a conceptually rich choice, but it can also lead to confusion. Students who are not familiar with the notion of factors as divisor numbers may not easily establish the relationship between factor and geometric dimension. In languages such as French (Facteur) or Italian (Fattore), the word takes on different meanings and, in some cases, very far from the mathematical meaning of factor.

Although the integration of diagrams and textual explanations is a positive step - as it offers different visual, linguistic and symbolic representations – it alone does not guarantee the internalization of concepts. It is essential that students recognize the correlation between factors and rectangle dimensions; otherwise, reading these representations can result in misinterpretations.

In this sense, it is proposed to incorporate examples of everyday use, such as calculating the dimensions of a floor or a garden, in order to make the approach more tangible and universally recognizable. Similarly, providing multilingual glossaries with detailed explanations of specific terms would help students from different linguistic backgrounds to build up an adequate lexical repertoire. In addition, investing in narratives that explain the origin and meaning of key words (for example, why square is used to describe certain roots) can help with the relationship between the concept and its geometric root.

These lexical challenges, present in the book analyzed, highlight the importance of considering language as a central aspect in the teaching of mathematics. Craig and Morgan (2015, p. 3) reinforce that “mathematical learning is mediated by language, which acts not only as a vehicle for knowledge, but also as an element that reflects and shapes pedagogical practices”.

When the specific vocabulary is not accessible, the process tends to be restricted to the memorization of procedures, without the effective construction of conceptual meanings. Curricular globalization, according to Akkari and Radhouane, (2022) intensifies the problems of cultural and linguistic adaptation of teaching materials, especially in multilingual environments. The simple transfer of foreign definitions and representations, without proper adaptation to the local context, often results in figures that function as mere “didactic decoration” (Rezat et al., 2021, p. 14) - attractive to the eye, but insufficient to promote effective appropriation of mathematical concepts.

To mitigate this cultural gap, Erath et al. (2021) and Valero and Zevenbergen (2004) propose the hybridization of content, i.e. the reinterpretation and adaptation of figures and definitions to reflect the cultural and linguistic reality of the learner.

From this perspective, the teacher takes on the role of cultural mediator (Ladson-Billings, 1995; Lerman, 2003) acting as an interlocutor capable of relating visual representations and explanations to the students' contexts, as well as building contextual glossaries. This mediation includes both the elaboration of examples situated in the student's daily life and dialog about the differences in mathematical vocabulary in each language (Rezat et al., 2021). It is also necessary to explore the logic behind specific terms, such as square root, so that students can associate the content with geometric structures.

With regard to the concept of square root, diagrams illustrating subdivided squares can be particularly useful for demonstrating how factors connect to geometric dimensions. According to Tan and Xun (2023), the creation of localized examples and the reflexive translation of key terms are decisive for the construction of meanings in multiculturally diverse environments.

Finally, Caligari et al. (2021), Craig and Morgan (2015) and Erath et al. (2021), point out that the critical articulation between theory, practice and culture is indispensable when teaching concepts such as square root in multilingual settings. In addition to visual clarity and conceptual foundations, it is essential to consider the learners' linguistic and cultural background, their previous repertoire and their perceptions of different visual representations. Only through this integration is it possible to offer a math teaching experience that is truly meaningful and inclusive, especially in contexts of linguistic diversity.

6.2 Sociocultural and Sociopolitical Dimensions in Mathematics: Sports, Art and Festivities as Learning Scenarios

This category addresses the ways in which everyday contexts, sport, art and architecture, and cultural festivities – can reflect and convey the sociocultural dimension of Canadian mathematics textbooks. The aim here is to examine how these references might be understood or adapted in multilingual Brazilian classrooms, thereby responding to the overarching research objective of exploring sociocultural differences in foreign textbooks and their implications for local educational scenarios.

By looking at examples such as ice hockey alongside football, or Canadian winter festivals alongside regional Brazilian celebrations, this category investigates the extent to which textbook content aligns (or not) with the cultural and linguistic realities of students in Brazil. This approach resonates with the specific objectives of analyzing sociocultural differences (Objective a) and investigating cultural and linguistic aspects in dialogue with local contexts (Objective b), while acknowledging that sociopolitical elements also permeate these analyses (Objective c). Through the subcategories of sport, art and architecture, and cultural festivities, we observe how the textbook's representations potentially broaden or constrain the ways students experience mathematical concepts within their own cultural framework.

6.2.1 From field hockey to soccer: the sports world as a learning scenario

Ice hockey is a sport widely practiced in Canada and strongly connected to the country's cultural identity. Recognized at all levels, from amateur to professional, by the National Hockey League (NHL)[31]it is present in the lives of many Canadian children from an early age, integrating school and community

[31] A NHL é a principal liga de hóquei no gelo da América do Norte, composta por times dos Estados Unidos e Canadá. Ela é conhecida por seu alto nível de competição e por ser um dos esportes mais populares nesses países. Avaiblabe at: https://www.nhl.com/info/about. Acesso em: 11 de agosto de 2024.

activities. Thus, for Canadian students, ice hockey represents a familiar sport in their daily lives, widely known and practiced by many (eTA Canada, website[32]).

It is therefore pertinent that ice hockey appears in Canadian textbooks as an everyday context close to the students' reality. This insertion is evident in the analysis of these materials, which shows the presence of the sport in various proposals.

Figure 11 shows an example taken from the book, relating the content of whole numbers to the statistics of field hockey players. It can also be seen those 38 proposals in materials MMS6 to MMS9 mention sport; however, in 27 of them, the approach is restricted to visual references, without going into relevant mathematical concepts.

As an illustration, tasks 5(a) and 5(b)(Figure 45) mention a significant sporting event (the Grey Cup) and provide data on the average attendance and the percentage change (166%).

Although they involve proportional calculation, the sporting context only serves as a backdrop for the mathematical application, and does not promote further reflection on sports statistics, such as performance analysis or comparison between teams. Similarly, in task 6(Figure 45), the subject of field hockey is introduced in a secondary way, limited to calculating percentages (20% of a set of cards) and without exploring the sport's specific characteristics.

[32] Avaiblabe at: https://www.visacanadaeta.org/pt/ice-hockey-canadas-favourite-sport. Acesso em: 09 de janeiro de 2025.

Figure 45 - Applying Percentage Calculations to Sporting Events

5. The average attendance at a regular season home game of the Winnipeg Blue Bombers in 2006 was 26 988. The attendance at the 2006 Grey Cup game in Winnipeg was about 166% of the regular season attendance.

a) How many people attended the 2006 Grey Cup game?

b) Estimate to check your answer is reasonable.

6. Joline collects hockey cards. She needs 5 cards to complete a set. This is 20% of the set. How many cards are in the set? Justify your answer.

Source: MMS8 (2009, p.308)

These tasks show that, although ice hockey and other sports appear in the material, they do little to explore the specific mathematical potential of these sporting contexts. A more in-depth approach could include statistical analysis, proportions and other concepts directly linked to the dynamics of ice hockey, making learning more contextualized to students' everyday lives. The unit before Data Analysis focuses on Equations, introducing fundamental concepts for solving equations using models and operations with integers.

Subunit 6.2, Using a Model to Solve Equations, and Subunit 6.3, Solving Equations Involving Whole Numbers, emphasize visual and practical methods for exploring algebraic relationships (Lamon, 2020), providing the basis for understanding and transitioning to more abstract approaches in Subunit 6.4, Solving Equations Using Algebra, where students begin working with symbolic representations. Subunit 6.5, Using Different Methods to Solve Equations, expands the repertoire of solving strategies.

Interactive components, such as the Equation Baseball game, are proposed as opportunities for social learning, in which students participate in collaborative activities to reinforce their understanding (Vygotsky, 1978). However, the reliance on these games raises questions about the balance between engagement and the depth of mathematical understanding achieved. In the Review and practice sub-unit, the examples and applications lead students to solve mathematical problems associated with everyday situations, although this approach does not always create opportunities for reading and interpreting mathematical language, a skill that continues to require reinforcement in later units (Erath et al., 2021).

The unit begins with an overview (Figure 46) of basic statistical measures - mean, median, mode and range - presented in subunits 7.1 and 7.2. These concepts provide an understanding of central tendency and variability, enabling students to assess dispersion and trends in a set of data (Garfield & Ben-Zvi, 2008).

The discussion of outliers in Subunit 7.3, The effects of outliers on the mean, is essential for students to recognize how extreme values can distort data and influence statistical calculations. The applications of averages in Subunit 7.4 bring these concepts closer to the real world, illustrating how the average is used in scenarios such as budgeting, resource allocation and scientific research (Garfield & Ben-Zvi, 2008).

This unit also incorporates technology, through activities such as Using Spreadsheets to Investigate Averages (Subunit 7.4), which introduce digital tools for data analysis. Subunits 7.5 and 7.6 focus on probability, exploring methods such as tree diagrams to model probabilistic scenarios. In these sections, students work with structured exercises and the All the Sticks game, experiencing probability concepts in a visual way. The unit ends with a problem-solving activity involving board games, contextualizing the probabilistic content in a familiar way.

Subunit 7.1, mean and mode, presents these two measures of central tendency, as illustrated in Figure 47, emphasizing the analysis and summary of data in practical situations. The Practical exercises allow students to calculate mean and mode in various contexts, such as weekly subsidies and age groups of video renters, highlighting the applicability of central tendency concepts in everyday scenarios.

One example, shown in Figure 53, features the field hockey statistics of Jordin Tootoo, an Inuk First Nation athlete. This approach places data analysis in a broader sociocultural perspective, highlighting how mathematical concepts relate to the real world and emphasizing the importance of including different cultural backgrounds in teaching materials.

The mathematical content related to statistics is in the MMS7 textbook, in Unit 7, entitled Data Analysis. According to the book, in this unit students work with calculating the mean, mode, median and range in a data set, as well as analyzing the impact of outliers. The unit also discusses choosing the most appropriate measure to represent the data collected. Other topics include expressing probabilities in proportions, fractions and percentages, identifying the sample space in experiments with two independent events and comparing theoretical and experimental probability (Figure 46).

As the book points out, these contents are essential for students to learn to interpret statistical and probabilistic data in the media (Figure 46).

Thus, Unit 7 begins with an overview of the basic statistical measures – mean, median, mode and range - covered in sub-units 7.1 and 7.2, laying the foundations for understanding central tendency and variability, which makes it possible to analyze dispersion and patterns in a set of data (Garfield & Ben-Zvi, 2008).

Figure 46 - An Overview of Data Analysis in the MMS7

Source: MMS7 (2009, p.156-157)

Subunit 7.3, The effects of outliers on the mean, discusses how extreme values (outliers) can influence statistical calculations (Figure 47) and distort results (Figure 48). Subunit 7.4, Applications of Averages, then investigates which average best describes a set of data.

The applications of averages presented in Applications of Averages connect these mathematical operations to practical scenarios, highlighting how they are used in real-world situations such as budgeting, resource allocation and scientific research (Garfield & Ben-Zvi, 2008).

Figure 47 - When the Outlier Changes Everything: Reflections on Discrepant Values

Connect

A number in a set of data that is significantly different from the other numbers is called an **outlier**.

An outlier is much greater than or much less than most of the numbers in the data set.

Outliers sometimes occur as a result of error in measurement or recording. In these cases, outliers should be ignored.

Sometimes an outlier is an important piece of information that should not be ignored. For example, if one student does much better or much worse than the rest of the class on a test.

Outliers may not always be obvious.
Identifying outliers is then a matter of choice.

Source: MMS7 (2009, p. 267)

Figure 48 - Example of Statistical Distortion by Extreme Values

6. Assessment Focus A Grade 7 class wanted to find out if a TV advertisement was true.
The ad claimed that *Full of Raisins* cereal guaranteed an average of 23 raisins per cup of cereal.
Each pair of students tested one box of cereal.
Each box contained 20 cups of cereal.
The number of raisins in each cup was counted.

a) Assume the advertisement is true.
   How many raisins should there be in 1 box of cereal?
b) Here are the results for the numbers of raisins in 15 boxes of cereal:
   473, 485, 441, 437, 489, 471, 400, 453, 465, 413, 499, 428, 419, 477, 467
   i) Calculate the mean, median, and mode numbers of raisins.
   ii) Identify the outliers. Explain your choice.
   iii) Calculate the mean, median, and mode without the outliers.
   How do the outliers affect the mean?
   iv) Should the outliers be used when reporting the average number of raisins? Explain.
   v) Was the advertisement true? Justify your answer.

Source: Source: MMS7 (2009, p. 270)

Subunit 7.4 also incorporates the use of technology through a proposal entitled Using Spreadsheets to Investigate Averages, which focuses on investigating averages using spreadsheets (Figure 49).

Figure 49 - Investigating Averages with Spreadsheets

Technology

Using Spreadsheets to Investigate Averages

Focus Investigate averages using a spreadsheet.

You can use spreadsheet software to find the mean, median, and mode of a set of data.
A spreadsheet program allows us to calculate the averages for large sets of data values quickly and efficiently.
You can also use the software to see how these averages are affected by outliers.

Here are the heights, in centimetres, of all Grade 7 students who were on the school track team: 164, 131, 172, 120, 175, 168, 146, 176, 175, 173, 155, 170, 172, 160, 168, 178, 174, 184, 189

In some spreadsheet software, the mean is referred to as the average.

Use spreadsheet software.

- Input the data into a column of the spreadsheet.
- Use the statistical functions of your software to find the mean, median, and mode. Use the Help menu if you have any difficulties.
- Investigate the effect of an outlier on the mean, median, and mode. Delete 120. What happens to the mean? Median? Mode? Explain.

Source: MMS7 (2009, p. 276)

Subunits 7.5 and 7.6 focus on the content of probability. Subunit 7.5 discusses the expression of probabilities in the form of proportions, fractions and percentages, while Subunit 7.6 deals with the investigation of the results of probabilistic experiments. It is in the latter that tree diagrams appear, used to model different probability scenarios (Figure 51).

This Unit also includes the game All the Sticks, which allows students to visually explore probability concepts, a subject that will be returned to in Category 3. To conclude Unit 7, an activity based on board games is presented, offering a practical and familiar context for applying the probability concepts studied (Figure 52).

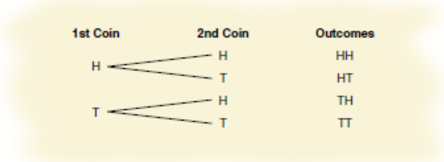
Figure 50 - Branches of Possibility: Probability Tree Diagrams

Connect

Two events are **independent events** if the result of the one event does not depend on the result of the other event.
Tossing two coins is an example of two independent events.
The outcome of the first toss does not affect the outcome of the second toss.
The outcome of the second toss does not depend on the outcome of the first toss.
We can use a **tree diagram** to show the possible outcomes for an experiment that has two independent events.

When 2 coins are tossed, the outcomes for each coin are heads (H) or tails (T).
List the outcomes of the first coin toss.
This is the first branch of the tree diagram.
For each outcome, list the outcomes of the second coin toss.
This is the second branch of the tree diagram.
Then list the outcomes for the coins tossed together.

You could also use a table to list the outcomes.



		First Coin	
		H	T
Second Coin	H	HH	HT
	T	TH	TT

There are 4 possible outcomes: HH, HT, TH, TT

Source: MMS7 (2009, p. 285)

Figure 51 - Boards and Tasks: Learning Probability in a Playful Way

Unit Problem **Board Games**

Many board games involve rolling pairs of dice, labelled 1 to 6.
Suppose you are on a particular square of a game board.
You roll the dice. You add the numbers on the uppermost faces.
How likely are you to roll 7?
Investigate to find out.

Part 1

Work with a partner.
Copy and complete this table.
Show the sums when two dice are rolled.

When one die shows 1 and the other die shows 4, then the sum is 5.

Sum of Numbers on Two Dice						
Number on Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2				6		
3						
4	5					
5						
6						

How many different outcomes are there?
In how many ways can the sum be 6? 9? 2? 12?
Why do you think a table was used instead of a tree diagram?
Find the theoretical probability for each outcome.

Source: MMS7 (2009, p. 296)

After this brief presentation of Unit 7, let's return to the discussion of the sports context present in the book's proposals. As already mentioned, ice hockey appears as an element associated with statistics content, and this approach is also present in the unit specifically intended for this study.

Tasks 6(a), 6(b), 6(c) and 6(d) explore the concepts of mean and mode in the context of ice hockey player Jordin Tootoo, an Inuk First Nation athlete (Figure 52). In these activities, students calculate the average of the Games Played, Goals, Assists and Points categories, analyzing the average performance over several seasons.

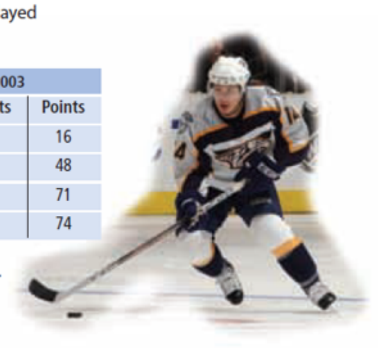
In addition, the concept of mode is worked on by identifying the most frequent values in the data set. However, the use of only four records per category limits the depth of the statistical analysis, as the mode may not be identified or have little relevance due to the small sample size, restricting its interpretative potential.

Figure 52 - Jordin Tootoo's Statistics: Hockey and Cultural Diversity

6. Jordin Tootoo is the first Inuk athlete to play in the National Hockey League. On October 9, 2003, he played his first game for the Nashville Predators. This table shows Jordin's statistics when he played junior hockey for the Brandon Wheat Kings.

Jordin Tootoo's Scoring Records 1999-2003

Year	Games Played	Goals	Assists	Points
1999-2000	45	6	10	16
2000-2001	60	20	28	48
2001-2002	64	32	39	71
2002-2003	51	35	39	74



Find the mean and mode for each set of data.

a) Games Played
b) Goals
c) Assists
d) Points

Source: MMS7 (2009, p. 261)

The use of Jordin Tootoo 's statistics in these tasks adds a culturally relevant element to the Canadian classroom by highlighting the achievements of a remarkable Inuit athlete. In Canada, the Inuit are indigenous peoples whose cultural and historical ties span the country's Arctic regions, including Canadian territories such as Nunavut, the Northwest Territories, Northern Quebec and Labrador.

Renowned for their resilience and ability to adapt to extreme climates, the Inuit maintain traditions based on hunting, fishing and a rich cultural heritage of storytelling, art and crafts (Inuit Tapiriit Kanatami, 2025). In recent years, the Inuit have actively participated in political movements for the recognition of their rights to land, cultural preservation and environmental management, especially in the face of climate change (Kulchyski & Tester, 2008).

Thus, this context offers an opportunity for meaningful discussions about the visibility of minority groups in sports familiar to Canadian students and how this representation can shape individual and collective aspirations. The presence of athletes from diverse ethnic backgrounds, such as Jordin Tootoo, reinforces the importance of inclusion and diversity in society (Abtahi & Planas, 2024; Erath et al., 2021; Gutstein, 2012).

In relation to the context adopted by the task, specifically field hockey, it can be seen that, although it is a recognized sport, its practice involves high costs and depends on specialized infrastructure, such as ice rinks, which are uncommon in Brazil. This socio-economic and cultural distance suggests that the textbook may not take into account the diversity of Brazilian students' experiences, which could result in barriers to meaningful appropriation of the contexts.

This choice raises questions about the student profile envisioned by the authors of the material and the sociocultural realities considered relevant. From the perspective of intercultural education, the textbook is expected to value the plurality of contexts present in the school universe, promoting the recognition of different life trajectories and cultural practices (Banks, 2014; Candau, 2012).

By resorting once again to sports commonly associated with more privileged socio-economic segments, the book possibly reinforces narrow worldviews, making it difficult to build an inclusive educational environment that incorporates cultural diversity and ensures more equitable conditions for participation and learning (Gomes, 2003). Thus, the tasks in Figure 52 allow the teacher to critically address issues associated with ice hockey in Canada, such as unequal access to the sport, high costs and social pressure to participate, which end up excluding economically disadvantaged groups. These reflections can deepen students' understanding of the cultural and social implications of this sport.

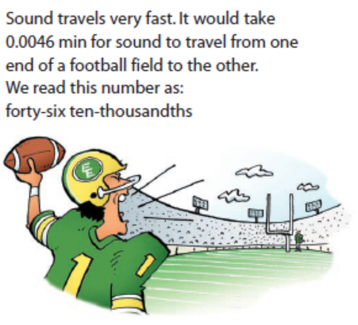
Therefore, tasks with this approach can promote inclusion and awareness of sports cultural diversity, encouraging students to value diversity in sports and, by extension, in other professional and social spheres.

The centrality of ice hockey in the activities in the MMS collection can be viewed critically when considering its relevance in different contexts, such as Brazil. For Brazilian students, the choice of this sport as a familiar and everyday context can seem distant and disconnected from their cultural, social and economic reality. The lack of familiarity with ice hockey can make it difficult for students to get involved and interested, resulting in proposals that, instead of promoting identification and motivation, present an inaccessible and unfamiliar context.

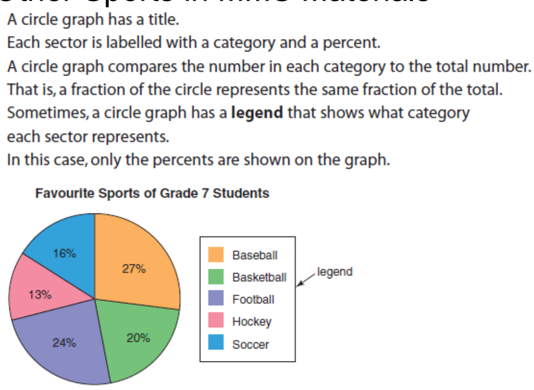
In Brazil, field soccer plays the same central role as ice hockey does in Canada, and is recognized as an integral part of Brazilian cultural identity. The sport is present in the streets, schools and sports leagues, making it widely familiar and deeply connected to students' daily lives. In contrast, ice hockey is practically non-existent in Brazil due to factors such as climatic, cultural and structural differences. Due to its strong connection with Brazilian culture, field soccer is an example of a sport capable of generating greater identification and engagement among students (Celi, 2014; Macagnan & Betti, 2014). Its inclusion in textbooks could bring the teaching and learning process closer to their experiences, by aligning it with local experiences and students' familiarity with sport.

In the MMS collection, it can be seen that there is very little mention of soccer as a sporting context. It's important to note that the collection uses the terms soccer to refer to American soccer and soccer to refer to field soccer. In total, the context related to football appears four times throughout the collection, in books MMS6, MMS7 and MMS9 (Figure 53). Although football does not depend on specific climatic conditions, like ice hockey, it is not practiced culturally in Brazil, which makes us reflect on its relevance in the Brazilian educational context.

Figure 53 - Football, Soccer and Other Sports in MMS Materials



Source: MMS6 (2009, p. 90)



Source: MMS7 (2009, p. 157)

7. Trinity wants to find out how football fans feel about building a new indoor football stadium for a Canadian Football League team. She goes to the stadium to survey fans after a winning game on a warm August evening.

a) Describe how the timing of her question may influence the responses.

b) In what setting might the responses be different than those Trinity received?

Source: MMS9 (2009, p. 435)

7. Which students in your school would you survey for their opinions on each topic?

a) the quality of cafeteria service

b) the cost of a gym uniform

c) the number of student parking spaces

d) the school spirit at football games

Source: MMS9 (2009, p. 444)

The soccer context appears twice in MMS6 and twice in MMS7. In Task 7 (Figure 54), the soccer context is used as a backdrop to work on the mathematical concept of proportions. The situation depicts children on a soccer field, which, despite being a familiar representation, is set in a Canadian context of the Winnipeg Youth Soccer League.[33] Although it is a valid use of soccer as a sporting theme, the relationship between the players and the training balls seems to present the sport in a restricted and specific way, without delving into the cultural and/or daily importance it has in countries like Brazil, see Figure 55.

Figure 54 - Proportions on the Field: Soccer as a Backdrop for Calculations

7. Atiba plays for the Linden Woods Vipers in the Winnipeg Youth Soccer League.
The ratio of players to soccer balls at practice sessions is 5 : 2.
How many soccer balls are needed for 20 players?

Source: MMS6 (2009, p. 182)

[33] The Winnipeg Youth Soccer Association (WYSA) was created in 1982 to coordinate the operation of youth soccer within the City of Winnipeg and its surrounding areas. Through the assistance of its member organizations, WYSA has created and operates league structures for participants aged 9 to 18. Avaiblabe at: https://winnipegyouthsoccer.com/content/about-wysa. Accessed at: 15/01/2025

The proposal in Figure 55 presents a list of sports for students to choose their favorite to watch on television. Among the options is soccer, but it appears as a secondary choice, competing with other sports more associated with Canadian culture, such as hockey and baseball. This perception is reinforced by the visual representation, in which soccer appears only as a thought, while the other sports are presented as concrete choices. Thus, although soccer is present in the proposal, its inclusion can also be considered superficial and does not reflect its centrality in contexts such as Brazil, where it would be the dominant choice.

Figure 55 - Measuring Popularity: A Comparison of Sports

➤ Each person should find an answer she would choose.

Suppose you want to find out people's favourite sports to watch on TV.
You think of asking:
What is your favourite sport to watch on TV?
☐ *Hockey* ☐ *Baseball*

Some people may prefer a different sport.
Others may not watch any sports on TV.
So, add more choices.
A better question would be:
What is your favourite sport to watch on TV?
☐ *Hockey* ☐ *Baseball* ☐ *Soccer*
☐ *Other (please specify)* ____________ ☐ *None*

Source: MMS6 (2009, p. 249)

The three tasks shown in Figure 56 (7(a), 7(b) and 7(c)) refer to the bar graph of the most popular sports among Wesley's 13- to 15-year-old. Soccer appears with intermediate prominence, but other sports, such as golf, hockey and baseball, are significantly more popular. This representation highlights the cultural weight of sports other than Brazil, relegating soccer to a less popular position. In a Brazilian context, soccer would certainly be at the top of the list, reinforcing the need to adapt the examples to the cultural profile of the students.

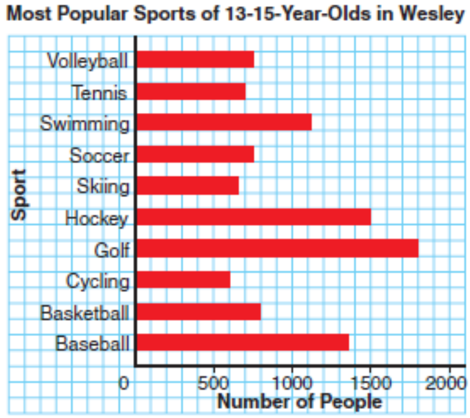
Figure 56 - Searching Preferences: TV Sports and Bar Graphs

7. Assessment Focus The graph shows the most popular sports of 13–15-year-olds in Wesley.

a) Which sports are equally popular?

b) How could you use the bar graph to find the mode? Explain and show your work.

c) Calculate the mean. Use estimated values from the graph.

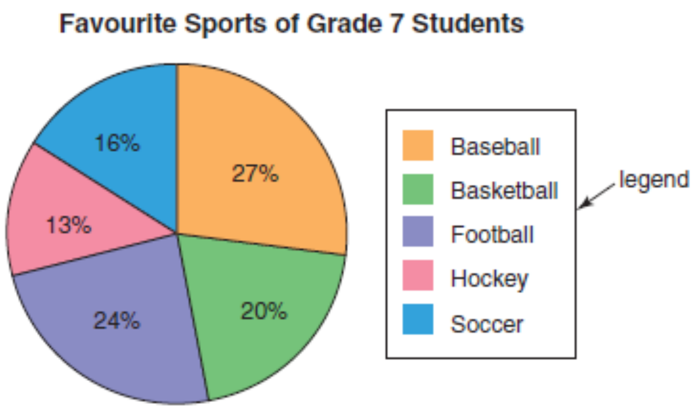


Source: MMS7 (2009, p. 261)

The circular graph in Figure 57 shows the same analysis. It shows the students' favorite sports, with soccer occupying 16% of the preferences, second only to hockey (13%), but with little difference in percentage. The predominance of sports like baseball (27%) and football (24%) again reflects a reality that is not Brazilian. For Brazilian students, this distribution would be different, with soccer occupying the largest share of the graph, in line with the country's sporting culture.

Figure 57 - Sports Chart: Distribution of Preferences in a Pie Chart

A circle graph has a title.
Each sector is labelled with a category and a percent.
A circle graph compares the number in each category to the total number.
That is, a fraction of the circle represents the same fraction of the total.
Sometimes, a circle graph has a **legend** that shows what category each sector represents.
In this case, only the percents are shown on the graph.



Source: MMS7 (2009, p. 157)

The presence of the soccer context becomes more recurrent in books MMS8 and MMS9. In both books, it can be seen that the sport is more referenced and, at times, more discussed and situated. In MMS8, the soccer context appears in 16 tasks. Of all these analyzed, the majority (12 tasks) allude to soccer in a secondary and superficial way, using the sport only as a backdrop to address mathematical concepts, such as ratio calculations (Figure 59), polyhedra (Figure 60), tax (Figure 61) or performance (Figure 62). In one case, the focus is on calculations or commercial contexts (Figure 63), without significantly exploring the sport or its cultural relevance.

In other tasks, soccer appears as one of the options in graphs showing the participation of boys and girls in sports (Figure 54). Although it is represented, the main focus of the proposal is the comparison between genders and not the sport itself, limiting in-depth study of soccer, including the discussion of equity in its practice versus genders and what this data shows in terms of cultural, social and economic issues.

Figure 58 - Percentage Calculations with Soccer: An Example in MMS8

2. A glass holds $\frac{2}{3}$ cup of juice. A jug contains 8 cups of ju
How many glasses can be filled from the juice in the jug

Source: MMS8 (2009, p. 157)

Figure 59 - Polyhedra in Play: Relating Geometric Concepts and Soccer

8. A soccer ball is not a sphere.
It is a polyhedron. Explain which polygons are joined to make the ball.
How are the polygons joined?

Source: MMS8 (2009, p. 181)

Figure 60 - Calculating Taxes: A Commercial Glimpse into the Sports Scenario

5. Suppose you are in Fort Simpson, Northwest Territories. Find the GST on each item.

c) a soccer ball that costs $17.99

Source: MMS8 (2009, p. 260)

Figure 61 - Performance and Statistics: Soccer as a Discussion Trigger

7. Which sentences use ratios? Which sentences use rates? How do you know?

d) The soccer team won 25 games and lost 15 games.

Source: MMS8 (2009, p. 298)

Figure 62 - Soccer in Business: Exploring Commercial Operations

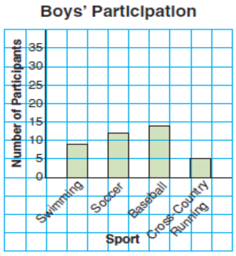
5. A school soccer team rented a bus for the day.
The bus cost $200 for the day, plus $14 for each person on the bus.
The total cost of the bus rental was $424.
How many people were on the bus?

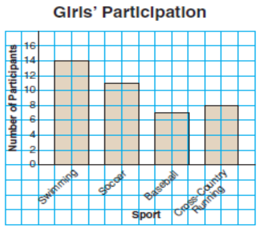
a) Write an equation you can use to solve the problem.
b) Solve the equation.
c) Check your answer. Write to explain how you know it is correct.

Source: MMS8 (2009, p. 375)

Figure 63 - Gender and Sports Practice: Comparing Boys and Girls

10. Assessment Focus Giada surveyed her classmates to find out which sports they participated in. She drew these graphs.





a) What impressions do these graphs give?
b) Describe how the graphs create a false impression.
c) What features of the graphs make it seem that the girls participate in sports more than the boys?
d) How could the graphs be changed to present the data accurately?
e) Suggest a different graph that could be used to accurately display these data.

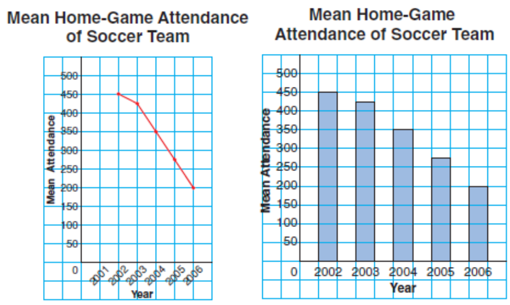
Source: MMS8 (2009, p. 401)

However, there were tasks in which soccer was approached in a broader and deeper way by presenting comparative graphs that analyzed the average attendance at matches over five years (Figure 64). Unlike the tasks mentioned above, which use the sport in a superficial way, here it becomes the main focus, serving as a basis for developing students' analytical and interpretative skills. The presence of two types of graph, a line graph and a bar graph – promotes comparison between different visual representations, allowing students to assess the advantages and limitations of each (Radford, 2009)

In addition, the tasks (9(a), 9(b), 9(c) and 9(d)) (Figure 64) involve students in critical reflection by questioning which type of graph is most appropriate for presenting data and why, encouraging them to make informed decisions.

By addressing the audience at soccer games, the tasks contextualize the sport within an everyday setting, connecting the mathematical concepts of average and graphical representations to situations familiar to the students. This approach broadens the scope of the tasks, as it not only works on statistical concepts, but also encourages an understanding of the social involvement, access and relevance of the sport.

Figure 64 - Interpreting Line and Bar Graphs: Average Soccer Attendance



9. Each graph shows the mean home-game attendance of the school's soccer team over the past 5 years.

a) What are the strengths of each graph?

b) What are the limitations of each graph?

c) Which type of graph is more appropriate to display these data? Justify your choice.

d) Could you use a circle graph to display these data? Why or why not?

Source: MMS8 (2009, p. 389)

Following Radford's (2009) reflections on the construction of meaning and the role of semiotic representations in mathematical learning, MMS9 incorporates the context of soccer into different parts of its content. Unlike previous books, this feature appears not only in tasks (Figure 66), but also in the introductory panel of subunit 8.3, Properties of Angles in a Circle (Figure 67), and in an example (Figure 68). Despite the increased presence of sport, the analysis shows a predominantly superficial use of the topic.

In tasks such as 15(bi) and 15(bii) (Figure 65), soccer is used to contextualize the production of postage stamps, even mentioning the U-20 World Cup. However, despite referring to a significant sporting event, the focus is on numerical calculation, restricting soccer to a historical (the U-20 World Cup) and cultural (the making of special postage stamps by Canada) scenario, without delving deeper into these dimensions. From the perspective of Radford (2009), this suggests that visual and textual representations are not fully exploited as semiotic resources capable of generating richer cultural meanings.

When thinking about these two dimensions in the Brazilian context, we can see a cultural misalignment. Although the importance of soccer in Brazil is undeniable – a country that has hosted World Cups and where the sport exerts a significant social and cultural influence – the emphasis on Canada's special label does not find the same cultural echo. Thus, the mention of soccer is restricted to the role of a backdrop, without really mobilizing the students' sociocultural background.

In this sense, the presentation of soccer as a mere historical and cultural backdrop does not deepen the symbolic or semiotic connections that could enrich the students' understanding – something that Radford (2009) highlights as essential for creating more meaningful learning contexts. By evoking an element (the seal) that is not a relevant cultural marker in Brazil, the material remains on a superficial level, failing to take advantage of the sport's potential as a means of constructing meaning.

Figure 65 - Special Stamps, Cups and Curiosities: Soccer in International Context

15. Canada Post often creates special postage stamps to celebrate important events and honour famous people.

b) In July 2007, Canada hosted the FIFA U-20 World Cup Soccer Championships. Canada Post issued a 52¢ stamp to honour all the players and fans.

i) How many stamps are in a 4 by 4 block? Write the number of stamps as a power.

ii) What is the value of these stamps?

Source: MMS9 (2009, p.56)

5. a) In each case, will the selected sample represent the population? Explain.

iii) To find out if the soccer league should buy new uniforms for the players, 20 parents of the students in the soccer league were surveyed.

b) If the sample does not represent the population, suggest another sample that would. Describe how you would select that sample.

Source: MMS9 (2009, p.448)

Discuss the ideas

1. How can the circle properties in this lesson help you decide where soccer players need to stand to have the same shooting angle on goal?

Source: MMS9 (2009, p.409)

Figure 66 - Properties of Angles in Circles: When Soccer Becomes Geometry

8.3 Properties of Angles in a Circle

FOCUS

- Discover the properties of inscribed angles and central angles, then solve related problems.

A soccer player attempts to get a goal. In a warm-up, players line up parallel to the goal line to shoot on the net. Does each player have the same shooting angle? Is there an arrangement that allows the players to be spread out but still have the same shooting angle?

Source: MMS9 (2009, p. 404)

Figure 67 - Bias in Research: Example of Recent Influence in Soccer

Example 1 Identifying and Eliminating Potential Problems

For each survey question, explain why a problem may occur and the effect it would have on the results. Suggest how each problem could be avoided.

b) A survey is conducted to find out the level of school spirit. Students are polled about their level of school spirit after the soccer team wins the championship.

A Solution

b) The timing of the survey question could be problematic.
Since the school's soccer team just won the championship, the level of school spirit would be higher than usual.
The results of the survey may show a higher level of school spirit than if the survey was conducted at another time.
Asking students the same question a month later, when no school event is occurring, should produce more accurate results.

Source: MMS9 (2009, p.433)

Following Radford's (2009) reflections on the sociocultural dimension in the construction of mathematical meanings, it can be seen that the superficial use of sporting themes can be seen in tasks 5(aiii) and 5(b) (Figure 65). In these activities, soccer appears to justify research into the need for new uniforms, associating it with the study of sampling, but without delving into social or cultural dimensions that could give greater relevance to this context.

In Example 1 (Figure 67), also linked to the sampling study, the sport is invoked to illustrate possible problems in statistical research - such as the influence of recent results (the team's victory) – but does not promote broader reflections on the social or cultural significance of soccer. This use reinforces what Radford (2009) describes as attributing a form of apparent contextualization to the themes, because although there is mention of a cultural phenomenon, its potential to generate deeper reflections or discuss the collective values and practices of the participants is not explored.

On the other hand, in Task 1, Discussion of Ideas (Figure 68) and in the introduction to Subunit 8.3, Properties of Angles in a Circle (Figure 67), soccer appears in a broader and deeper way. On both occasions, sport is the axis for exploring geometric concepts (angles, properties of circles), relating mathematics directly to game situations. In this sense, the practical and visual integration of soccer creates a meaningful connection between the mathematical content and everyday sports life, which, in line with Radford (2009), enhances the construction of meaning as students relate mathematical representations to their concrete experiences.

The positioning of the players to obtain the same angle of kick, for example, refers to a familiar use of the sport, allowing students who practice or follow soccer to give greater meaning to the concepts being worked on. From this perspective, familiarity with the sport brings mathematical knowledge closer to everyday experiences, broadening the relevance of the concepts presented, as well as reflections that are more contextualized and coherent with the student's sporting experience (Reis, 2019). Although soccer appears several times in the MMS collection, a detailed analysis of the mentions of the sport reveals that, in most cases, its potential as a cultural and historical element remains little explored.

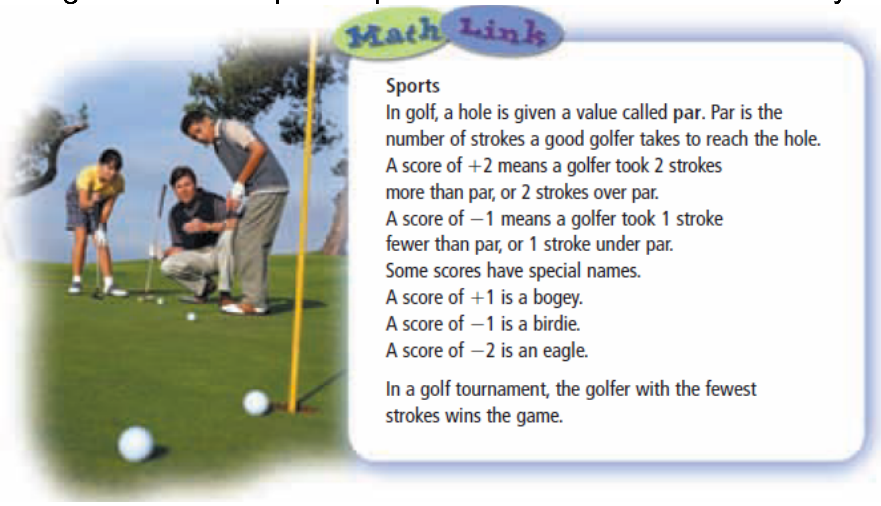
In most of the activities, it is only used as a backdrop for the application of mathematical concepts, without mobilizing, as Radford (2009) suggests, the sociocultural dimension that could enrich the learning process. This mostly superficial approach to soccer indicates a cultural perspective centered on the Canadian context, disregarding other realities, such as Brazil, where soccer plays a central role in social and cultural identity. As the books are produced in Canada, it is understandable that they prioritize sports such as field hockey and baseball, which are familiar to Canadian students.

However, it is problematic to assume equivalence of cultural, social and economic contexts, since in Brazil, soccer enjoys widespread recognition, transcends socio-economic and geographical barriers, and is a cultural symbol for a large part of the population. In addition, the MMS collection covers other sports, such as golf, baseball, snowboarding, climbing, ice skating, lacrosse[34]

[34] Lacrosse boasts a captivating history that spans centuries. The sport's genesis can be traced back to the Indigenous peoples of North America, who played variations of the game as early as the 12th century. These early iterations were more than recreational, they held significant cultural and spiritual importance, often serving as rituals and training for war. The game's evolution saw it adapt and grow in popularity, with various tribes and communities developing their unique rules and playing styles. European settlers in the 17th century observed and adopted lacrosse, which eventually helped lead to its widespread recognition. Over time, organized leagues and clubs emerged, cementing

Paralympic cycling, biathlon and horse riding (Figure 68), which have little adherence to the Brazilian cultural context. In some cases, such as ice hockey and snowboarding, climatic factors make these sports unfeasible in most regions of the country.

Figure 68 - Infrequent Sports: Reflections on Accessibility


Math Link

Sports

In golf, a hole is given a value called **par**. Par is the number of strokes a good golfer takes to reach the hole.

A score of +2 means a golfer took 2 strokes more than par, or 2 strokes over par.

A score of −1 means a golfer took 1 stroke fewer than par, or 1 stroke under par.

Some scores have special names.

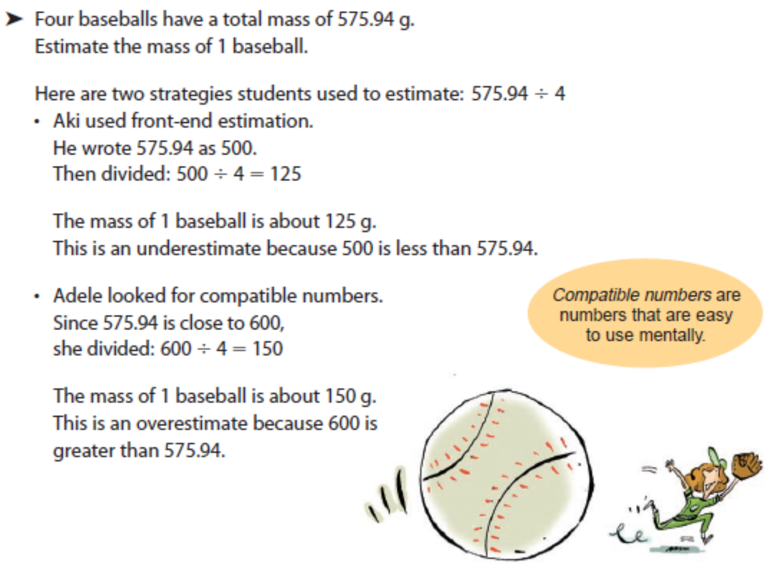
A score of +1 is a bogey.

A score of −1 is a birdie.

A score of −2 is an eagle.

In a golf tournament, the golfer with the fewest strokes wins the game.

Source: MMS7 (2009, p. 54)


➤ Four baseballs have a total mass of 575.94 g.
Estimate the mass of 1 baseball.

Here are two strategies students used to estimate: 575.94 ÷ 4

- Aki used front-end estimation.
  He wrote 575.94 as 500.
  Then divided: 500 ÷ 4 = 125

  The mass of 1 baseball is about 125 g.
  This is an underestimate because 500 is less than 575.94.

- Adele looked for compatible numbers.
  Since 575.94 is close to 600,
  she divided: 600 ÷ 4 = 150

  The mass of 1 baseball is about 150 g.
  This is an overestimate because 600 is greater than 575.94.

Compatible numbers are numbers that are easy to use mentally.

Source: MMS6 (2009, p. 93)

lacrosse's position as one of the most thrilling and competitive team sports in the world. With its rich heritage and continuous global expansion, lacrosse remains a symbol of tradition, athleticism and cultural unity, reflecting both its origins and the diverse communities that have embraced it. Link: https://worldlacrosse.sport/the-game/origin-history/. Accessed at: 18/01/2025

Example 3 **Using a Probability to Support Opposing Views**

Jon wants to learn how to snowboard but does not want to take lessons. His mother insists that Jon take lessons. Jon and his mother find an article that claims:

68% of snowboarding injuries occur during beginner lessons

Explain how both Jon and his mother can use this statistic to support their opinions.

Source: MMS9 (2009, p. 426)

Figure 68 - Infrequent sports: Reflections on Accessibility

22. Three hikers are returning to base camp after a mountain climbing expedition. Hiker A is 26.4 m above base camp, hiker B is 37.2 m below base camp, and hiker C is 15.7 m below base camp.

a) Represent each hiker's distance above or below base camp as a rational number.

b) Sketch and label a vertical number line to show the base camp and the positions of the hikers.

c) Which hiker is closest to base camp? Explain your reasoning.

d) Which hiker has the lowest altitude? How do you know?

Source: MMS9 (2009, p. 103)

11. Taho plays shinny every 2 days. He plays lacrosse every 3 days. Suppose Taho plays shinny and plays lacrosse on October 1. What are the next 3 dates on which he will play shinny and play lacrosse? Explain how you know.

Source: MMS6 (2009, p. 57)

Jen and Rodrigo are planning a surprise skating party for their friend Lacy. They use a secret code to send messages to each other.

To create their code, Jen and Rodrigo wrote the position number of each letter in the alphabet.

Source: MMS6 (2009, p. 42)

The Paralympic Games are an international sports competition for athletes with disabilities. They are held in the same year and city as the Olympic Games. Vancouver was named host of the 2010 Paralympic Games.

For most paralympic sports, the athletes are grouped into classes according to their balance, coordination, range of motion, and skills required for the sport.

Chantal Petitclerc, French-Canadian Paralympian and 5-Time Gold Medalist, Beijing 2008

Source: MMS6 (2009, p. 103)

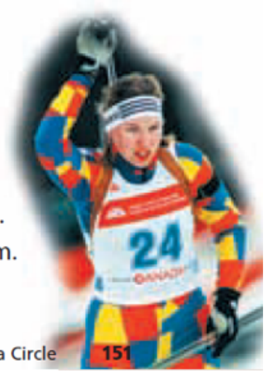
Figure 68 - Infrequent sports: Reflections on Accessibility

5. In the biathlon, athletes shoot at targets. Find the area of each target.

a) The target for the athlete who is standing is a circle with diameter 11.5 cm.

b) The target for the athlete who is lying down is a circle with diameter 4.5 cm.

Give the answers to the nearest square centimetre.



4.5 Area of a Circle 151

Source: MMS7 (2009, p. 151)

14. An equestrian was penalized a total of 24 points over a number of performances. The mean number of points lost per performance was –6. How many performances did the equestrian make?

a) Write this problem as a division expression using integers.

b) Solve the problem.

Source: MMS8 (2009, p. 88)

In the books in the MMS collection, it is understandable that sports familiar to Canadian students, such as field hockey and baseball, are prioritized. However, it is inappropriate to assume that this sociocultural scenario is equivalent to the Brazilian context. In Brazil, soccer is widely recognized, representing a cultural symbol that transcends socio-economic and geographical barriers. In addition, the collection contains references to various other sports (golf, baseball, snowboarding, climbing, ice skating, lacrosse, Paralympic cycling, biathlon, horse riding) that are not very well connected to the Brazilian reality. In some cases, climatic factors make it impossible to practice these sports in Brazil, while in others, socio-economic aspects restrict access.

The question therefore arises: what is the cultural, social and economic profile of the students covered by the teaching materials? Which cultures and cultural contexts are really being valued? Although books can open up horizons to new sporting realities, the massive presence of sports that are unfamiliar to Brazilian students can create distance, making it difficult for them to identify and engage with the content. On the other hand, the occasional inclusion of lesser-known sports can enrich students' cultural repertoire, as long as it is balanced with examples close to their real context.

In line with Radford (2009), whose work reinforces the importance of considering the cultural signs that mediate learning, it is essential that the teaching material not only presents mathematical content, but does so through contexts that dialog with the students' lives. After all, when sports contexts reflect the daily lives and experiences of those who learn, they become meaningful and relevant, promoting greater engagement and understanding.

Therefore, although it is not a question of excluding contexts such as ice hockey from Brazilian textbooks, it is essential that such examples are not used predominantly to develop mathematical concepts. Overvaluing sports that are far removed from the local reality can compromise students' understanding and interest, while a balance between familiar and unfamiliar contexts can stimulate both learning and the cultural broadening of students.

This balance is particularly important for aligning the use of sports contexts with a sociocultural perspective of math teaching that simultaneously values students' familiarity and the expansion of their cultural horizon.

6.2.2 Art, monuments and architecture: when geometry comes to life

Architecture encompasses the design and planning of physical spaces - houses, buildings and landscapes - taking into account functional, aesthetic, environmental and cultural aspects. Based on this understanding, this category proposes to investigate how the architectural context is presented in the books in the MMS collection and to what extent this architectural diversity is reflected in the content proposed.

A culturally rich and personally meaningful approach, as argued by Bishop (1988) and Ladson-Billings (2014), emphasizes that mathematics is inseparable from cultural perceptions and social norms. Thus, when we situate mathematics teaching in architectural contexts, we not only illustrate concepts with concrete examples, but also enable discussion about how different cultures and eras produce spaces that reveal varying perspectives on form, function and aesthetics. In this way, mathematics ceases to be isolated knowledge and becomes a living part of cultural practices, which reinforces its educational potential.

In the MMS collection, the connection between art, architecture and mathematics appears explicitly, as illustrated in the introduction to Unit 8 of the MMS8 book (Figure 69). In this section, there are images of various buildings and architectural monuments: a metal truss bridge – reminiscent of large railway or highway bridges - modern urban constructions, the Geodesic Dome at Science World in Vancouver (Canada)[35] a contemporary, possibly commercial-style building and glass pyramid structures reminiscent of the Louvre Pyramid in Paris.

These references show the intersection between mathematics - especially in the field of geometry – and architecture, highlighting shapes such as domes, pyramids and triangular structures.

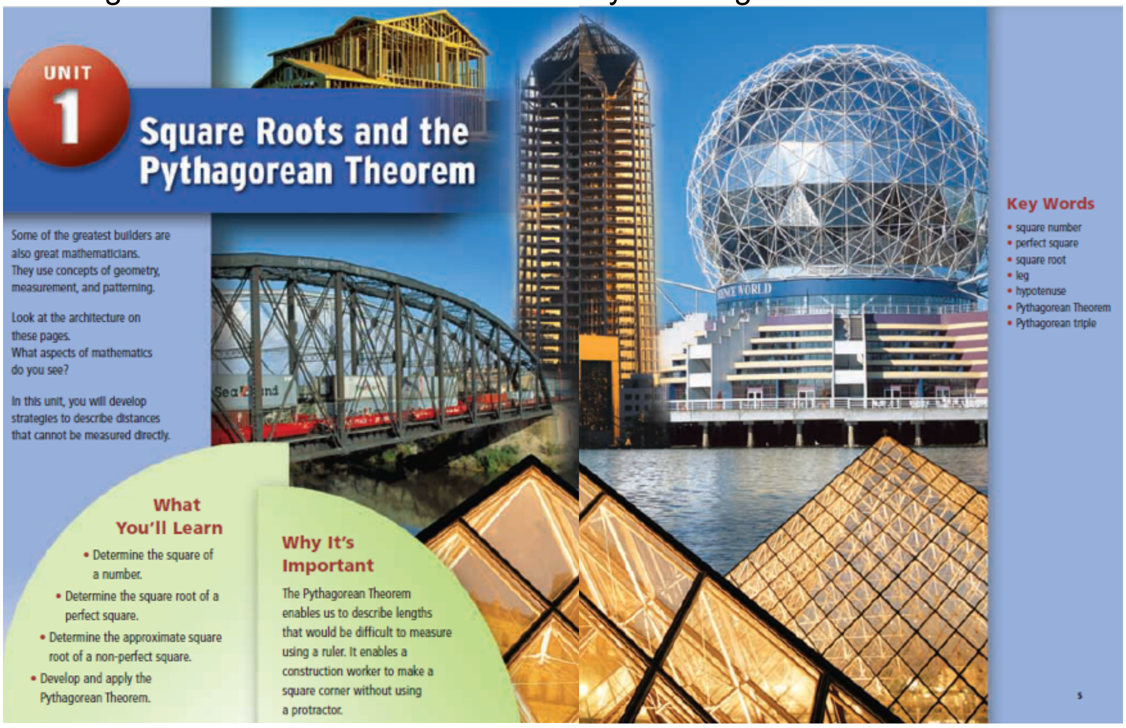
These concrete examples contribute to the understanding of fundamental concepts, such as the Pythagorean Theorem and operations involving square roots. In addition, the introduction of the unit itself highlights the value of studying geometry based on real constructions, in line with the critical perspective of (Bishop, 1988) for whom mathematics should be approached in a way that is

[35] Science World's Geodesic Dome, located in Vancouver, Canada, is an architectural icon designed for Expo86. Its design was inspired by the geodesic structures developed by Buckminster Fuller, providing an efficient and resistant construction. Science World British Columbia (2020). *Built to be torn down, dreamed to last: The history behind Science World's dome*. Science World. Retrieved January 29, 2025, from https://www.scienceworld.ca/stories/built-to-be-torn-down-dreamed-to-last-the-history-behind-science-worlds-dome/

sensitive to diverse cultural realities, providing more contextualized and meaningful learning.

> Some of the greatest builders are also great mathematicians.
> They use concepts of geometry, measurement, and patterning.
> Look at the architecture on these pages.
> What aspects of mathematics do you see?
> In this unit, you will develop strategies to describe distances
> that cannot be measured directly (MMS, 2009, p. 4).

Figure 69 - Architecture and Geometry: Starting Unit 8 in the MMS8



Source: MMS8 (2009, p. 4-5)

Therefore, the analysis of the collection revealed two distinct scenarios related to the category in question. The first concerns situations in which art and architecture are only mentioned or illustrated, while the second deals with a context of art and architecture that, at first glance, is given greater prominence.

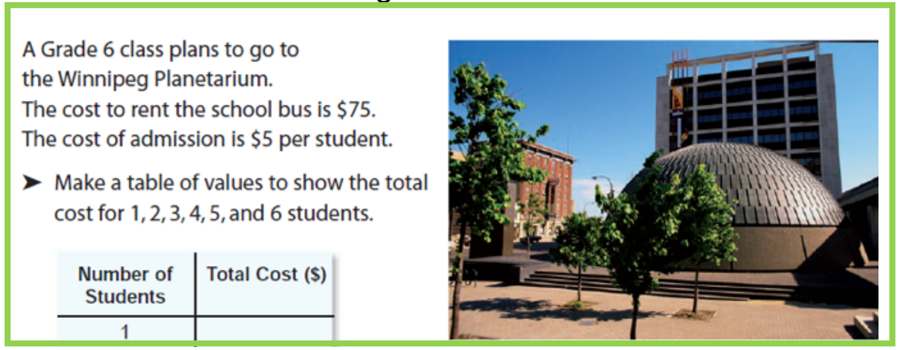
In the first scenario, there are several references to elements such as planetariums, houses, towers and buildings (as shown in Figure 69). These mentions appear mainly through images - some accompanied by brief textual descriptions (as in the case of the planetarium and the tower, highlighted in Figure 69), but others are presented in isolation, without any contextual mention (as in the case of one of the houses, also highlighted in Figure 69).

It can be seen that these examples do not explore the potential local, social, economic, architectural, cultural and/or historical aspects of the elements depicted. Instead, these references appear in an ad hoc manner, with the main aim of linking the proposed activities to a real-world context.

In this sense, Radford (2010) points out that the insertion of images or everyday examples should transcend mere illustration, as the different representations (visual, verbal or symbolic) only make full sense when articulated to broader discussions, encompassing cultural and historical meanings. Similarly, Bishop (1988) and Ladson-Billings (2014) emphasize that the practices of teaching and learning mathematics can benefit from delving deeper into the social and cultural dimensions of the examples presented, enhancing the understanding of concepts.

However, in the case analyzed, the superficial use of art and architecture elements indicates a limited approach, which does little to explore the multiple possibilities of contextualization pointed out in the literature. If, on the one hand, the presence of these references may signal an attempt to bring mathematical content closer to everyday situations, on the other hand, their restricted treatment ignores historical, cultural and social layers that can enrich the teaching and learning process. In this way, the scenario found lacks greater depth, both in the selection of what is presented and in the way mathematical concepts are related to contexts that are actually meaningful to the student.

Figure 70 - Planetarium and Urban Towers: Brief Illustrations in MMS6 and MMS7


A Grade 6 class plans to go to the Winnipeg Planetarium.
The cost to rent the school bus is $75.
The cost of admission is $5 per student.

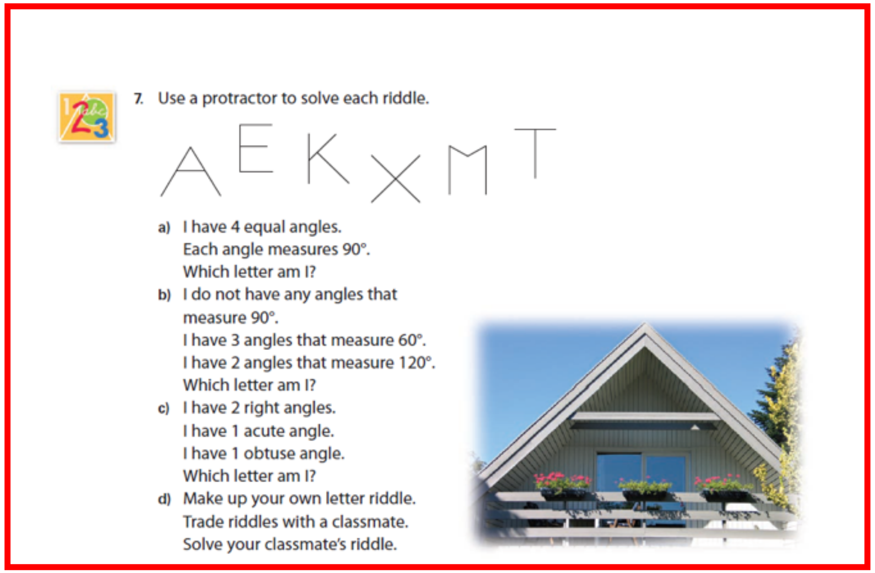
➤ Make a table of values to show the total cost for 1, 2, 3, 4, 5, and 6 students.

Number of Students	Total Cost ($)
1	

Source: MMS6 (2009, p. 19)


5. Here is the truss of the Burrard Street Bridge in Vancouver, BC.
Which types of triangles do you see in the truss? How could you check?

Source: MMS6 (2009, p. 202)


7. Use a protractor to solve each riddle.

A E K X M T

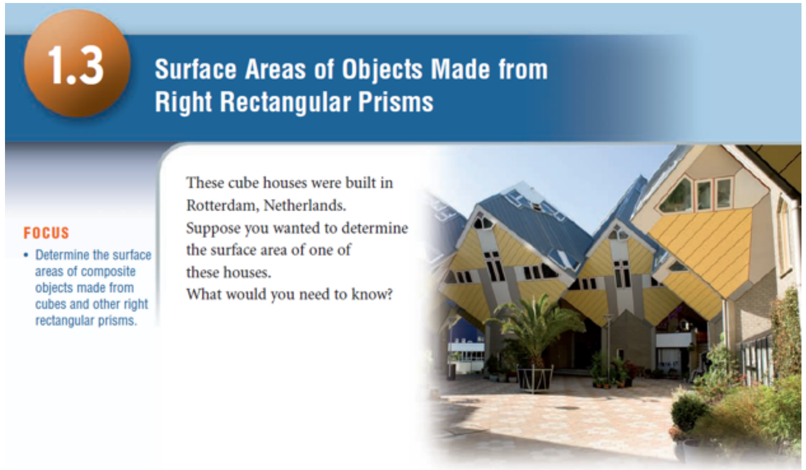
a) I have 4 equal angles.
Each angle measures 90°.
Which letter am I?

b) I do not have any angles that measure 90°.
I have 3 angles that measure 60°.
I have 2 angles that measure 120°.
Which letter am I?

c) I have 2 right angles.
I have 1 acute angle.
I have 1 obtuse angle.
Which letter am I?

d) Make up your own letter riddle.
Trade riddles with a classmate.
Solve your classmate's riddle.

Source: MMS7 (2009, p. 138)


1.3 Surface Areas of Objects Made from Right Rectangular Prisms

FOCUS
- Determine the surface areas of composite objects made from cubes and other right rectangular prisms.

These cube houses were built in Rotterdam, Netherlands.
Suppose you wanted to determine the surface area of one of these houses.
What would you need to know?

Source: MMS9 (2009, p.25)

Example 2 **Using a Scale Factor to Determine Dimensions**

This photo of longhouses has dimensions 9 cm by 6 cm.
The photo is to be enlarged by a scale factor of $\frac{7}{2}$.
Calculate the dimensions of the enlargement.

Source: MMS9 (2009, p. 320)

9. We measure time in hours.
Suppose 12 noon is represented by the integer 0.
a) Which integer represents 1 P.M. the same day?
b) Which integer represents 10 A.M. the same day?
c) Which integer represents 12 midnight the same day?
d) Which integer represents 10 P.M. the previous day?
Describe the strategy you used to find the integers.

Clock Tower, Calgary's Old City Hall

Source: MMS6 (2009, p. 77)

2. Waldo paid $29.85 for 3 admission tickets to the Calgary Tower.
Estimate the cost of one admission ticket.

Source: MMS6 (2009, p. 94)

2. The tallest building in the world is the Taipei 101 in Taiwan. Its height is 0.509 km. The tallest building in North America is the Sears Tower in Chicago, USA. Its height is 0.442 km. What is the difference in the heights of the buildings?

Source: MMS7 (2009, p. 98)

On some occasions, even within this first scenario, the material analyzed not only mentions art and architectural monuments, but also explores - albeit briefly - historical and cultural aspects related to these constructions (Figures 69 and 70). This approach helps to enrich the reader's understanding, as it places these works in their respective contexts of origin, history and cultural relevance (MMS9; Fiorin, 2008; Saito & Dias, 2013).

The Mesopotamian ziggurats [36]highlighted in blue in Figure 70, illustrate this intersection between mathematics, architecture and the culture of ancient peoples. Built by Assyrians and Babylonians, these monuments display architectural layers that incorporate geometric and structural concepts, showing the practical application of mathematics in large-scale works. At the same time, they reflect the spiritual dimension of these buildings, since they functioned as religious temples.

The red highlight in Figure 70 refers to Egyptian civilization, presenting the pyramids and hieroglyphs as cultural and historical elements intrinsically linked to the mathematical development of that society. As well as having a strong visual and cultural impact, the pyramids are examples of precise calculations and advanced construction techniques, demonstrating remarkable geometric mastery in the execution of their shapes.

The hieroglyphs, on the other hand, show the complexity of the Egyptian writing system, whose elaboration required organized knowledge to transmit messages in different languages, including Greek. According to the material: “Over 2000 years ago, the Egyptians carved the same message in stone in different languages, including hieroglyphics and Greek. By comparing the texts, scholars were able to solve the puzzle of Egyptian hieroglyphics” (MMS, 2009, p. 199).

[36] ziggurat, pyramidal stepped temple tower that is an architectural and religious structure characteristic of the major cities of Mesopotamia (now mainly in Iraq) from approximately 2200 until 500 bce. The ziggurat was always built with a core of mud brick and an exterior covered with baked brick. It had no internal chambers and was usually square or rectangular, averaging either 170 feet (50 metres) square or 125 × 170 feet (40 × 50 metres) at the base. Approximately 25 ziggurats are known, being equally divided among Sumer, Babylonia, and Assyria. Avaiblabe at: https://www.britannica.com/technology/ziggurat. Accessed at: 18/01/2025

This excerpt illustrates how interdisciplinarity between mathematics, history, culture and linguistics has fostered significant advances in understanding ancient systems of communication and recording (Saito & Dias, 2013; Fiorin, 2008). Although the material gives visibility to these aspects, it is important to emphasize the importance of further deepening this historical and cultural perspective, in order to highlight not only the practical application of mathematical concepts, but also the social, political and economic dynamics that underpinned such architectural and symbolic productions.

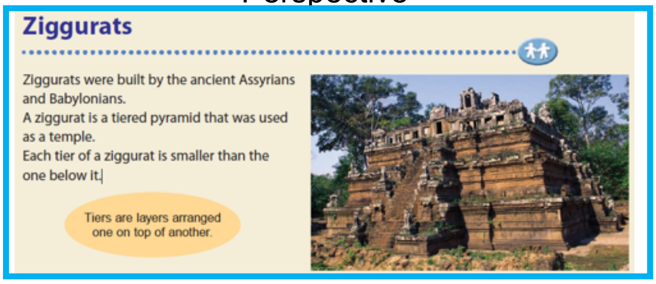
In this way, the critical view of the subject is broadened, showing that mathematics, combined with historical and cultural knowledge, offers multiple opportunities for reflection and learning. On some occasions, even within this first scenario, the references to art and architectural monuments in the material analyzed go beyond their mere mention, thus addressing, even if briefly, historical and cultural aspects linked to them. These mentions enrich understanding by placing the art and/or monument in the context of its origin, history and cultural relevance. Examples can be seen in Figure 71.

The Mesopotamian ziggurats, highlighted in blue in Figure 71, exemplify the intersection between mathematics, architecture and the culture of ancient peoples. Built by Assyrians and Babylonians, these monuments have architectural layers that incorporate geometric and structural concepts, showing the practical application of mathematics in large-scale works. In addition, these constructions are deeply related to spirituality, since they were used as religious temples.

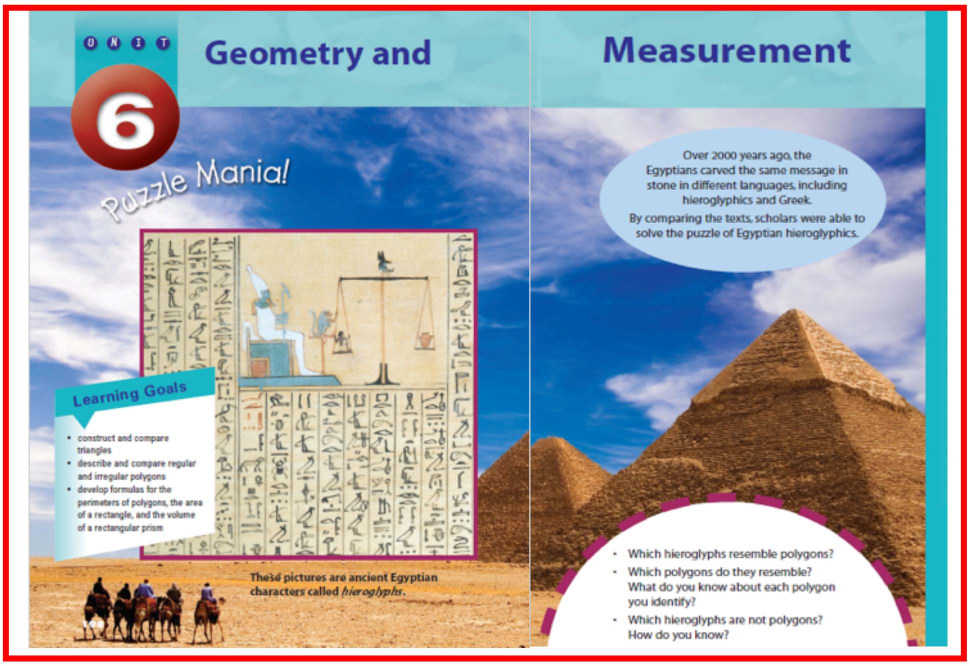
The red highlight in Figure 71 highlights Egyptian civilization, presenting the pyramids and hieroglyphs as cultural and historical elements intrinsically linked to the mathematical development of that society. The pyramids, in addition to their visual and cultural impact, are examples of precise and advanced calculations, demonstrating the geometric mastery required for their construction. The hieroglyphs, in turn, reflect the complexity of the Egyptian writing system, which required organized knowledge to transmit messages in different languages, including Greek.

As described in the material: “Over 2000 years ago, the Egyptians carved the same message in stone in different languages, including hieroglyphics and Greek. By comparing the texts, scholars were able to solve the puzzle of Egyptian hieroglyphics" (MMS, 2009, p. 199). This example shows how interdisciplinarity between mathematics, history, culture and linguistics has enabled significant advances in understanding ancient systems of communication and recording (Fiorin, 2008; Saito & Dias, 2013).

Figure 71 - From Ziggurats to Totems: Mesopotamia, Egypt and First Nations in Perspective

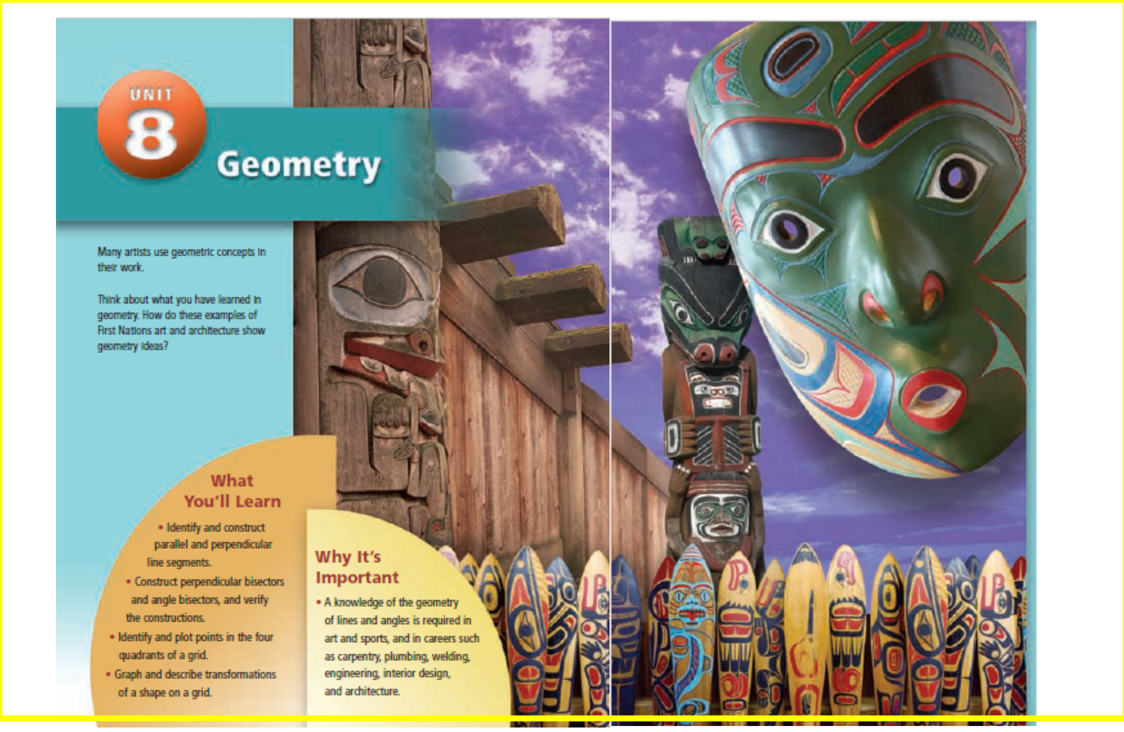


Source: MMS6 (2009, p.158)

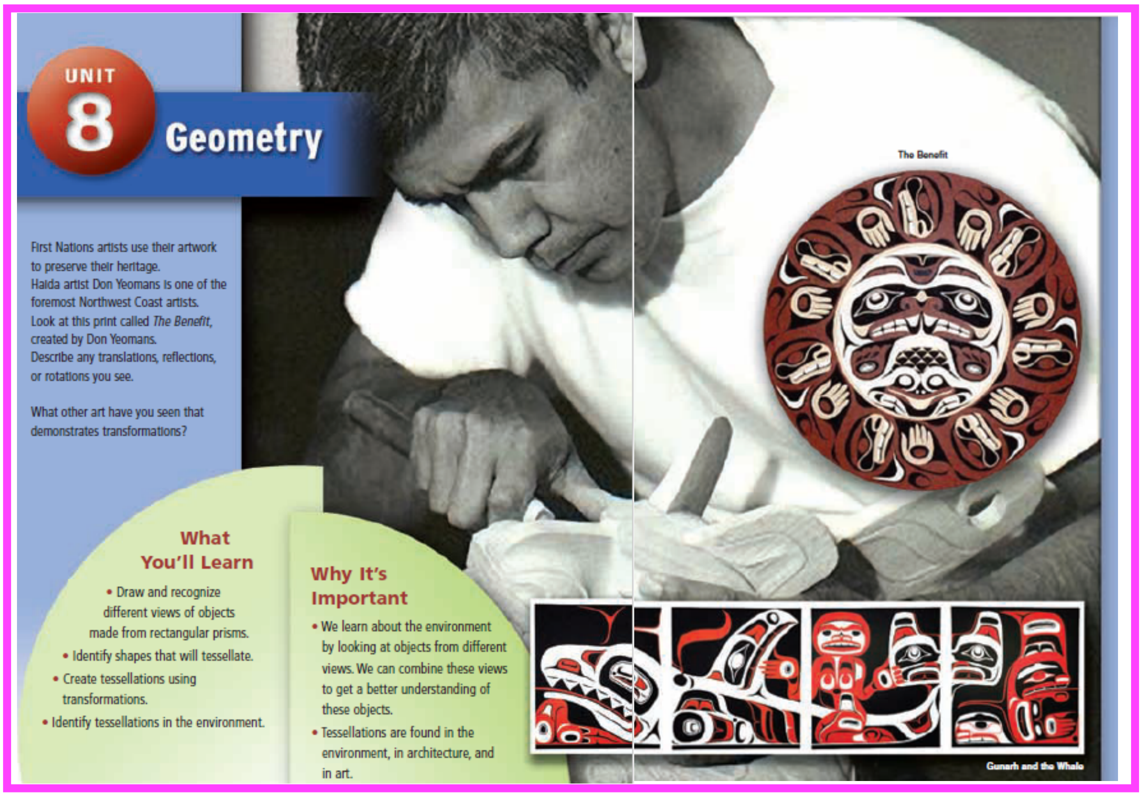


Source: MMS6 (2009, p.198-199)

Figure 72 - Thunderbird Park e K'san Village: Tradições Indígenas em BC



Source: MMS7 (2009, p.298-299)



Source: MMS7 (2009, p.432-433)

The following two examples (highlighted in yellow and pink in Figure 72) show the richness of First Nations artistic and architectural production by linking art, geometry and cultural preservation. In the example highlighted in yellow in Figure 72, we see sculptures and tribal masks that integrate geometric principles, revealing not only artistic skill, but also mathematical knowledge applied to the creation of symmetrical and proportional shapes. The architecture of these communities, as exemplified by the totems, balances functionality and symbolism, conveying ancestral narratives and spiritual traditions.

These totems, in particular, illustrate the cultural depth and symbolic complexity of the First Nations, as they are used to tell stories, demarcate territories and honor ancestors. Their construction involves mastery of carefully planned three-dimensional shapes to convey balance and visual harmony. On the other hand, the colorful and ornate masks demonstrate the use of rotational and reflective symmetries, aspects that reinforce the intertwining of aesthetics, spirituality and mathematical knowledge.

In the last example (pink highlight in Figure 72), the work The Benefit, created by Haida artist Don Yeomans, reaffirms the continuity of the First Nations artistic tradition while at the same time dialoguing with contemporaneity. The presence of tessellations and geometric transformations in his composition underscores the structuring role of mathematics in art

Furthermore, as Silva (2016)(Silva, 2016), indigenous art functions as a strategy of resistance, preserving cultural heritage in the face of constant economic and social pressures that threaten these groups. These examples, although brief, indicate how First Nations cultural and artistic manifestations transcend purely aesthetic value, carrying historical, spiritual and identity meanings.

In contexts marked by transformation and adversity, art emerges as a vehicle of resilience and resistance, keeping the collective memory of these populations alive and reinforcing the importance of mathematical knowledge deeply rooted in sociocultural practices. After discussing this first scenario, the second deals with a context of art and architecture that, at first glance, receives more attention.

This is the last unit of the MMS6, entitled Transformations (see Figure 73), in which concepts of geometric transformations are explored. According to the MMS6, it is in this unit that students are invited to:

- draw shapes in the first quadrant of a Cartesian plane
- draw and describe images on a plane after single transformations
- draw and describe images after combinations of transformations,
  with and without technology
- create a design by transforming one or more shapes
- identify and describe transformations used to produce an image or a design (MMS6, 2009, p. 288).

Figure 73 - Lesson 1: The First Step in Geometric Transformations

UNIT 8 **Transformations**

Source: MMS6 (2009, p. X)

These learning objectives are apparently linked to the context of art and architecture. Here, the book introduces the content by contextualizing the title Art and Architecture by showing three images of monuments located in British Columbia, Canada (Figure 64).

The first image (highlighted in red in Figure 64) features Thunderbird Park in Victoria. According to the book:

> Longhouses have long been the centre of social activity in West Coast First Nations communities. The longhouse is usually built from large cedar posts, beams, and boards. The outsides of the longhouses are often decorated with art, and there is always a totem pole in front (MMS6, 2009, p. 288).

According to the Vancouver Island Bucket List website[37]website, Thunderbird Park "is more than just a green space in the heart of Victoria; it is

[37] Avaiblabe at: https://vancouverislandbucketlist.com/experiences/thunderbird-park/. Accessed at: 16/12/2024.

loaded with culture and history that offers a deep dive into British Columbia's rich indigenous heritage."

The site briefly presents its history. According to it, the:

> Thunderbird Park está ligado ao Royal BC Museum e foi estabelecido em 1941 para exibir totens das coleções provinciais. Em 1952, um programa de restauração de quarenta anos começou com o chefe Martin como o escultor chefe. Em 1992, todos os totens foram substituídos por réplicas, e os originais foram movidos para dentro para preservação (Vancouver Island Bucket List, site).

Figure 74 - Geometric Art: Designs Generated by Transformations

Source: MMS6 (2009, p. 288-289)

The second image (highlighted in yellow in Figure 74) shows K'san Village, located in Hazelton. K'san is a historic village and living museum of the indigenous Gitxsan people in Skeena country in northwestern British Columbia, Canada.

K'san is situated near the confluence of the Bulkley and Skeena rivers, located close to the ancient village of Gitanmaax. According to The Hazeltons website[38]website, K'san is a place where the rich Gitxsan cultural heritage has been celebrated for over 8,000 years, preserving traditions and paying homage

[38] Avaiblabe at: http://www.hazeltonstourism.ca/ksan-historical-village-and-museum.html. Accessed at: 16/12/2024.

to generations past and present. K'san “is a premier showcase of Aboriginal culture” (The Hazeltons, website).

Finally, the third image (highlighted in orange in Figure 74) illustrates the First Nations Longhouse, located at the University of British Columbia. Next to it, the book mentions that: “In 1993, the University of British Columbia opened The First Nations Longhouse. It is a meeting place and library for First Nations students. The construction was overseen by First Nations elders, and it reflects the architectural traditions of the Northwest Coast” (MMS6, 2009, p. 289).

It is considered that the monuments presented in the introduction to Unit 8 exemplify, albeit not explicitly, the integration between mathematics and cultural heritage. By exploring geometric transformations – such as reflections, translations and rotations - in artistic and architectural contexts, students have the opportunity to realize that it is possible to understand mathematics in different cultures and historical contexts (Leme Da Silva & Jahn, 2024).

In addition to developing skills such as spatial perception and geometric visualization, this approach allows for a critical and reflective look at the role of mathematics in preserving cultural traditions.

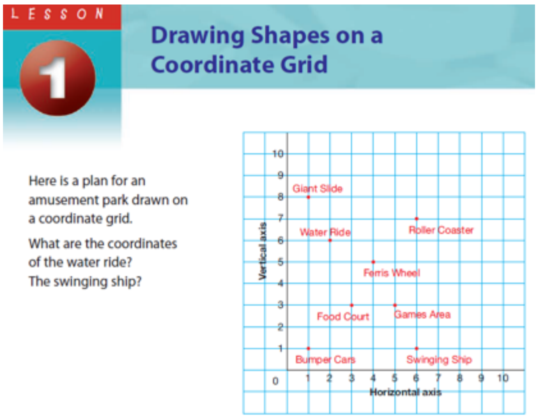
The architectural monuments, with their symmetrical structures and geometric patterns, not only enrich learning, but also encourage an appreciation of cultural diversity and an understanding of its relevance in the contemporary world.

The content of Unit 8 begins with Lesson 1, which focuses on developing skills related to basic transformations on a coordinate grid (Figure 75).

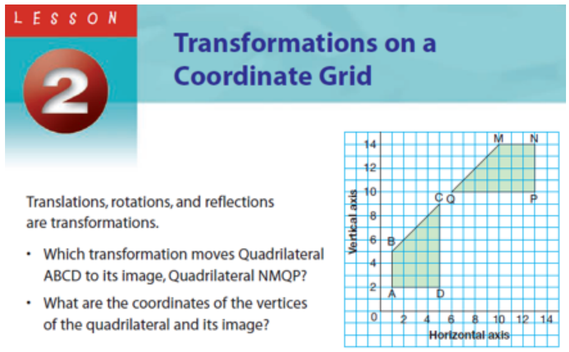
The aim here is to introduce students to the exploration of fundamental concepts, such as the location of points and displacement in a Cartesian coordinate system.

Lesson 2 begins by working concretely with geometric transformations - reflections, translations and rotations - using the Cartesian coordinate system (Figure 75).

Figure 75 - Lesson 1: The First Step in Geometric Transformations



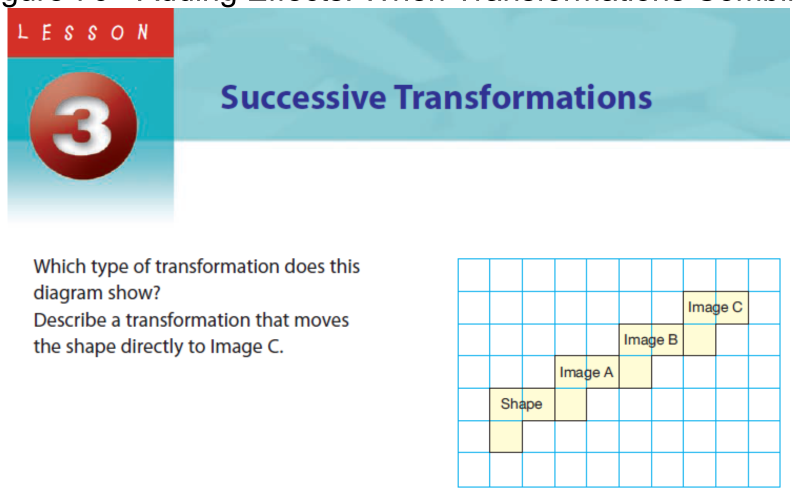
Source: MMS6 (2009, p. 290)



Source: MMS6(2009, p. 295)

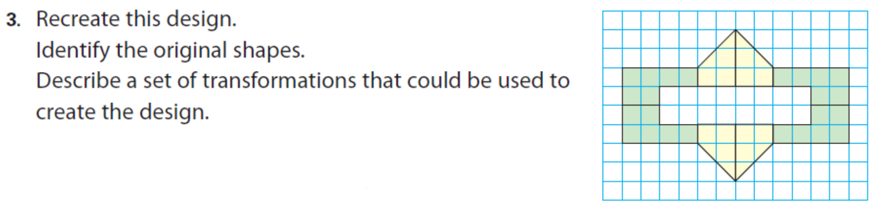
Subsequent Lessons deepen these concepts by incorporating successive transformations (Figure 76) and the combination of different types of transformations (Figure 77), culminating in proposals that explore the creation of designs (Figure 78).

Figure 76 - Adding Effects: When Transformations Combine



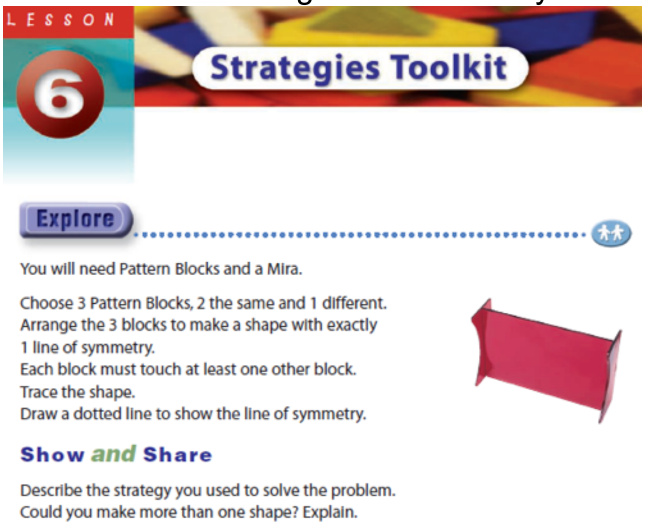
Source: MMS6 (2009, p. 303)

Figure 77 - Creating Complex Patterns: New Layers of Transformation



Source: MMS6 (2009, p. 316)

Figure 78 - Geometric Art: Designs Generated by Transformations


LESSON 6

Strategies Toolkit

Explore

You will need Pattern Blocks and a Mira.

Choose 3 Pattern Blocks, 2 the same and 1 different.
Arrange the 3 blocks to make a shape with exactly 1 line of symmetry.
Each block must touch at least one other block.
Trace the shape.
Draw a dotted line to show the line of symmetry.

Show and Share

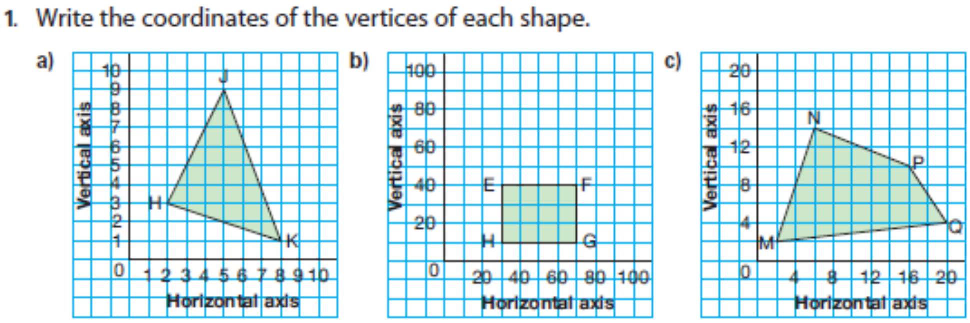
Describe the strategy you used to solve the problem.
Could you make more than one shape? Explain.

Source: MMS6 (2009, p. 318)

An analysis of the Lessons reveals that, throughout their content, references to art and/or architectural monuments were practically non-existent, only occurring on two occasions.

Visualizations of geometric transformations were predominantly limited to working with regular polygons represented in Cartesian coordinate systems or on grids. This pattern can be illustrated by Figure 79, which shows examples of tasks proposed in these circumstances.

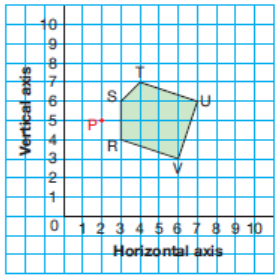
Figure 79 - Polygons in Action: Practical Transformations Exercises



Source: MMS6 (2009, p. 293)

7. Copy this pentagon on a coordinate grid.
Write the coordinates of each vertex.
For each transformation below:
- Draw the image.
- Write the coordinates of the vertices of the image.
- Describe how the positions of the vertices of the pentagon have changed.

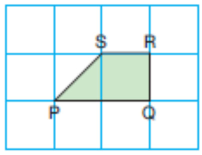
a) a translation 2 units right and 3 units up
b) a reflection in the vertical line through the horizontal axis at 5
c) a rotation of 90° counterclockwise about P



Source: MMS6 (2009, p. 300)

You will need grid paper, tracing paper, and a Mira.

1. Copy this quadrilateral on grid paper. Make:
   a) 3 successive translations of 1 square right and 2 squares up
   b) 3 successive reflections in the line through SR
   c) 3 successive rotations of 180° about vertex R



Source: MMS6 (2009, p. 306)

The references to art and/or architectural monuments in the Lessons can be considered occasional and isolated in relation to the rest of the content developed in this unit. One occurrence is in Lesson 4, entitled Combining Transformations.

Here, the reference appears specifically in a box called Math Link, as illustrated in Figure 80. This box focuses on First Nations art, highlighting cultural elements and geometric transformations observed in the bead patterns used in artifacts such as mukluks.

Figure 80 - Mukluks and Beads: Indigenous Art and Symmetrical Transformations

Math Link

First Nations Art

Many First Nations artists use beads and braiding in their work. They produce many items, including jewellery, belts, purses, moccasins, and mukluks. We can often see transformations in the designs used by these artists.
What transformations do you see in the beading on these mukluks?

Source: MMS6 (2009, p. 312)

The image of the mukluks presents a rich cultural manifestation, evidenced by the detailed use of geometric patterns in the beads. This art not only reflects the technical skill of these artists, but also reveals how mathematical concepts, such as geometric transformations, are intrinsically present in cultural expressions. The symmetries, translations and rotations identifiable in the drawings demonstrate how mathematics can be present in everyday contexts, particularly in craft and traditional practices.

The inclusion of this proposal in the textbook illustrates the interdisciplinary potential of mathematics, connecting formal content to cultural and historical contexts. However, inserting this reference in isolation suggests that the subject is underused. By exploring the relationship between mathematics and art, it would be possible to significantly enrich learning, valuing First Nations cultural knowledge, as well as the history and culture of the Mukluks and, at the same time, deepening the understanding of geometric concepts. (Santos & Gonçalves, 2020).

First Nations make up one of the three main groups of indigenous peoples in Canada, along with the Métis and Inuit, as described by the Encyclopedia Canada website.[39] Unlike the Métis and Inuit, most First Nations communities own land on reserves. Members of these communities can reside both inside and outside these areas.

According to the 2021 census carried out by Statistics Canada, 1,048,405 people in Canada identified themselves as belonging to First Nations. The country is home to 630 communities of these indigenous nations, which preserve more than 50 distinct languages (Encyclopedia Canada, website). First Nations are mentioned several times in the collection: four times in MMS6 and MMS7, seven times in MMS8 and three times in MMS9. This suggests that Canadian students may be familiar with this community or have some prior knowledge of it.

This assumption is reinforced by the absence of explanations or in-depth explanations throughout these mentions, indicating that the presence of First Nations in the collection is treated as an everyday context for Canadian students.

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[39] Avaiblabe at: https://www.thecanadianencyclopedia.ca/en/article/first-nations. Accessed at: 18/12/2024.

However, this same reflection cannot be developed in the Brazilian scenario, where this context does not have the same cultural and historical relevance, thus requiring an introductory and explanatory approach for students to understand. For Brazilian students who use this collection, the lack of familiarity with First Nations can generate a certain strangeness, since the historical and cultural context presented is specific to Canada and does not dialogue directly with the reality of Brazilian indigenous peoples.

In Brazil, history and culture are deeply marked by the presence of distinct indigenous peoples, such as theGuarani, Yanomami, Xavante, Tikuna,among others, who form an essential part of the country's cultural, social and historical formation. Contact with these indigenous references is possibly more common and significant for Brazilian students, as these peoples are part of the national narrative and are present in debates about cultural identity, territory, history and the preservation of traditions. Thus, the absence of a specific contextualization of First Nations can limit the understanding of Brazilian students, highlighting the need for pedagogical adaptations or supplements that connect the content presented in the collection with the national reality.

In this sense, it is essential that Brazilian educators use these mentions as opportunities to broaden the discussion, contemplating/comparing the Canadian indigenous context with the Brazilian one in relation to its singularities. This can promote reflections on the similarities and differences between the historical, cultural and social processes of indigenous peoples in both countries, contributing to a more critical and globalized education for students (Ribeiro & Urt, 2017). One of the similarities between indigenous contexts can be seen in the use of mathematical concepts, such as geometric transformations, in the production of artifacts that are culturally demarcated by their traditions. For example, the context shown in Figure 66 refers to the patterns/transformations of beads used in artifacts such as mukluks.

Traditionally, mukluks were made from animal skins, such as reindeer, seal, moose or bear, and were commonly used by indigenous peoples from the generally arctic regions of Alaska, Canada, Greenland and eastern Siberia. The decoration of a mukluks with beads reveals geometric patterns/transformations characteristic of their culture. This also happens in the production of artifacts by Brazilian indigenous peoples, which is the case discussed and evidenced by Silva da Silva and Lorenzoni (2008) in which they studied geometry in the practices and artifacts of the Tupinikim and Guarani ethnic groups of Espírito Santo.

For example, in their analysis, the authors establish the first links between geometry and the cultural practices of the Indians of Aracruz, through body painting and the manufacture of basketry (Figure 81).

Figure 81 - Tupinikim-Guarani Body Painting and Basketry: Between Tradition and Geometry

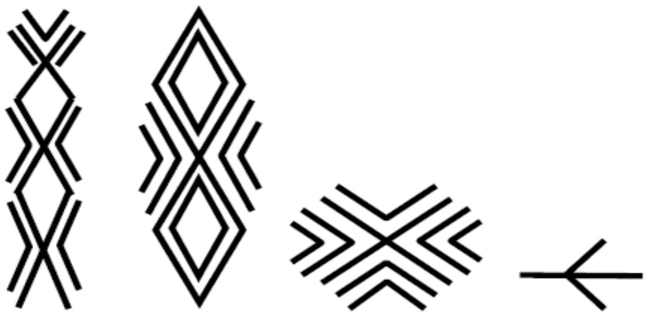


Ilustração 1 – Padrões de pintura Tupinikim. Da esquerda para a direita: dois desenhos utilizados na pintura masculina de tronco e membros e dois desenhos utilizados na pintura facial feminina

Legend: Illustration 1 – Tupinikim painting patterns. From left to right: two designs used in male torso and limb painting and two designs used in female face painting (our translation)

Source: Silva da Silva & Lorenzoni (2008, p. 11)

Figure 82 - Bear Paw Quilt Block: Culture, History and Geometry in Patchwork

Foto 6 – Base de cesto Guarani tipo 1 [Exposição de artesanato: Aldeia Pau-Brasil em 19 de abril de 2008]. Foto da investigadora.

Legend: Photo 6 – Base of Guarani basket type 1 [Crafts exhibition: Aldeia Pau-Brasil on April 19, 2008]. Photo by the researcher (our translation)

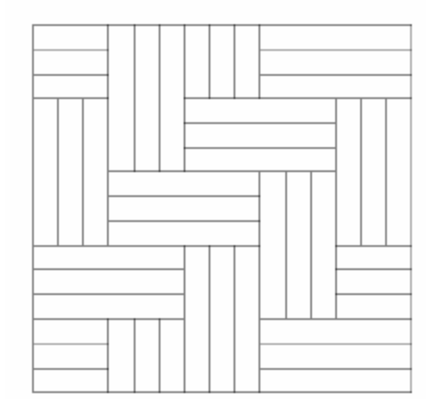


Ilustração 5 – Padrão de entrelaçamento de cesto Guarani tipo 1

Legend: Illustration 5 – Basket weave pattern Guarani type 1 (our translation)

Source: Silva da Silva & Lorenzoni (2008, p. 15)

The second occurrence is in Lesson 5 and corresponds to tasks contextualized in a Bear Paw quilt block (Figure 82). The blog Sherriquiltsalot[40]blog, in its publication entitled The Muddled History of the Bear's Paw, describes the Bear Paw as a classic pattern widely used in quilting.

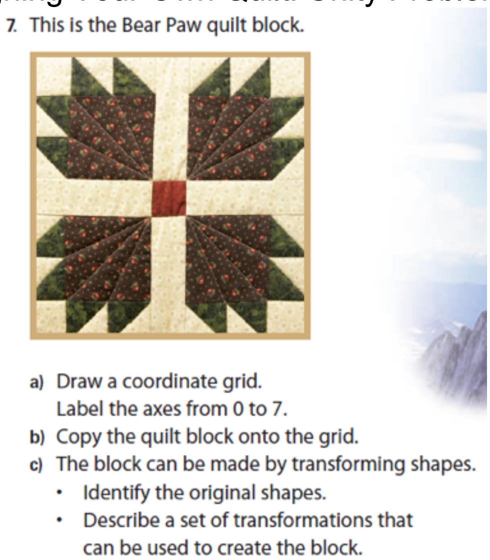
It is a geometric design that refers to the shape of a bear's paw, with claws represented by triangles and a square-shaped center. This pattern is widely recognized in traditional American quilting, with historical origins dating back to the 18th and 19th centuries.

According to the blog,

> As time and creativity progressed, by 1823, we see the first "Bear's Paw" block in Ohio. Quilt historians believe the block was named "Bears Paw" due to all the stories about bears in that area of the United States. The same block appears on Long Island, New York a few years later and it was called "Ducks-Foot-in-the-Mud" (Sherriquiltsalot, site).

He also points out that other names have been given to this pattern, varying according to location.

Figure 83 - Designing Your Own Quilt: Unity Problem with Geometry


7. This is the Bear Paw quilt block.

a) Draw a coordinate grid.
Label the axes from 0 to 7.
b) Copy the quilt block onto the grid.
c) The block can be made by transforming shapes.
- Identify the original shapes.
- Describe a set of transformations that can be used to create the block.

Source: MMS6 (2009, p. 317)

Historically, o blog Sherriquiltsalot narra que,

> According to legend, the appearance of a Bear's Paw quilt meant the runaway slaves were on the right track to freedom. The sight

[40] Avaiblabe at: https://sherriquiltsalot.com/2024/04/24/the-muddled-history-of-the-bears-paw/. Accessed at: 19/12/2024

> of a Bear's Paw block in a quilt could also mean that in order to find food, they would need to follow a bear's trail by looking for bear scat (bear poop). Along this trail, they would supposedly find enough berries, fresh water, etc., to sustain them until they arrived at the next Underground Railroad post.
> Along with Bear's Paw, there were other quilt blocks which held secret meanings (Sherriquiltsalot, site).

On the historical side, the Sherriquiltsalot blog reports that, according to oral tradition (legend), the presence of a quilt with the Bear's Paw pattern indicated that escaped slaves were on the correct route to freedom.

The visualization of the block in a quilt could also signal that, in order to find food, the fugitives had to follow the trail of a bear, identifying faeces as a landmark. Along the way, they would find fruit, drinking water and other resources necessary for their survival until they reached the next point on the Underground Railroad. In addition to the Bear's Paw pattern, other quilts contained blocks with secret meanings, associated with the same context of escape and resistance (Sherriquiltsalot, website).

With regard to the tasks involving the Bear Paw quilt block (Figure 83), it can be seen that they are all restricted to the field of mathematics, without promoting questioning/reflection on the culture and history associated with this context. This limited approach is repeated elsewhere in the collection. Although the mention of Bear Paw is restricted to the tasks shown in Figure 65, the quilt block is mentioned three times in MMS6, four times in MMS8 and once in MMS9.

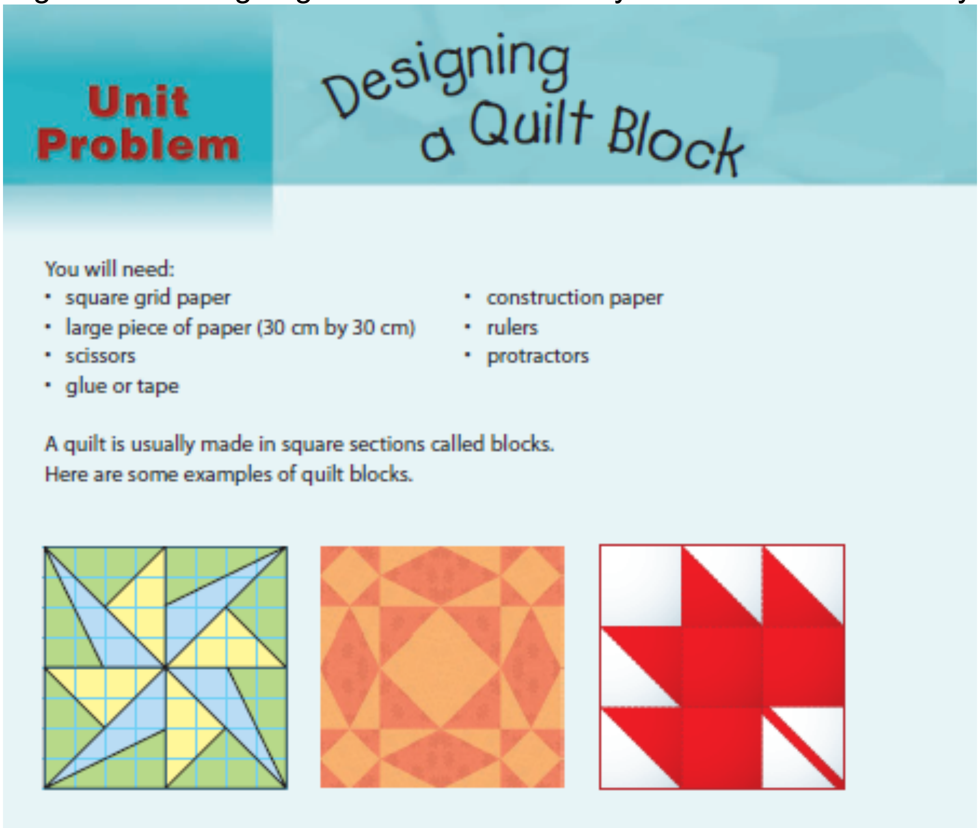
In MMS6, the mention appears in the introduction to Unit 4 (Figure 84), where it is stated that "The Heritage Park Historical Village in Calgary, Alberta, hosts A Festival of Quilts each May. It is Western Canada's largest outdoor quilt show" (MMS6, 2009, p. 125).

Despite this introduction, Unit 4 is developed without returning to the context of the quilt block until the end, when it reappears in the Unit Problem section entitled Designing a Quilt Block (Figure 84).

Figure 84 - Quilts Festival in Calgary: An Invitation to Create Math Blocks

Source: MMS6 (2009, p, 124-125)

Figure 85 - Designing Your Own Quilt: Unity Problem with Geometry



Source: MMS6 (2009, p, 156)

In MMS8, the mentions of the quilt block are linked to the tasks (Figure 85). Similarly, to the previous section, there are no references to the cultural and historical dimensions associated with the quilt block context, even in task 16 (Figure 86), where there is also a mention of the First Nations. In MMS9, the topic is covered in Unit 9, in the Project section, entitled Constructing a Math Quilt (Figure 87). However, in this instance too, the cultural and historical dimensions related to the context of the quilt block are not explored.

Figure 86 - Quilt Blocks and First Nations: Exploring Shapes and Patterns in MMS8

16. This First Nations quilt is a square, with area 16 900 cm^2. How long is each side of the quilt?

Source: MMS8 (2009, p, 56)

10. Use transformations to create your own quilt design. Describe the transformations you used.

Source: MMS8 (2009, p, 477)

9. Here is a quilt design. Use a copy of the design.

Find as many transformations in the design as you can. Ignore the different patterns on the material. Consider only the shapes.

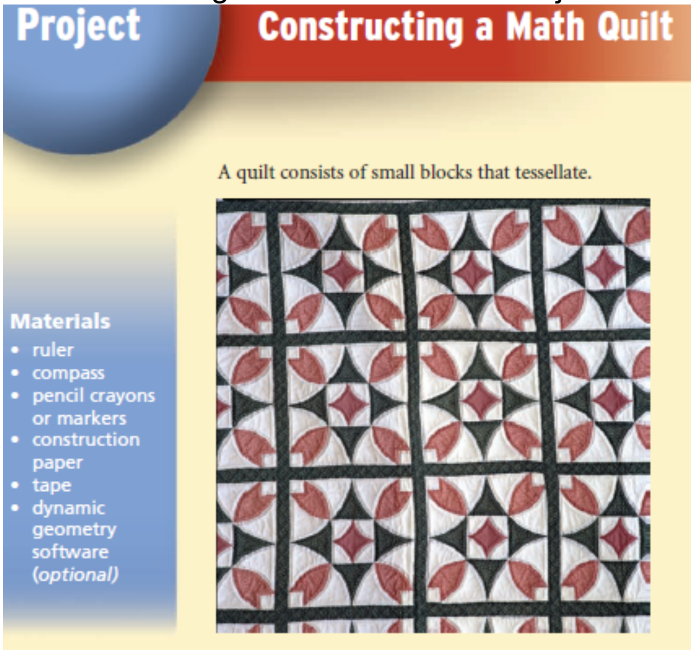
Source: MMS8 (2009, p, 477)

14. Here is a quilt design.

Use as many different transformations as you can to describe the design. Ignore the different patterns, colours, and textures on the material. Consider only the shapes.

Source: MMS8 (2009, p, 485)

Figure 87 - Building a Math Quilt: Final Project in MMS9



Source: MMS9 (2009, p, 462)

The mentions of the quilt block, as presented, lead us to reflect on the proximity of Canadian students to this context, especially the Bear Paw pattern. As discussed, the Bear Paw quilt block is intrinsically linked to the culture and history of traditional American quilting. This analysis leads us to ponder the degree to which Brazilian students are familiar with this context and its possible implications for teaching and learning processes.

The integration of quilt blocks into the Brazilian scene can be observed through events and artistic productions that highlight patchwork and quilting in the country. An example of this is the International Quilt and Patchwork Festival, held in Gramado[41]which brings together artists and enthusiasts to exhibit their creations and share techniques. In the field of Brazilian indigenous arts, although there may not be a direct correspondence with traditional American quilt blocks, there is a rich tradition of geometric and symbolic patterns evidenced in artifacts such as ceramics and basketry.

Studies linking indigenous art to mathematics, especially with regard to geometric patterns and transformations, such as those by (Medrada, 2020; R. C. da Silva, 2016; Silva da Silva & Lorenzoni, 2008) highlight the complexity and diversity of these artistic expressions among the indigenous peoples of Brazil,

[41] Avaiblabe at: https://quiltbrasil.com.br/?utm_source=chatgpt.com. Accessed at: 19/12/2024.

emphasizing their deep cultural connection and functionality in preserving traditions.

For example, (Medrada, 2020) describes the Kai, a basket used in the daily life of the Gavião people, as well as the mat that also makes up the daily practices of this community (Figure 88). According to the author:

> The Kai is used by the Gavião Indigenous community in their daily village activities. It serves to carry açaí, cupuaçu, Brazil nuts, and other food items. To make this basket, they use Akrô (a type of piassava vine found in the forest). The mat, on the other hand, is woven from palm leaves and used as bedding by the Indigenous people. Both items are crafted by the elders of the community. Typically, the elders collect the vine and fresh palm fibers ("palha nova"), which are still in their formative stage on the palm tree, to use for weaving. These materials are then scraped until they become smooth and flexible enough to be woven. Practically all members of the community use these items Medrada, 2020, p. 139, our translation).

Figure 88 - Kai and Mats of the Gavião People: Geometry in Everyday Artifacts

Fonte: Acervo do Leite (2019).

Source: Medrada (2020, p.139)

The reflections presented above allow us to consider that, although there are similarities between the context of quilt blocks in Canada and Brazil, it is essential to take into account the cultural and historical specificities of each location.

For Brazilian students who use textbooks covering topics such as the Canadian Bear Paw quilt block, it is essential to situate the content in a way that also includes quilt blocks in the Brazilian context. This perspective not only brings teaching closer to the students' cultural and historical experiences, but also fosters a deeper identification with the content, broadening the understanding of the multiple cultural and mathematical dimensions of these patterns. This promotes a dynamic that values and integrates cultural diversity into the educational process.

Continuing the analysis of this Unit 8, it becomes clear that, after the Lessons, the last occurrence of references to art or architectural monuments takes place in the Unit Problem section, entitled Art and Architecture.

The Art and Architecture references are contextualized in the image about Hatley Castle, Victoria, British Columbia and the locations in Saskatchewan (Moose Jaw and Assiniboia, see figure 89).

Figure 89 - Hatley Castle e Saskatchewan: Paisagens Canadenses no Unit Problem

Source: MMS6 (2009, p. 324)

The Unit Problem section (Figure 89) makes use of geometric transformations to support students in understanding the real-life applications of mathematical concepts in architectural design (Figure 85).

As highlighted by Ladson-Billings (1995) in her conception of culturally relevant pedagogy, the exploration of patterns and arrangements, such as translations, rotations and reflections found in herringbone masonry or decorative tiles, broadens students' engagement by relating mathematics to their culture and daily lives.

In this sense, the analysis of local Saskatchewan architecture contextualizes these mathematical concepts in a specific socio-environmental panorama, strengthening the recognition of the different forms of knowledge present in the community and favoring culturally responsive mediation.

Figure 90 - Art and Architecture: Uniting Transformations and Building Projects

Source: MMS6 (2009, p. 324-325)

In Part 1 (highlighted in red in Figure 90), students are asked to analyze architectural patterns in Saskatchewan buildings, identifying transformations such as rotation in brick patterns or reflection in symmetrical designs.

For example, the herringbone pattern observed in the former Bank of Toronto illustrates rotational transformations that create visual complexity and structural rhythm (Guerreiro, 2017). In the Brazilian context, similar analyses could be applied to the iconic Portuguese stone mosaic sidewalks, such as those in Rio de Janeiro (Figure 91).

Figure 91 - Portuguese Sidewalks in Brazil: An Analysis of Patterns and Symmetries

Source: Google Imagens (2024)

These mosaics often show symmetries of reflection and rotation, displaying undulating patterns that evoke natural elements such as ocean waves and end up becoming important cultural symbols. In this way, students can recognize geometric transformations as universal design principles, connecting architectural elements from different cultures.

In Part 2 (highlighted in orange in Figure 90), the challenge is to design the external pattern of a new building, drawing shapes and applying geometric transformations. This activity highlights the creative application of patterns and transformations, while bringing theoretical knowledge closer to practical design.

In the Brazilian context, for example, students could be inspired by the traditional colonial tiles found in historic buildings in Salvador and São Luís (Figure 92). These tiles have rotational and reflective symmetry, combining aesthetic functions with the preservation and communication of cultural heritage (Guerreiro, 2017).

By reproducing these transformations, students gain a more concrete understanding of how geometry can be used to tell cultural stories and reinforce identities.

Figure 92 - Colonial Tiles: Between History, Identity and Transformations

Source: Brasil (2024, site[42])

Source: g1 (2022, site[43])

In Part 3 (highlighted in purple in Figure 90), students describe the designed patterns and specify the transformations used. They might, for example, combine translations and rotations to create a repetitive design that runs along the façade of a building.

In the Brazilian context, an architect could apply these transformations to reinterpret traditional patterns in contemporary projects, integrating cultural motifs with modern design principles. In addition, the tasks encourage students to explore how geometric transformations are employed in other professions - such as bricklayers, landscapers and fashion designers - who often use these concepts to create functional and aesthetically appealing designs (Guerreiro, 2017).

[42] Avaiblabe at: https://www.gov.br/iphan/pt-br/assuntos/noticias/restaurados-azulejos-portugueses-do-claustro-da-igreja-e-convento-de-sao-francisco-em-salvador-ba. Accessed at: 19/12/2024

[43] Avaiblabe at: https://g1.globo.com/ma/maranhao/noticia/2022/09/02/azulejos-coloniais-atraem-a-atencao-de-moradores-e-turistas-no-centro-historico-em-sao-luis.ghtml. Accessed at: 19/12/2024

Despite the potential of relating geometric concepts to design, history and culture, there is a noticeable lack of a critical approach that goes beyond mathematical application. Although cultural aspects of Saskatchewan are covered, the reflection does not expand to other realities, such as Brazil, where geometry is intrinsically linked to unique cultural expressions.

A more in-depth study, through the analysis of architectural examples in Brazil and Canada, could show that mathematics, although a universal language, manifests itself in unique ways in each historical and social context.

In this sense, Radford (2012) points out that mathematics is not just made up of abstractions, but is formed and re-signified according to cultural contexts and social interactions.

This perspective promotes not only the understanding of mathematical content, but also cultural sensitivity, emphasizing that geometric principles are not just tools for investigation; they are fundamental elements in the construction of cultural identities and narratives.

In this way, the proposed tasks would gain greater relevance if they provoked reflection on how cultural norms and values influence the application of geometry in different architectural contexts, both national and international.

Valuing Brazilian cultural specificities, for example, reinforces the importance of bringing geometric concepts closer to students' daily lives and experiences, making teaching more meaningful and culturally relevant (Guimarães, 2024).

This approach not only improves the learning of mathematical content, but also helps to form citizens who are more aware of and respectful of cultural diversity - something that Radford (2012) and Banks, (2014) consider to be the basis for a mathematical education that takes into account the plurality of human experiences.

In concluding the analysis of the two scenarios presented, it can be seen that, although the second initially seems to emphasize art and architecture more, the investigation shows that these references, as in the first, appear in a punctual way. Throughout the content of the unit examined, there is no effective link between mentions of art or architectural monuments and the proposed mathematical objectives.

Although the mentions of art and architectural monuments show that mathematical concepts do not arise in isolation, but as part of a broader process linked to the history, culture and economy of various societies, this focus remains restricted.

In fact, the collection presents mathematics as a living science, shaped by the demands and contexts of civilizations over time. However, it can be concluded that these quotations, made in a sporadic and dissociated way - often inserted in boxes or specific sections - end up recognizing the diversity of cultural contexts without, however, fully valuing it. The lack of integration makes it impossible to relate to or exchange different cultures in the material (Banks, 2014; Kirsch & Duarte, 2020).

In order not to reiterate a cultural hierarchization - in which one dominant culture takes precedence over others - these references should not be treated as mere curiosities, but as an integral and equitable part of the content.

In summary, a multicultural approach, along the lines proposed by Banks (2014), Caligari et al. (2021) and Radford (2012), implies considering the diversity of cultural and historical contexts as a structuring element of the mathematics curriculum.

The ideal is not just to recognize different cultures, but to value and integrate their contributions in a balanced way, promoting exchange and reflection between them. This would allow the richness of the various mathematical traditions to be highlighted and deepened, helping to educate students who are more aware of the multiple facets of mathematics and the cultural realities with which it is related.

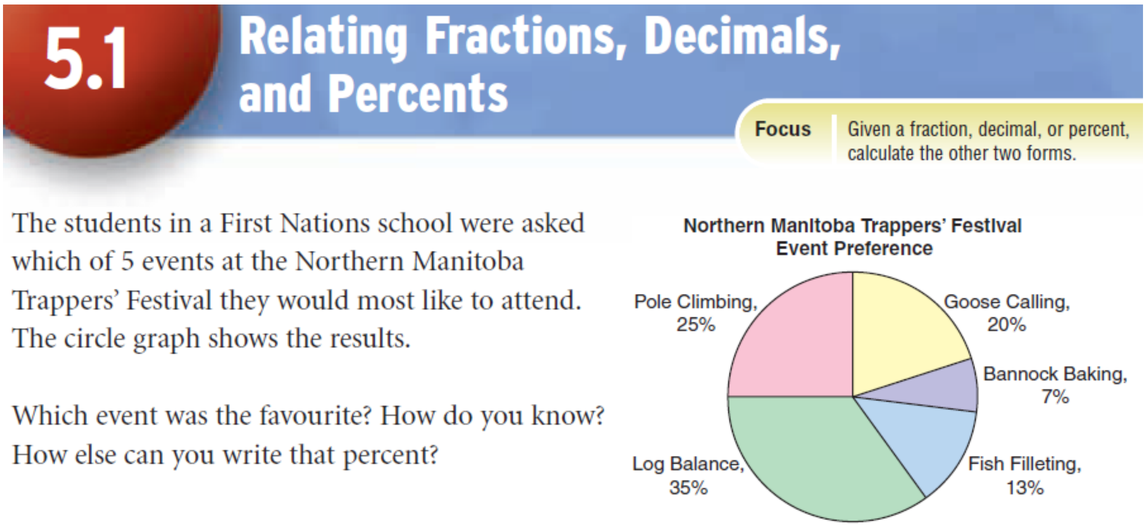
6.2.3 Cultural festivals and regionalisms: when mathematics dresses up as a party

Festivals play a significant role in celebrating and valuing cultural diversity. By associating mathematics with these contexts, there are possibilities for bringing together cultural practices and learning linked to everyday situations. These events are not just festive moments, as they bring with them historical, social and cultural elements that can broaden the pedagogical approach (Ladson-Billings, 2014; Presmeg & Radford, 2008).

The Northern Manitoba Trappers' Festival, founded in 1916, takes place annually in The Pas, Manitoba (Figure 83). Recognized as one of Canada's oldest winter festivals, it highlights the traditions and culture of northern pioneers. Its origins are linked to The Pas Dog Derby and the initiative to promote better sled dogs, as well as publicizing development opportunities in the region.

Over time, it has included activities such as muskrat skinning, tea boiling, bannock making and the World Championship Dog Race, which continues as the main attraction (Lambert, 2024, website[44]).

Figure 93 - Northern Manitoba Trappers' Festival: Winter Legacy and Local Culture



Source: MMS8 (2009, p. 234)

The textbook's proposal, when dealing with preferences in festive events (log balancing, fishing and typical cuisine), suggests analyzing fractions, decimals and percentages: "Which event was your favorite? How do you know? How else can you write this percentage?"(MMS8, 2009, p. 234).

The questions encourage the interpretation of quantitative data, while inviting the student to reflect on the cultural values of these events.Elsewhere in the collection, there are references to celebrations such as the Fancy Shawl Rega lia and the Powwow (Figure 94). The Fancy Shawl Regalia corresponds to the

[44] Avaiblabe at: https://www.mhs.mb.ca/docs/mb_history/15/trappersfestival.shtml. Accessed at: 22/12/2024.

clothing used in the Fancy Shawl Dance, a contemporary dance that incorporates elements historically associated with women (ICT News, 2011). According to the ICT News website (2011):

> [...] não há dúvida de que, para os dançarinos do fancy shawl, a dança pode ser uma ferramenta contemporânea para criar laços dentro do país indígena – era intertribal desde suas origens. Quando perguntada sobre seu ritual pré-competição, Pratt se concentra no aspecto comunitário e em quão essencial isso é para ela. "Sempre penso nas pessoas, minha família, que não sabem dançar. Espero que isso os deixe felizes quando eu danço. Não consigo me imaginar dançando outra coisa (ICT News, 2011, site[45]).

Also, the illustration related to Fancy Shawl Regalia refers to the Aboriginal Fancy Dance.

Figure 94 - Fancy Shawl Dance and Powwow: Expressions of Indigenous Identity

4. Shona cut a ribbon into 8 equal lengths to finish sewing her Fancy Shawl Regalia. Each piece was 0.158 m long.

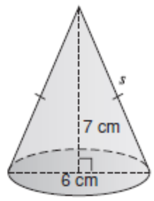
a) How long was the ribbon before Shona cut it?

b) How many cuts did she make?

Woman Dancing an Aboriginal Fancy Dance

Source: MMS6 (2009, p. 102)

19. Take It Further A Powwow is a traditional practice in some First Nations cultures. Women dancers have small cone-shaped tin jingles sewn onto their dresses, one for each day of the year. A typical jingle has a triangular cross-section. Suppose the triangle has base 6 cm and height 7 cm. Use the diagram to help you find the slant height, *s*, of the jingle. Give your answer to one decimal place.



Source: MMS8 (2009, p. 51)

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45 Avaiblabe at: https://ictnews.org/archive/the-evolving-beauty-of-the-fancy-shawl-dance. Accessed at: 21/12/2024.

The Powwow is an indigenous celebration that brings together music, dances, traditional costumes, crafts and cuisine. Held both in First Nations communities on reserves and in urban areas, it is usually open to other participants, including non-indigenous people, as well as Métis and Inuit.

Contemporary powwows, which originated in the Great Plains region at the end of the 19th century, have grown in number and popularity since the 1950s, according to The Canadian Encyclopedia (website[46]). According to this source:

> Powwows serve an important role in many Indigenous peoples' lives as a forum to visit family and friends, and to celebrate their cultural heritage, while also serving as a site for cross-cultural sharing with other attendees and participants. Indeed, powwows provide the opportunity for visitors to learn about, and increase their awareness of, traditional and contemporary Indigenous life and culture (Enciclopédia Canadense, site[47]).

Given the cultural events mentioned in both examples in Figure 94, we understand that by dealing with topics such as the length of sewing tapes and calculations related to geometric elements of costumes, such as conical-shaped jingles, the tasks allow students to explore mathematical concepts such as perimeter, area and volume in traditions deeply rooted in spirituality and cultural identity.

It's also worth bringing up other examples that deal with cultural festivities. The Raven Mad Days celebration, represented in a task about distributing money (Figure 95), was a festival that took place in Yellowknife, Canada, to celebrate the longest day of the year.

The event took place on Franklin Avenue, which was closed to the public, and included music, food, shopping and shaving cream. The festival took place on the Friday closest to the summer solstice, in June, for more than 30 years. However, the event disappeared about five years ago due to a lack of community support (Edge Yellowknife, website[48]).

The task brought in a cultural element from Yellowknife that reflected the local economy and social interactions during the festival. This approach connects

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[46]Avaiblabe at: https://www.thecanadianencyclopedia.ca/en/article/powwows-editorial. Accessed at: 21/12/2024.

[47] See reference 15, pag.102.

[48] Avaiblabe at: https://edgenorth.ca/article/yk-past-blast-raven-mad-daze/#:~:text=For%20over%2030%20years%2C%20this,to%20lack%20of%20community%20support. Accessed at: 21/12/2024.

students to specific economic and social practices, showing how mathematical concepts, such as systems of equations, can model everyday situations and promote appreciation of regional particularities (D'Ambrósio, 2005).

Figure 95 - Raven Mad Days in Yellowknife: Economy and Celebrations

3. Marsha and Ivan have money to spend at the Raven Mad Days Celebration in Yellowknife.
If Marsha gives Ivan \$5, each person will have the same amount.
If, instead, Ivan gives Marsha \$5, Marsha will have twice as much as Ivan.
How much money does each person have?

Source: MMS8 (2009, p. 369)

Another example is the Ice Magic Festival at Chateau Lake Louise in Banff National Park, where a large castle is built out of blocks of ice. The Ice Magic Festival is an annual event that takes place at Chateau Lake Louise in Banff National Park, Alberta, Canada.

According to the Banfflakelouise website, during the festival, teams of ice sculptors from around the world compete to create imposing and detailed ice sculptures, including castles and other structures, using blocks of ice taken from Lake Louise itself.

The festival, which usually takes place in January, is part of SnowDays, a winter celebration in Banff and Lake Louise, attracting tourists and ice art enthusiasts to enjoy the magnificent creations (Banfflakelouise, website[49]).

The context of building the ice castle at the Ice Magic Festival (Figure 96), held in Banff, connects mathematical concepts to cultural practices in a particular

49 Avaiblabe at: https://www.banfflakelouise.com/ice-magic-festival. Acesso em: 13/08/2024.

way. Students are asked to sketch a roofless castle using 30 blocks of ice, each measuring 25 cm by 50 cm by 100 cm.

In addition, the request to determine the total surface area of the castle built, considering both the interior and exterior (task 13(b) in Figure 96), challenges students to apply geometry concepts in their productions. These calculations encourage spatial perception and the integrated application of mathematical knowledge in a real-world and culturally situated context.

Figure 96 - Ice Magic Festival: Ice Castles and Frozen Mathematics

13. Every January, the Ice Magic Festival is held at Chateau Lake Louise in Banff National Park. An ice castle is constructed from huge blocks of ice.

a) Suppose you have 30 blocks of ice measuring 25 cm by 50 cm by 100 cm. Sketch a castle with no roof that could be built with some or all of these blocks.

b) Determine the surface area of your castle, inside and out.

Source: MMS9 (2009, p. 32)

This integration of mathematics and a cultural event suggests how experiential learning brings students closer to abstract concepts by linking them to the dynamics of the physical world. Ice, used as a building material, is not limited to a structural function; it reveals cultural and environmental symbolism, highlighting the link between local communities and the harsh climate and natural landscape. The analysis of the castle's structure, including its geometry and aesthetic form, encourages reflections on ecological preservation and the conscious use of natural resources.

As argued by Ladson-Billings (2014), Lerman (2003), Presmeg & Radford (2008) and Silva (2019), mathematical learning transcends the acquisition of technical knowledge, involving an understanding of the sociocultural dimensions that permeate its application.

In this sense, building an ice castle not only fosters the development of mathematical skills, but also encourages the recognition of the connections between mathematics, culture and the environment. By relating mathematics to cultural and ecological practices, the tasks not only broaden students' understanding, but also invite them to reflect critically on the impact of human actions on their surroundings.

Two other examples of cultural events mentioned are Christmas and Hanukkah (Figure 97). In both, the emphasis is on cultural sensitivity. With regard to Christmas, the book notes that it is not celebrated by all cultures, suggesting that certain research questions may not resonate with some groups (see red highlight in Figure 97).

As for Hanukkah, the task asks students to examine why questions about this festival can be culturally sensitive. Also known as Chanukah or the Festival of Lights, Hanukkah is a Jewish celebration that commemorates the rededication of the Holy Temple in Jerusalem, which took place after a period of conflict.

As mentioned by the CNN Brazil website[50]although Hanukkah takes place in December, “it is not the equivalent of Christmas for Judaism. The Festival of Lights celebrates the Jewish community's struggle for freedom to practice their religion”.

This proposal invites students to reflect on how mathematics can address social and cultural issues in a respectful way, going beyond the mere numerical

[50] Avaiblabe at: https://www.cnnbrasil.com.br/lifestyle/o-que-e-o-hanukkah-festa-judaica-comeca-nesta-quinta-feira-7/#:~:text=Festa%20judaica%20come%C3%A7a%20nesta%20quinta%2Dfeira%20(7),-%E2%80%9CFestival%20das%20Luzes&text=Mesmo%20que%20aconte%C3%A7a%20todo%20m%C3%AAs,sexta%2Dfeira%20(15). Accessed at: 21/21/2024.

dimension and stimulating debates on the formulation of questions and the interpretation of data without reinforcing stigmas or prejudices.

The inclusion of these topics signals the relevance of an educational approach that integrates the development of mathematical concepts with cultural awareness, in line with the perspectives advocated by Ladson-Billings (2014) and Presmeg and Radford (2008).

Figure 97 - Christmas and Hanukkah: Respecting Cultural Differences and Sensitivities

Example 3 **Overcoming Potential Problems of Data Collection**

Antonia wants to find out if there is a relationship between household income and how much people spent on Christmas presents.
Identify potential problems Antonia may encounter, and explain how she could deal with the problems.

A Solution

Christmas is not celebrated by all cultures, and so the survey question does not apply to everyone. An appropriate opening question for the survey might be: "Do you celebrate Christmas?" If a person responds "No", then he or she will not need to answer the other question in your survey.

Information about income and spending habits is personal, so people may be uncomfortable revealing it. An anonymous survey would be appropriate.

The use of language may influence responses.
Examples of inappropriate or intrusive questions would be:
"How much do you make?" and "How much do you spend?"
A better question might be:
"Is the amount you spend on Christmas presents:
– greater than your weekly income?
– less than your weekly income?
– equal to your weekly income?"

Source: MMS9 (2009, p. 434)

16. a) Why might questions about Hanukkah be culturally sensitive?
b) Think of 3 more topics that might be culturally sensitive. Explain why.
c) Design a culturally sensitive survey question about one of the topics in part a or b. Explain how you would collect the data to address the cultural sensitivity.

Source: MMS9 (2009, p. 436)

The examples shown (Figures 93 to 97) and the reflections around them suggest the possibility of relating cultural aspects linked to festivities to the teaching of mathematical concepts, favoring debates about the sociocultural relevance of these events.

Even so, the materials analyzed show that cultural festivals are usually only mentioned, without a more in-depth development of their historical, religious or regional significance. As a result, there is a risk of limiting the understanding of cultural traditions to narrow perspectives, with little connection to the complexity of their practices and values.

In this sense, mathematics can function as a way for students not only to learn formal content, but also to critically reflect on how this content interacts with social practices. It can be considered that bringing in regional festivals opens up the opportunity to broaden students' cultural repertoire (Ladson-Billings, 2014; Presmeg & Radford, 2008).

According to Presmeg and Radford (2008), recognizing the historical and social diversity of mathematics is a step that goes beyond mastering techniques, as it involves interpreting contexts and dialoguing with local and global values. However, the use of cultural elements of a religious nature in textbooks can generate controversy or even conflicts with educational parameters.

In this case, a reflexive approach is relevant: it is possible to present religious festivities as historical and cultural phenomena, without necessarily promoting practices of faith, but recognizing their influence on the construction of collective identities (Ladson-Billings, 2014).

This strategy requires educators and textbook authors to take a critical and careful stance, so that religious references are treated as part of a sociocultural panorama, in line with curriculum guidelines.

Attention to cultural festivals in textbooks opens up a promising avenue for exploring mathematical knowledge in a contextualized way, although it can be seen that such references often appear in a one-off way. As Valero (2012) points out, mathematics should be interpreted as a cultural practice linked to social and historical relations, allowing students to understand the multiple layers of meaning present in each context.

Gutstein (2012) complements this view by emphasizing that, without a deeper involvement, cultural connections can be reduced to mathematical abstractions, losing the richness of the stories and values that constitute them.

With this in mind, it is suggested that educators continuously articulate the sociocultural significance of festive events, avoiding treating them as simple mentions or decontextualized illustrations.

This attitude expands the possibilities for teaching and learning, as students come to recognize mathematics as a component of living and situated practices, rather than a mere set of algorithms disconnected from everyday life (D'Ambrósio, 2005; Gutstein, 2003). In this way, we can promote a view that values cultural diversity, while at the same time studying mathematics in a way that is linked to concrete social references (Ladson-Billings, 2014; Presmeg & Radford, 2008; Valero, 2012).

When we look at the MMS collection, aimed at Canadian students, there is an emphasis on local festivals, such as the Northern Manitoba Trappers' festival, Raven Mad Days and the Ice Magic Festival. This choice is consistent with the context for which the collection is intended.

However, when considering the adoption of this material in other countries, such as Brazil, such festivals may not produce the same engagement, as they are not known or easily imagined by the students.

This point signals the importance of understanding how, in ice festivals in Canada, structural design involves calculations of mass, weight and stability at low temperatures; whereas in festivals in Brazil, such as Carnival or the Parintins Festival, the construction of floats or allegories requires other forms of planning (Ladson-Billings, 2014; Presmeg & Radford, 2008)

In both cases, there are challenges of a mathematical nature, but cultural practices, available materials and climatic conditions differ, requiring solutions adapted to local realities (Gutstein, 2003).

To make these proposals more meaningful in Brazilian environments, activities could be adapted by considering celebrations such as Carnival, the

June Festivals, the Parintins Festival, the Folia de Reis or the Festa do Divino, among others.[51].

Some of these celebrations are widely known, while others have a more limited regional scope, such as the Grape Festival in Rio Grande do Sul, reinforcing the idea that integrating regional festivals that are not widely known can be a way of bringing students closer to different cultures (Ladson-Billings, 2014) and valuing the plurality of traditions that exist in the national territory.

On the other hand, the suggestion of also keeping examples from Canada or other parts of the world is in line with what Presmeg and Radford (2008) discuss about the importance of linking the local and the global.

As the authors note, "the transfer of mathematical ideas across cultural boundaries should not ignore the specific meanings and practices attached to local contexts. A mixed approach that integrates local and global scenarios can promote critical reflection on the universality and cultural specificity of mathematical knowledge" (Presmeg & Radford, 2008, p.270).

In fact, by comparing, for example, the challenges of building ice sculptures at the Ice Magic Festival with making floats at the Brazilian Carnival, students can see that there are different ways of dealing with spatial, weight and durability issues, which deepens their understanding of how mathematics manifests itself in different contexts (D'Ambrósio, 2005).

These practices also allow for the development of skills related to probability analysis in real situations, such as estimating the chances of rain in an outdoor parade or assessing risks in temporary structures. When students calculate production costs and sales values for typical foods at June Festivals, for example, they establish links between mathematics, economics and culture.

[51] O Carnaval é considerado uma festividade nacional, celebrado em todo o Brasil com expressões regionais distintas, como o Carnaval de Salvador, o desfile das escolas de samba no Rio de Janeiro e São Paulo e o frevo em Pernambuco. O Festival de Parintins ocorre na região Norte do Brasil, especificamente no Amazonas, onde se celebra o Boi-Bumbá com as disputas entre os bois Garantido e Caprichoso. O Maculelê é uma manifestação cultural tradicional do Nordeste, mais comum na Bahia, caracterizando-se como uma dança de origem afro-brasileira. A Festa do Divino é celebrada nas regiões Sudeste e Centro-Oeste, sendo uma festividade religiosa com forte influência católica, especialmente em estados como Minas Gerais, São Paulo e Goiás. Por fim, a celebração de Nossa Senhora dos Navegantes ocorre no Sul do país, principalmente no Rio Grande do Sul, em cidades como Porto Alegre, em homenagem à padroeira dos navegantes.

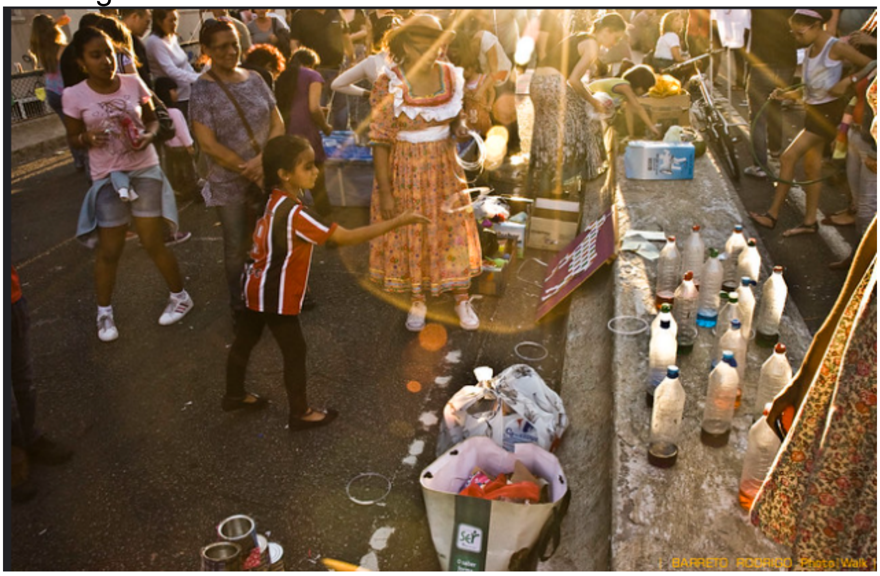
These activities, supported by participatory methods, reinforce Valero's conception of mathematical learning as a process situated in social practices.

In short, by integrating cultural festivals from both Brazil and other contexts, textbooks can foster mathematical learning that dialogues with multiple realities. Mathematics manifests itself as a tool for reading and interpreting the world, going beyond calculation and reaching historical, symbolic and social dimensions (D'Ambrósio, 2005; Gutstein, 2003; Valero, 2018).

This attitude values the diversity of cultural practices (Figure 98) and encourages a more open vision, stimulating reflection on how different communities solve structural and symbolic problems.

In this way, mathematics teaching can contribute to the formation of individuals who recognize the plurality of contexts and understand science as an integral part of social and cultural fabrics, in line with the ideas of these authors.

Figure 98 - Ring Games at the June Festivities: Statistics in the Arraial Climate



Source: Barreto[52] (2012)

Ladson-Billings (2014) proposes that a culturally relevant pedagogy incorporates cultural contexts into teaching, involving students in ways that value and recognize their identities.

Even so, there is a risk that these contexts are approached in a superficial way, focusing on solving tasks rather than highlighting the cultural meaning

52 Avaiblabe at: https://www.flickr.com/photos/barretorodrigo/7491234406/in/photostream/. Accessed at: 09/01/2024

behind each community's festive practices. This type of approach can unintentionally limit festivities to quantitative aspects (e.g. numerical data), leaving aside their historical and cultural depth.

As Presmeg and Radford (2008) warn, turning cultural experiences into mere data erases their own value and takes away their meaning. To avoid this reduction, Ladson-Billings (2014) points out that a culturally relevant pedagogy should propose forms of teaching that engage with students' cultural backgrounds, promoting not only the learning of school concepts - in this case, mathematics - but also a broad understanding of their traditions.

These reflections lead us to question the regional particularities present in the collection. As mentioned earlier, this was exemplified by addressing cultural festivities which, in the MMS collection, are often related to Canadian regionalism.

This point also appeared in section 1.2.1 The sporting context: ice hockey and its ilk , which discussed the cultural relationship with sports, illustrated by ice hockey. In addition, there are several other moments in which the collection highlights elements linked to the Canadian context or characteristics of the local environment, such as the presence of snow. Here are some examples of these references.

The tasks in Figure 99 refer to the flags of the Canadian provinces British Columbia, Saskatchewan, Nunavut and the national flag of Canada. The cultural and geographical variety of these provinces and territories is highlighted, as each flag features symbols that refer to local identity and history, such as the inuksuk on the Nunavut flag.

In this case, there is also an environmental feature, as the flags, especially that of Nunavut, refer to natural aspects of Canada, including the tundra and the inuksuks, stone structures traditionally used as orientation points by indigenous peoples in the Arctic. The tundra, represented on the Nunavut flag by the inuksuk, is characterized by the absence of trees, undergrowth, permanently frozen ground(permafrost) and harsh climatic conditions.

Figure 99 - Flags of the Provinces: Canada's Symbols and Identities

3. Your teacher will give you a large copy of these flags.
List the flags with:
a) a right angle
b) an acute angle
c) an obtuse angle
d) a reflex angle
On each flag, label an example of each type of angle you find.

British Columbia | Saskatchewan | Nunavut | Canada

Source: MMS6 (2009, p. 128)

The inuksuk has a significant connection with the tundra, as it was used by the Inuit as a reference for navigation, orientation and communication in large, uninhabited areas, where the homogeneous landscape made it difficult to locate and move around.

Its presence on the flag of Nunavut refers not only to the historical coexistence of indigenous peoples with the environment, but also to ways of adapting and respectfully using the available resources. By including the inuksuk, the flag of Nunavut highlights both the cultural identity of the region and the close relationship between local communities and the characteristics of the tundra.

The tasks in Figure 100 are set in the context of dogsled racing, a practice that remains ingrained in Canada's cold areas, especially in the north, where the use of sleds is often used for transportation on snow-covered terrain.

This scenario dialogues directly with the harsh climatic conditions of these locations and the historical role of dogs in the subsistence of communities. In this environment, snow appears as a central element, highlighting the specific character of the country's icy regions.

Figure 100 - Sled Dog Racing: A Tradition in Canada's Snowy Regions

4

3. In a dogsled race, teams of 6 dogs race to the finish.
   a) Make a table to show the numbers of dogs in a race when 2, 3, 4, 5, and 6 teams are entered.
   b) Write a pattern rule that relates the number of dogs to the number of teams entered.
   c) Write an expression to represent this pattern.
   d) Use the expression to find the number of dogs when 13 teams are entered. How can you check your answer?

Source: MMS6 (2009, p. 40)

Figure 101 shows an excerpt from the introduction to subunit 2.1 of the MMS7 collection, which proposes the use of colored pieces to represent whole numbers. In this excerpt, there is a reference to Antarctica, even though it is not part of Canada, by mentioning very low temperatures (such as -58°C), approaching the conditions found in regions such as the Yukon and Nunavut.

Although this choice of context involves a location outside of Canadian territory, the example can stimulate comparisons with temperatures in Canada's Arctic zones, where ice and cold are characteristic of the more northerly areas. Figure 101 also features an illustration of penguins, animals associated with icy environments, especially in the Southern Hemisphere, which reinforces the discussion about different landscapes and cold climates.

Figure 101 - Cold Diving: Negative Integers and the Antarctic Scenario

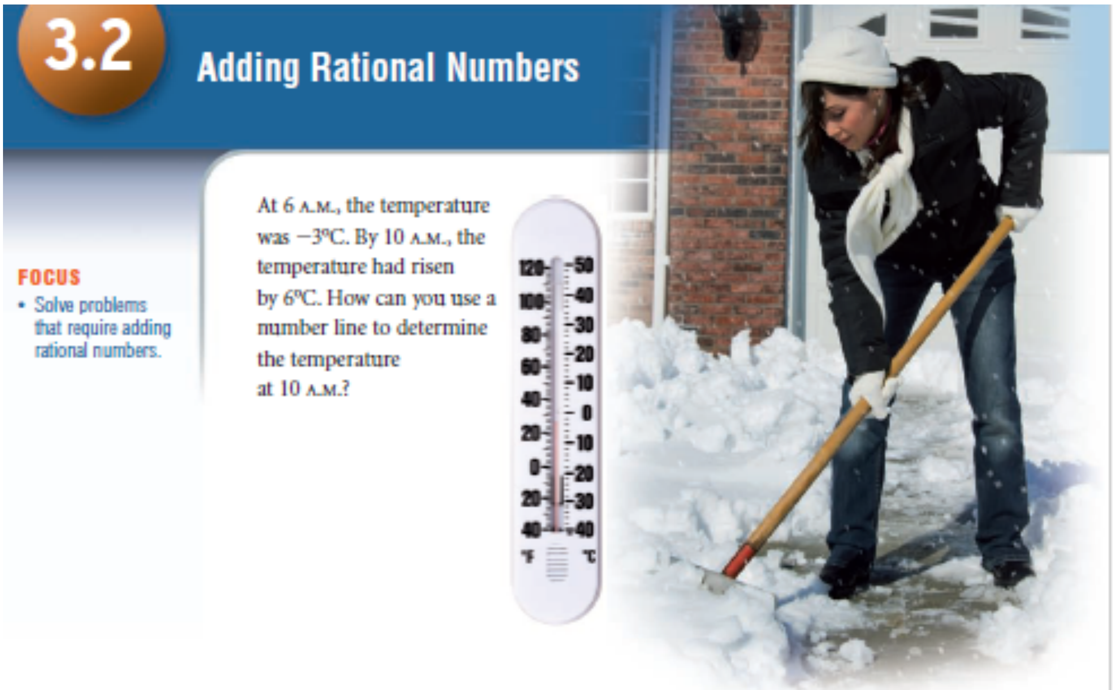
One of the coldest places on Earth
is Antarctica, with an average annual
temperature of about −58°C.
This is a **negative integer**.

Source: MMS7 (2009, p. 52)

Figure 102 is an excerpt from the introduction to subunit 3.2, entitled Adding Rational Numbers, from the MMS collection9. In this figure, the scenario of temperatures rising and falling during the winter appears in different Canadian regions and in other places with similar climatic conditions.

The mention of the harsh winter connects this proposal to the daily lives of part of the country's student public. The illustration with accumulated snow and the process of removing it, also shown in Figure 88, points to recurring practices in urban and rural areas of Canada, although it privileges a specific experience by representing the experience of dealing with large amounts of snow.

Figure 102 - Subindo e Descendo Temperaturas: O Inverno Canadense em Foco



Source: MMS9 (2009, p. 106)

The tasks shown in Figure 103 refer to Iqaluit, the capital of Nunavut, and Inuvik, located in the Northwest Territories. Both are located in regions classified as Arctic areas, known for their very low temperatures and snow-covered expanses. By highlighting these locations, the material suggests a look at geographical and climatic aspects that are often dealt with superficially in teaching resources.

The emphasis on negative temperatures and the mention of ice signal the typical conditions in the Canadian Far North, where the landscape is often frozen for much of the year. In this sense, the image associated with this context shows well-dressed children interacting with blocks of snow, which can reinforce the perception of a daily life markedly influenced by the cold.

Even so, it is important to question how these representations can limit understanding of the ways of life in these regions, since local culture goes beyond icy scenarios and requires attention to social, historical and economic aspects that are not always highlighted in study materials.

By presenting these locations, the text demonstrates an attempt to bring the reader closer to distant realities, but it is worth considering whether this is done in a way that promotes an in-depth view or whether it merely reproduces stereotypes about the Arctic.

The focus on extreme temperatures can cover up other dimensions of life in Iqaluit and Inuvik, such as customs, artistic expressions, languages and community practices, which are rarely highlighted outside of these contexts. Thus, the choice to mention these regions and their climatic peculiarities may arouse interest, but it also invites reflections on the breadth of information offered to those who study or read about these places.

Figure 103 - Iqaluit and Inuvik: Ice Adventures and Mathematical Concepts

8. On January 25th, 2008, the lowest temperature in Iqaluit, Nunavut, was −28.5°C.
On the same day, the lowest temperature in Inuvik, Northwest Territories, was −33.1°C.

a) What is the difference in these temperatures?

b) Why are there two possible answers to part a?

Source: MMS9 (2009, p. 119)

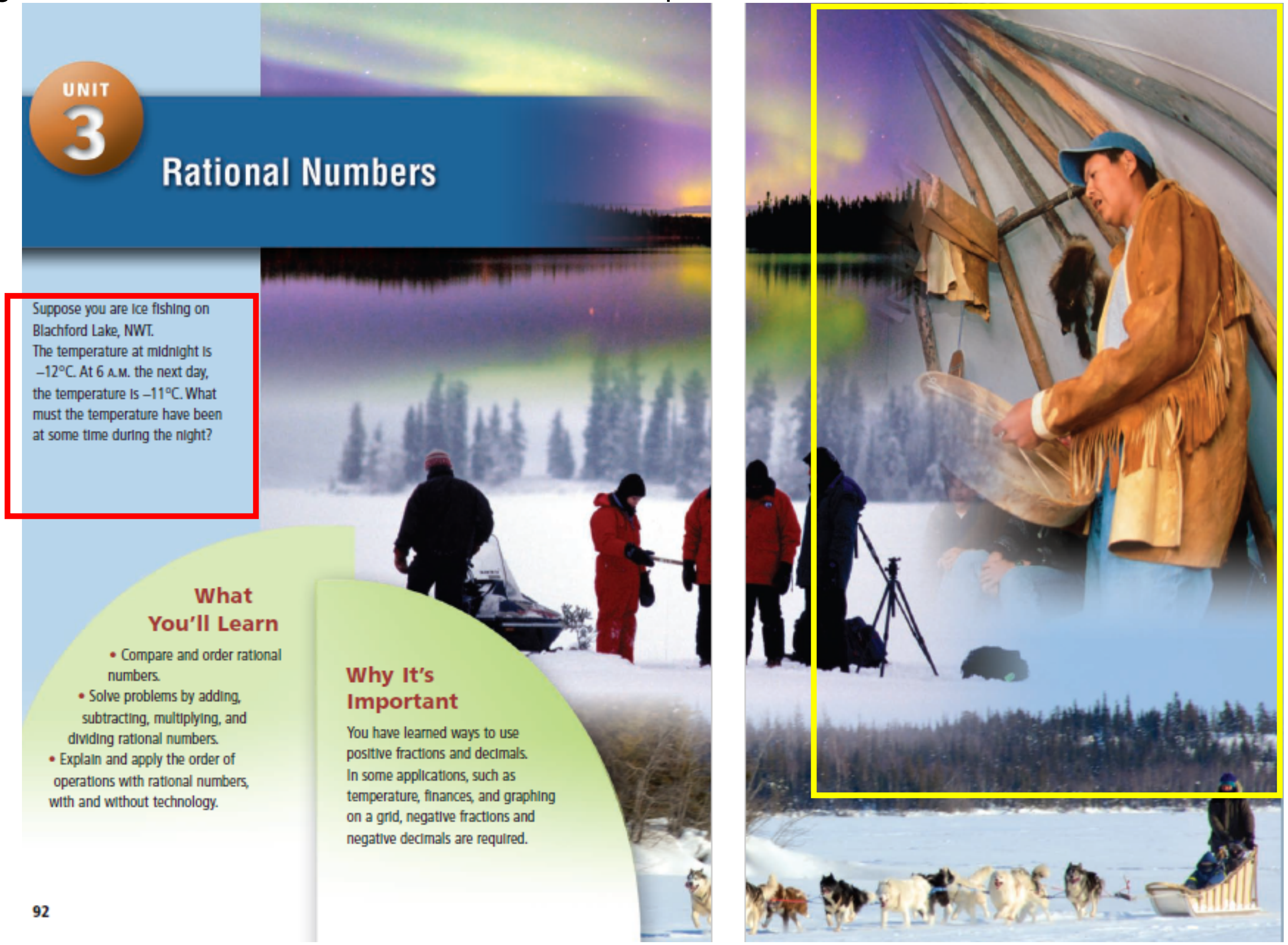
Finally, the last example can be found in the MMS9 book, as an introduction to Unit 3, entitled rational numbers (Figure 104). In this initial section, we see a representation marked by different elements associated with the Canadian locality and its environmental characteristics, such as the Blackford Lake region, NWT; the Aurora Borealis; Indigenous traditions; the Arctic and Subarctic climate; the snowy landscape and; outdoor activities.

The mention of Lake Blackford (highlighted in red in Figure 104), located in the Northwest Territories(NWT), reinforces the image of an area located in the northernmost portions of Canada, where temperatures reach very low levels, the landscapes are largely frozen and there is a strong presence of indigenous communities. Groups such as the Dene Peoples, Inuit Peoples, Métis Peoples, Cree Peoples and Inuvialuit have historically inhabited these lands and maintain cultural practices that demonstrate their relationship with the environment.

The aurora borealis phenomenon - mentioned in the text and illustrated by the intense coloration of the sky - makes up the scenery of Arctic and sub-Arctic regions. Places like Yukon, Nunavut and the Northwest Territories are known for the recurrent occurrence of this natural spectacle, which is deeply embedded in the construction of cultural identities. Thus, the sky lit up by the auroras symbolizes more than a meteorological event, taking on a prominent place in the lives of the communities that inhabit these areas.

The image of the man next to a tipi (traditional tent) (highlighted in yellow in Figure 104) makes direct reference to the ancestral housing practices of Canada's indigenous peoples, such as the Dene and Inuit. This type of dwelling was designed to cope with extreme weather conditions, demonstrating how cultural and environmental relationships interact in adapting to contexts marked by snow, ice and strong winds.

Figure 104 - Blackford Lake, Aurora Borealis and Tipi: Culture and Environment in Northern Canada



Source: MMS9 (2009, p. 92-93

The Arctic and Subarctic climate[53] is characterized by very low temperatures (between -12°C and -17°C), a long winter and extensive areas covered in snow and ice. The presence of dog sleds highlights a traditional means of transportation that, throughout history, has served both indigenous communities and explorers who have traveled through remote regions of Canada.

People fishing on the ice, meanwhile, are another practice that demonstrates ways of interacting with this extreme environment - something that, in addition to being an economic or survival activity, carries the cultural values of different communities (Borealis Expedicoes, website[54]).

In general, the examples shown (Figures 99 to 104) emphasize the effort to bring mathematical content closer to the Canadian cultural reality. This pedagogical stance is consistent with the collection's target audience of students in Canada.

However, when we try to bring the same collection to the Brazilian context, there are limits to the discussion and reflection on the topics covered, since these elements from the Canadian north do not directly relate to the daily lives of most students in Brazil.

By exploring local contexts to teach mathematical concepts - as shown in the Figures - the intention is to connect the school with the cultural reality of Canadian students. In this move, there is an attempt to bring learning closer, by inserting mathematics into situations that are part of everyday life.

Even so, the breadth of this proposal depends on how deeply these examples relate to the students' cultural identity, something that is relevant to conducting a pedagogy that is sensitive to diversity.

Ladson-Billings (2014) argues that a culturally relevant pedagogy encourages students to interpret school content in such a way that their cultural identities are not only recognized, but also valued.

Didactic material can then go beyond simple contextualization, broadening the possibilities for reading the world by articulating the voices, stories and experiences that make up the lives of each student group. By including cultural elements linked to the students' territory, textbooks can go beyond superficiality and foster a comprehensive, meaningful and respectful understanding of cultural diversity.

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[53] Regiões setentrionais são regiões que se encontram no Norte ou que se referem ao norte.
[54] Avaiblabe at: https://borealisexpedicoes.com.br/os-esquimos-e-a-cultura-dos-povos-originarios-do-artico/?utm_source=chatgpt.com. Acessa em: 09/01/2025.

In this sense, by integrating aspects related to local environments, as seen in Figures 99 to 104, math textbooks can address more than algorithms and formulas. They open up ways for students to reflect on the culture, geography and history of the place where they live.

However, when thinking about transposing these books from the MMS collection to Brazil, there is a mismatch between the typical scenarios of the Canadian Far North and the Brazilian reality. The challenge of developing appropriate mediations is a fundamental point in order to avoid the learner's experiences becoming disconnected from the material presented.

The images analyzed (Figures 99 to 104) show aspects linked to the intense cold, ice, the aurora borealis and practices such as ice fishing, dog sled races and buildings suitable for frozen environments.

These characteristics echo in everyday life in regions such as the Northwest Territories, Yukon and Nunavut, but are far removed from the repertoire of most Brazilian students, who live in a tropical climate, diverse vegetation and different cultural contexts. In addition, Brazil's physical landscape, full of rainforests, mountain ranges, wetlands and long coastlines, differs significantly from the scenery illustrated by Canada's snow-capped mountains and frozen surfaces.

At this point, reflections arise on how this content can make it difficult for Brazilian students to get involved. The narratives about ice fishing (Figure 90), for example, describe an activity that doesn't exist in most of Brazil, since the waterways don't freeze and the temperature variations don't reach the level of the polar regions.

If you wanted to create a more engaging activity for the Brazilian public, you could propose, for example, analyzing climatic variations in local ecosystems, such as the cerrado, whose temperatures can change throughout the day, or investigating the impacts of floods on rivers in the Amazon.

Similarly, the theme of dog sleds (Figures 87 to 90) could be rethought to align it with recognized means of transport in Brazilian regions, such as rafts in coastal areas of the Northeast or boats in riverside communities in the Amazon.

These reflections make it possible to bring the mathematical content closer to problems that integrate the environmental and cultural characteristics of Brazil, addressing, for example, the way in which populations deal with tides, floods or long distances on extensive rivers.

Another significant example is the use of Canadian provincial flags to explore angles (Figure 99). The idea of relating geometric content to national symbols can be maintained by using the flags of Brazilian states or even the Brazilian flag (Figure 105).

In this case, observing the angles formed by the stars or other geometric elements becomes more familiar, enabling debates about the meanings of the colors and symbols that make up national identity. Figure 105, for example, shows the Brazilian flag, the design of which can serve as a basis for investigating shapes, colors and geometry.

Figure 105 - The Brazilian Flag: Exploring Shapes, Colors and Geometry

Source: Brasil (2008, site[55])

The distance between the Canadian context and the Brazilian experience highlights the importance of critical mediation in textbooks, with attention to the environmental and cultural particularities of those who learn.

Presmeg and Radford (2008) point out that mathematics can be seen as a cultural and historical practice, which invites a perspective that goes beyond techniques and formulas. This view calls for the textbook to promote teaching and learning processes that include and give value to the students' trajectories in their realities.

Thus, proposing examples that are closer to Brazilian daily life, such as discussing the consequences of flooding in urban areas, analyzing temperature changes in the Pantanal or relating mathematical content to festivities such as Carnival, stimulates an immediate connection between students and the study of

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[55] Avaiblabe at: https://www.gov.br/planalto/pt-br/conheca-a-presidencia/acervo/simbolos-nacionais/bandeira/bandeira-nacional-brasil.jpg/view. Accessed at: 22/12/2024.

mathematics. This movement not only integrates the content with the local reality, but also strengthens the recognition of the country's cultural and environmental identity.

In this way, mathematics becomes an instrument for understanding and dialoguing with the world, as it is based on phenomena and situations that concern the students. It also stimulates critical reflection on the different realities that exist, encouraging comparative discussions between Brazil, Canada and other parts of the world, broadening the perception of cultural diversity and complexity.

6.3 Exploring Sociocultural and Sociopolitical Diversity through Games: Probability, Heritage and Intercultural Contexts

The MMS collection includes specific sections devoted to games, illustrated by the reference “Play a Game with your classmates or at home to reinforce your skills” (MMS7, 2009, p. XV). This invitation points to a pedagogical interest in fostering playful interaction and collaboration. In MMS7, for instance, board games appear in the Unit Problem or under a designated Game header. Although similar proposals surface at different moments in the series, Category 3 focuses on those instances published in a structured space explicitly labeled as Game. An overview of the volumes shows this section repeated systematically (eight times in MMS6, three in MMS7, eight in MMS8, and nine in MMS9), indicating a clear editorial choice to integrate playful activities throughout the materials.

When viewed in light of the final guiding question (what cultural aspects of the Canadian context are embedded and made explicit in mathematics textbooks, and how can these elements be interpreted in the context of multilingual schools in Brazil?) these games offer a way to investigate how foreign textbooks may embed diverse cultural practices. The general objective of examining how sociocultural differences in Canadian materials might influence or bring implications to Brazilian classrooms underlies this analysis, and the specific objectives (a), (b), and (c) guide the discussion in multiple ways. First, by exploring which sociocultural references inform the proposed games (Objective a), we see how playing cards or Blackfoot traditions intersect with the local realities that teachers and students navigate (Objective b). Second, discussions about the linguistic and historical backgrounds of these games may reveal alignments or tensions in educational practices (Objective c), especially in multilingual settings.

Thus, analyzing the Game sections helps illustrate how distinct cultural elements, ranging from European card decks to indigenous Blackfoot sticks, are presented to students, raising questions about adaptation and recontextualization in multilingual Brazilian schools. This perspective resonates with the broader concern of this thesis, which is to understand whether, and in what ways, Canadian textbooks can connect with local cultural practices, potentially broadening their scope or highlighting the challenges of employing foreign materials in different educational contexts. By examining these proposals, the discussion moves toward answering the main research question and reflecting on the complexities associated with bringing foreign textbooks, laden with their own cultural and linguistic traces, into new and varied learning spaces.

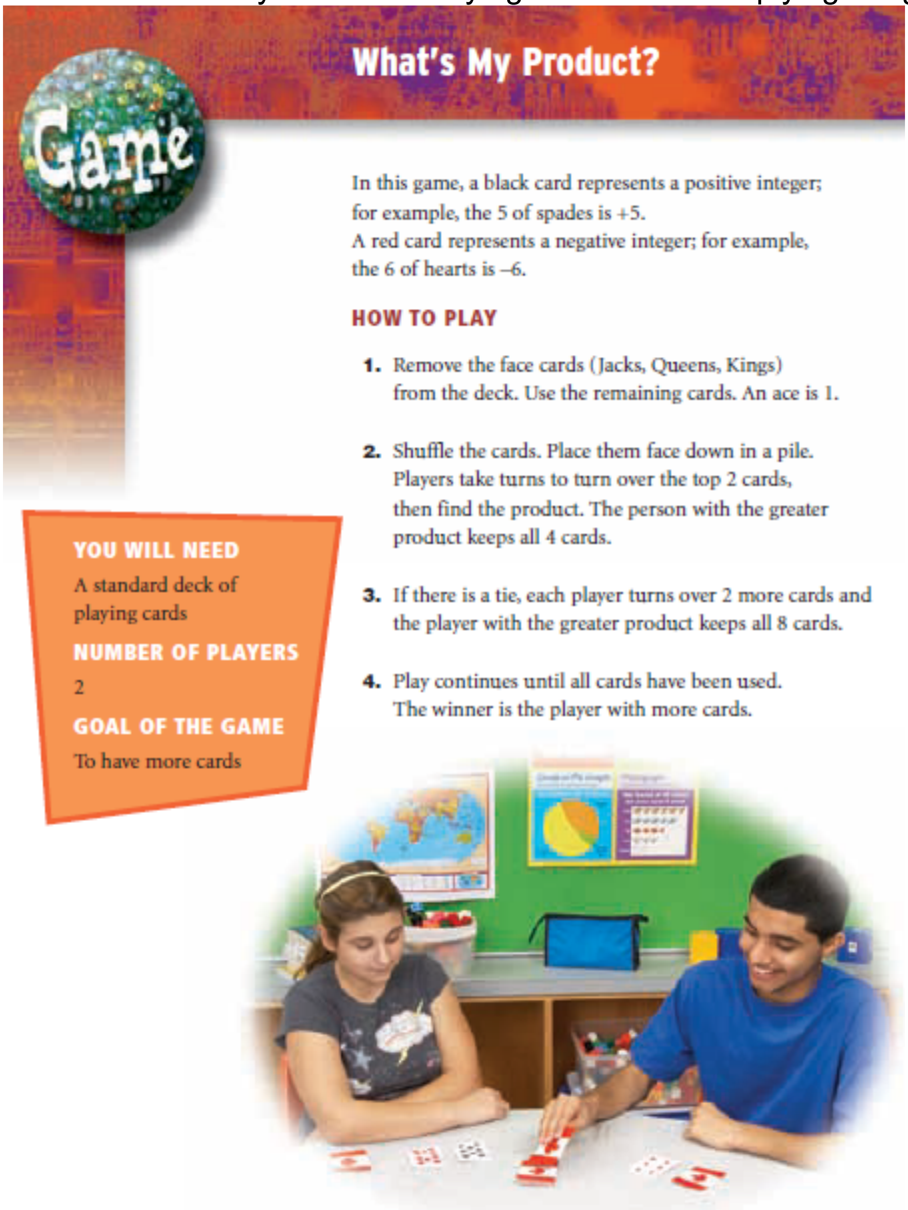
6.3.1 What's my product? Cards and whole number multiplication

The first example is entitled What's My Product? and uses playing cards to explore the multiplication of whole numbers in a practical and interactive way (Figure 106). In this proposal, the black cards (spade ♠ and clubs ♣) represent positive integers and the red cards (hearts ♥ and gold ♦) represent negative integers, leading participants to calculate products and accumulate cards by defeating their opponent.

As well as encouraging participation, this type of game encourages learning in pairs, inviting students to negotiate strategies and reflect on mathematical rules.

It can be seen that the mathematical goal revolves around consolidating understanding of integer multiplication and how signs behave when multiplying positive and negative numbers. During the game, students make quick calculations to determine products and identify which one is bigger, which can increase their familiarity with the sign rules.

Figure 106 - What's My Product? Playing Cards and Multiplying Integers



Game

What's My Product?

In this game, a black card represents a positive integer; for example, the 5 of spades is +5.
A red card represents a negative integer; for example, the 6 of hearts is –6.

HOW TO PLAY

1. Remove the face cards (Jacks, Queens, Kings) from the deck. Use the remaining cards. An ace is 1.
2. Shuffle the cards. Place them face down in a pile. Players take turns to turn over the top 2 cards, then find the product. The person with the greater product keeps all 4 cards.
3. If there is a tie, each player turns over 2 more cards and the player with the greater product keeps all 8 cards.
4. Play continues until all cards have been used. The winner is the player with more cards.

YOU WILL NEED
A standard deck of playing cards

NUMBER OF PLAYERS
2

GOAL OF THE GAME
To have more cards

Source: MMS8 (2009, P. 76)

In addition to the mathematical perspective, the proposal uses playful elements to engage students. Playing in pairs encourages communication and the negotiation of strategies, mobilizing social skills. Based on Vygotsky (1978, 1998), it is understood that interactions enhance cognitive development. In such activities, the teacher's mediation is decisive in balancing the relaxation of the game with mathematical exploration, fostering an understanding of the signal rules and not just the desire to win the game.

However, when a resource such as cards is adopted, cultural and political issues emerge (see Apple (2013), Ball (2012), Valero & Zevenbergen (2004)). In certain sociocultural contexts, cards have historically been associated with gambling, which can lead to resistance to their use in the classroom (Amaral, 2021).

This dimension illustrates that the adoption of an apparently neutral resource is not exempt from disputes and deep-rooted prejudices, linked to local history, religious traditions or collective representations about what the school can (or cannot) validate. From this point of view, the inclusion of cards to teach mathematics unfolds in several political layers.

Firstly, there is the cultural politics of the curriculum, since the choice of certain materials is not always received in the same way in different contexts (Valero, 2018). Then there are aspects of institutional regulation: in the MMS collection, published in Canada, the incorporation of cards appears to be commonplace, while in Brazil this same proposal may be objected to, given local conceptions of gambling. This contrast highlights that the acceptance (or rejection) of playful resources depends on how the school community interprets and legitimizes these practices (Apple, 2018; Valero, 2023).

On the other hand, from the perspective of collaborative teaching and encouraging active participation, What's My Product? opens up space for different voices to circulate. Students have the opportunity to construct knowledge in pairs, which activates the Zone of Proximal Development (ZDP) (Vygotsky, 1978).

However, it is worth pointing out the tensions inherent in working in small groups: while some students lead and take ownership of mathematical reasoning, others can take on more passive roles. Hence the importance of teacher mediation, which must be attentive to the balanced distribution of participation (Ribeiro et al., 2020).

Furthermore, the competitive aspect of the game can be controversial. If, on the one hand, competition increases the involvement and motivation of certain students, on the other hand it can reinforce internal hierarchies or promote meritocratic discourses in mathematics teaching (Rizvi & Lingard, 1995; Valero, 2023). In a socio-economic and culturally heterogeneous context, competition can reproduce inequalities, benefiting those who have already mastered the concepts quickly. It is therefore up to the teacher to monitor the process so that competition does not obscure conceptual depth or exclude students who find it more difficult.

As Ball (2012) suggests when discussing the neoliberal imaginary, pedagogical choices are crossed by values, beliefs and power structures, which can manifest themselves in the rejection or approval of certain methods. Thus, although What's My Product? exemplifies a way of approaching the multiplication of integers through play, it should be recognized that such an approach is not exempt from sociocultural and institutional negotiations.

The ability of teachers and managers to articulate these proposals with the values and representations of the school context is fundamental to achieving an inclusive practice. Ultimately, reflecting on the adoption of decks is not just a methodological or didactic discussion, but a debate about which visions of knowledge, culture and society are validated within the curriculum (Apple, 2018; Valero, 2004).

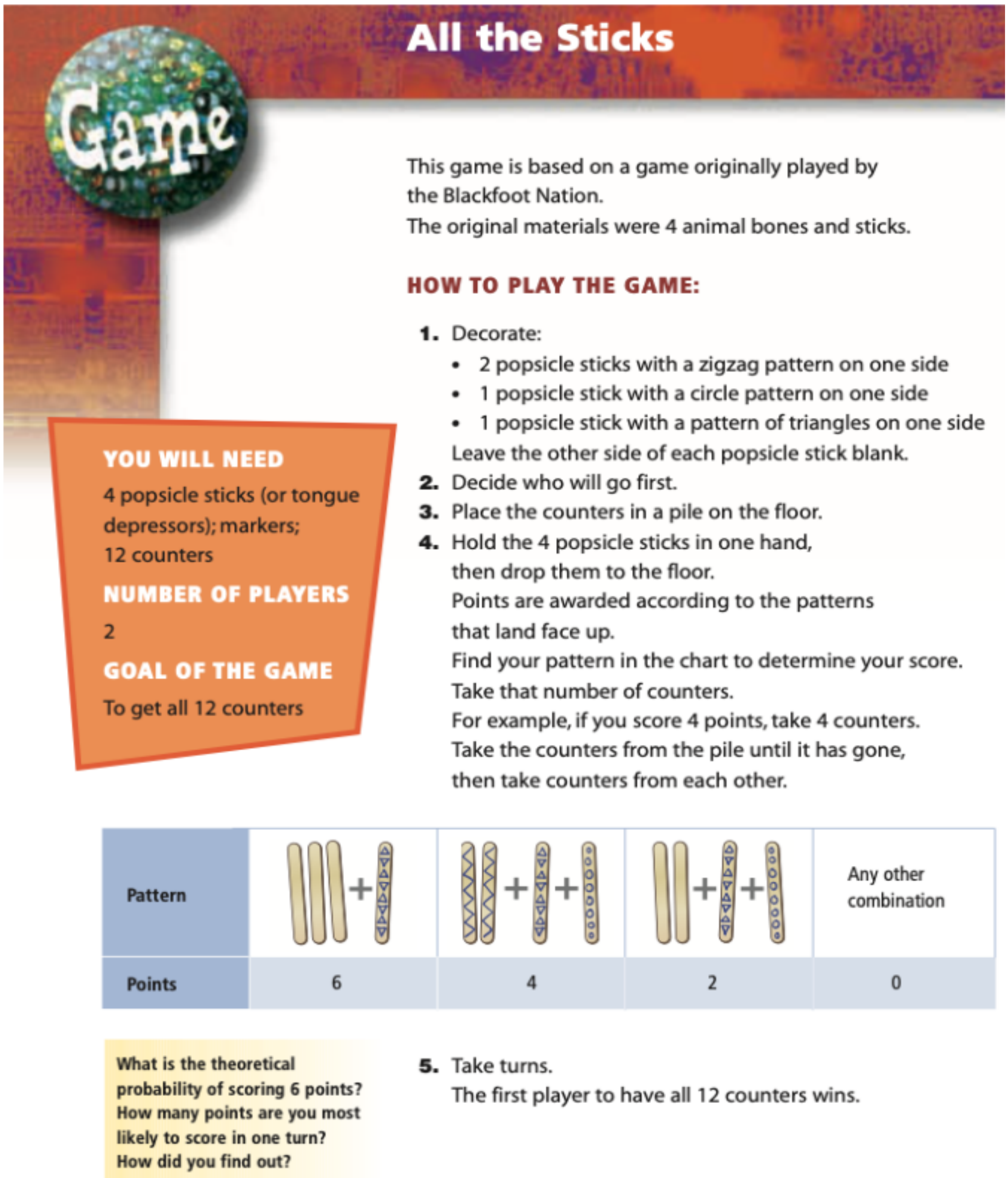
6.3.2 All the Sticks: probability and Blackfoot cultural heritage

The second example shows a game associated with the Blackfoot Nation, also known as the Blackfeet.

In this case, it is proposed to use popsicle sticks decorated by the students themselves as a substitute for the bones and twigs originally used by the native tribes, to explore concepts of probability and scoring.

As Figure 107 illustrates, by throwing the sticks on the ground, the combinations formed serve as a basis for calculating points and for discussions related to chance and descriptive statistics.

Figure 107 - All the Sticks: Probabilidade e Herança Cultural Blackfoot



Game

All the Sticks

This game is based on a game originally played by the Blackfoot Nation.
The original materials were 4 animal bones and sticks.

YOU WILL NEED
4 popsicle sticks (or tongue depressors); markers; 12 counters

NUMBER OF PLAYERS
2

GOAL OF THE GAME
To get all 12 counters

HOW TO PLAY THE GAME:

1. Decorate:
   - 2 popsicle sticks with a zigzag pattern on one side
   - 1 popsicle stick with a circle pattern on one side
   - 1 popsicle stick with a pattern of triangles on one side

   Leave the other side of each popsicle stick blank.
2. Decide who will go first.
3. Place the counters in a pile on the floor.
4. Hold the 4 popsicle sticks in one hand, then drop them to the floor.
   Points are awarded according to the patterns that land face up.
   Find your pattern in the chart to determine your score.
   Take that number of counters.
   For example, if you score 4 points, take 4 counters.
   Take the counters from the pile until it has gone, then take counters from each other.

Pattern	+	+ +	+ +	Any other combination
Points	6	4	2	0

What is the theoretical probability of scoring 6 points? How many points are you most likely to score in one turn? How did you find out?

5. Take turns.
   The first player to have all 12 counters wins.

Source: MMS7 (2009, p. 289)

The Blackfoot Nation, also called the Blackfeet, is a confederation of indigenous peoples of North America, traditionally inhabiting the regions that today correspond to Alberta in Canada and Montana in the United States of America. Made up of three main groups - the Siksika, the Kainai and the Piikani - this nation has a culture and heritage deeply rich in traditions, languages and historical practices.

They are widely recognized for their elaborate beadwork, ceremonial rituals and a spiritual connection to the land, especially the sacred Rocky Mountains (Encyclopedia Britannica, 2024, website[56]).

[56] Avaiblabe at: https://www.britannica.com/topic/Blackfoot-people. Acessa em: 15/05/2024.

In the game, students throw sticks on the ground and apply concepts of probability to calculate scores based on the patterns formed when they land. This game not only addresses mathematical aspects, such as calculating probabilities, but also promotes the appreciation of cultural practices, connecting school learning to the historical and cultural context of indigenous peoples.

The integration of cultural heritage into Mathematics Education, such as the incorporation of references to the artistic and mathematical practices of the Blackfoot people, reflects an attempt to enrich learning by connecting it to diverse cultural contexts.

While this approach highlights the potential of mathematics to transcend purely technical dimensions and engage with cultural narratives, it often runs the risk of presenting superficial representations of cultural diversity.

From a sociocultural perspective, such an approach can make learning more meaningful, as it brings mathematics closer to different contexts and favours the appreciation of historically marginalized practices. However, if this proposal is restricted to the mechanical execution of the game, there is a risk that the cultural dimension will be reduced to a curiosity.

In this sense, authors such as Rosa and Orey (2016) discuss the relevance of pedagogical strategies that promote critical dialog about the cultures involved, so that students understand why and how such practices originated, without reinforcing stereotypes or reducing traditions to mere decorations.

Valero (2004, 2023) points out that the presence of cultural elements in math tasks is part of a socio-political dispute about what should and should not be taught in school. Including the history and practices of an indigenous people broadens the debate on identity, diversity and inequality, inviting students and teachers to reflect on the power relations that permeate the curriculum. This approach opens up opportunities to think of mathematics not just as a set of neutral contents, but as part of a cultural field that may (or may not) engage in dialog with different ways of life.

To avoid the activity becoming just a superficial mention of the Blackfoot culture, it is up to the teacher to mediate an in-depth discussion that promotes reflections on the origin of the game, its relationship with the life of that people and the social implications of incorporating such a proposal into a school that often does not share this context.

According to Gutstein (2003, 2012), bringing cultural and social issues into math teaching allows students to develop a broader awareness of themselves and the world, giving new meanings to learning.

Furthermore, although the inclusion of indigenous games is potentially valuable for diversifying learning, it is prudent to reflect on the relevance of their use in certain national contexts.

In Brazil, for example, there is a plurality of indigenous cultures that could serve as inspiration for the creation or recreation of games traditionally practiced in local communities, such as peteca (Figure 108 and Figure 109).

This concern dialogues with Ladson-Billings (1995, 2014), who states that a culturally relevant pedagogy is based on experiences close to the student's universe, connecting domestic experience to the formal curriculum.

In contexts such as Brazil, where cultural references differ significantly from those of the Blackfoot Nation, it is crucial to critically evaluate the transferability of such games.

For example, instead of focusing on cultural narratives unknown to Brazilian students, one could consider adapting the game to reflect Brazilian traditions, such as the game of peteca (Figure 108 and 109), which has indigenous origins and which, according to the Brazilian Confederation of Peteca - CBP (Santos, 2020, website[57]) was a “practice spread in different ways by the peoples of America. There are records in the regions where Brazil, Peru and Argentina are located today, as well as Mexico”.

Among the Tupi-speaking peoples, the term Pe'teka referred to a game dynamically marked by the absence of rules or spatial boundaries, and the main objective was to extend the time the shuttlecock remained in the air (Santos, 2020). Conforme o autor, há menções de que o jogo era realizado em círculos, promovendo a interação social, ele ainda destaca que:

> [...] os povos de língua Tupi adotaram o termo Pe´teka que significa “bater com a palma da mão”. Dessa forma, a peteca era jogada sem regras rígidas, sem espaços delimitados, tendo como objetivo mantê-la no ar por mais tempo possível. Fontes indicam que era jogada em círculos favorecendo a integração entre os participantes (Santos, 2020, site).

[57]Avaiblabe at: https://cbpeteca.org.br/historia-da-peteca/#:~:text=Fontes%20indicam%20que%20era%20jogada,sentidos%2C%20inclusive%20em%20meios%20corporativos. Accessed at: 23/12/2024.

Figure 108 - The traditional shuttlecock: a bridge between games and mathematics

Source: Cruz (2020, site[58])

Figure 109 - Shuttlecock Varieties: Different Styles to Explore Concepts

Source: Santos (2020, site)

This traditional game, which originally involved making shuttlecocks out of palm or banana leaves, seeds and bird feathers (Figure 106), could be used to explore mathematical concepts such as:

a) Time and distance measurements. The time the shuttlecock remains in the air, working with counting, converting units (seconds to minutes) and estimates. The distance covered by the shuttlecock between the players, exploring units of measurement and also estimations.
b) Geometric concepts such as trajectory, area and perimeters and symmetry. Explore the parabolic trajectories that the shuttlecock describes in the air, introducing basic concepts of analytical geometry. Work on calculating areas and perimeters by delimiting playing spaces or creating tables for recording. Analyze symmetry in the shape and design of the shuttlecock, relating it to the concepts of axis and reflection.
c) Statistics and Probability, such as descriptive statistics, mean and median and probability. Collect data from the game, such as the number of passes without dropping the shuttlecock, and construct graphs (bars, lines, sectors) to represent the results. Calculate the average or median score for each player or group. Exploring the chance of keeping the shuttlecock in the air, taking

[58] Avaiblabe at: https://www.gentedeopiniao.com.br/colunista/montezuma-cruz/sim-porto-velho-tem-peteca. Accessed at: 23/12/2024.

into account the skill of the players or the environmental conditions (wind, space).

d) Ratio, Proportion and Percentage, when comparing the performance of different players or teams, exploring proportional relationships (e.g. passes made per player) and calculating the percentage of hits in relation to the total number of attempts or determining the percentage of time the shuttlecock remained in the air in a round.

e) Sequences and Progressions, such as numerical sequences to record and analyze sequences generated by the number of touches on the shuttlecock in each round, and arithmetic progression when analyzing the increase or decrease in hits over several rounds.

Following the sociocultural and political logic of Valero & Zevenbergen (2004), the adaptation of foreign games to other realities needs to consider the student's sociocultural universe.

The shuttlecock (Figure 110), for example, has indigenous roots in Brazil and can work as a resource for exploring concepts of time, distance, statistics and geometry, while respecting a cultural heritage that is more recognizable to Brazilian students.

So it's not a question of simply substituting characters or themes, but of reflecting on the cultural relevance of each resource and how it is introduced into the curriculum.

Figure 110 - Indigenous Peteca: Cultural Creativity and Mathematical Possibilities

Source: Torres (2021, youtube[59])

When this type of proposal is considered, the discussion about the cultural politics of the curriculum emerges (Apple, 2013; Valero, 2018). If the school opts for a pedagogy that favors hegemonic cultural references, it will limit students to a single repertoire, making other experiences invisible and making learning less connected to different social contexts.

On the other hand, making room for games from different peoples, including indigenous peoples, broadens the understanding that mathematics is not restricted to Eurocentric methods, but manifests itself in various sociocultural practices (D'Ambrósio, 2005; Rosa & Orey, 2016).

From Valero's perspective (2004, 2023), the analysis of mathematics teaching practices must necessarily take into account the social and political frameworks in which these practices are embedded.

When Canadian teaching materials, for example, present card games or games of indigenous origin, they reflect a set of values and representations that have been validated in a certain context, making up power relations that define which knowledge has legitimacy in the classroom.

The choice or rejection of certain games is also linked to the socio-economic and cultural conditions of school communities. In places where there are financial barriers, religious resistance or historical tensions in relation to games of chance, a playful approach based on cards can be viewed with suspicion.

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[59] Avaiblabe at: https://www.youtube.com/watch?v=XXrViu4TCW0. Accessed at: 23/12/2024. O vídeo demonstra a produção de uma peteca feita com embira de bananeira e penas. Também é possível acompanhar como o jogo da peteca é realizado.

In more open contexts, however, the same resource can be widely accepted and praised for its emphasis on active student participation (Apple, 2018). Thus, talking about games in mathematics is not something isolated: it implies understanding the mechanisms of regulation (Valero, 2012) and the power structures that manage what is taught and how it is taught.

The presentation of games such as What's My Product? and All the Sticks represents an effort to integrate playful and cultural aspects into the teaching of mathematics. If, on the one hand, they can motivate students by encouraging experimentation and collaboration, on the other, they raise questions about the appropriation (or not) of the cultures involved, possible institutional resistance and the need for teacher mediation that articulates social and political components.

When a game is introduced into the classroom, it opens up the possibility of connecting mathematical knowledge with students' experiences, bringing the subject closer to relevant sociocultural aspects.

This articulation, however, requires a critical and reflective stance from both teachers and the developers of teaching materials, so that cultural diversity is not reduced to an exotic appendage, but incorporated in a dialogical way (Rosa & Orey, 2016; Valero, 2018, 2023).

From this perspective, activities linked to games have the potential to shed light on identity issues, bringing new meanings to learning mathematics and generating spaces for discussion about representations, power and social participation (Valero & Knijnik, 2016).

In this way, the simple act of playing in the classroom becomes interpreted as part of a larger socio-political network, in which the legitimacy of certain content and methods is related to who decides, who participates and whose voices are heard in the process of curriculum construction.

6.4 Bridging Worlds: Reflections on Mathematics and Cultural Contexts

The multiple perspectives analyzed throughout this chapter suggest that the teaching of mathematics is far more than the transmission of numerical operations, algebraic expressions, or geometrical theorems. It is a tapestry where cultural practices, linguistic repertoires, and social interactions are interwoven, influencing both how students learn and how educators choose to teach. By examining the elements introduced in Category 1, Category 2, and Category 3, we are able to observe how each set of findings connects directly to the guiding question of this research: what cultural aspects of the Canadian context are embedded and made explicit in mathematics textbooks, and how can these elements be interpreted in the context of multilingual schools in Brazil? Moreover, these reflections dialog with the general objective of this thesis, which is to explore how sociocultural differences embedded in foreign mathematics textbooks, particularly those from the Canadian context, can influence Brazilian educational scenarios. They also speak to the specific objectives of analyzing sociocultural differences, investigating cultural and linguistic aspects, and discussing the potential sociopolitical implications that emerge from the adoption of such materials.

When the discussion in Category 1 turned toward the teaching of integers, fractions, and language, it became apparent that the relevance of modeling techniques and interactive practices extends well beyond simple pedagogical preference. The focus on visual models such as colored chips, number lines, fraction bars, and subdivided rectangles underscores how the presentation of mathematical content is culturally shaped. In Canadian textbooks, the emphasis on colored pieces or temperature lines can make complete sense in a North American environment where negative temperatures and the use of chips or counters are customary references. However, such practices may prompt educators in Brazil to ask whether these same examples resonate with local experiences.

Given that the first specific objective (analyzing sociocultural differences in Canadian textbooks) requires us to examine how these materials connect with Brazilian realities, we see a direct tension. On one hand, using a number line to model fractions or integers can be advantageous for students in any classroom, yet on the other hand, references to local climate conditions or everyday imagery might need to be adapted to ensure engagement. The possible mismatch between the textbook's context and that of Brazilian students demonstrates the bridging that teachers perform when taking a culturally embedded resource - like a hockey-related temperature example - and converting it into a concept that resonates with a local scenario, such as discussing the temperature ranges in a mountainous region of Brazil or referencing local uses of negative numbers.

The interplay among integers, fractions, and language also suggests that students in multilingual Brazilian schools may need additional linguistic mediation. If a textbook consistently uses English words or relies on specific local idioms from Canada, educators must find ways to introduce or compare those terms with Portuguese equivalents or local expressions, thereby building conceptual links that help students to interpret the book's tasks. From a sociopolitical perspective, which this research does not place at the center but acknowledges as an important factor, the adoption of a foreign textbook implies negotiations about which linguistic registers and cultural norms become dominant in the classroom. School communities, policymakers, and families may each have opinions about whether those registers should be embraced, translated, or even replaced by more localized expressions. The question of who decides and how those decisions are carried out is a sociopolitical matter, touching upon specific objective (c), which calls for a discussion of potential sociopolitical implications.

In Category 2, the analysis of sports, art, monuments, and festive events sheds light on how mathematics intersects with everyday life, cultural traditions, and even historical legacies. Sports such as ice hockey and golf, frequently seen in Canadian materials, might resonate with communities in Canada for whom those sports are strongly embedded in local identity. In Brazil, however, students may be more attuned to field soccer or regional games that hold greater cultural significance. The bridging process here is twofold.

First, teachers recognize the textbook's portrayal of certain sports as a form of cultural knowledge that might not match their students' daily experiences. Second, they either adapt the reference or supplement it with local sporting examples so that the essence of the mathematical concept remains intact, even if the cultural reference shifts. This practice ties to the guiding question of how cultural elements from Canadian textbooks can be interpreted in local contexts, emphasizing that the sports references are not simple decorations but elements that reflect cultural values and shape the way mathematical content is understood.

Artistic manifestations and architectural references, like quilts, totems, or indigenous totemic poles, introduce mathematics as a tool for understanding forms, patterns, and dimensions. In Canadian textbooks, the presence of quilts or references to geometric transformations in totemic art suggests a historical and aesthetic approach that is linked to certain communities in Canada, including First Nations. However, in a Brazilian classroom, images of totems might seem distant if students have never seen or heard of such structures before. This difference does not necessarily undermine the potential of the example. Instead, it opens an avenue for educators to relate geometry lessons to other artistic or architectural heritages that shape the Brazilian cultural landscape. Tile designs in historic city centers, indigenous baskets from different regions of Brazil, or even local craft traditions could serve the same purpose while leveraging geometry for pattern recognition and symmetrical analysis. This speaks to the bridging effort in which a teacher draws parallels between the Canadian example in the textbook and a locally meaningful counterpart, reinforcing the notion that mathematics is not limited to a single cultural expression.

Religious festivities and rites, such as Powwow, Hanukkah, or winter ice festivals, have appeared in the textbooks as contexts for tasks involving rational numbers, data analysis, or probability. These references carry ritualistic or community-based connotations for Canadian students, yet Brazilian students might interact with them differently. When asked to analyze numbers related to a Powwow or an ice festival, learners might find the idea intriguing, but they might also feel detached from the specifics of the tradition or the climate conditions involved.

Brazilian students could, for instance, more closely identify with local festivals that have parallel symbolic significance, like the Festa Junina or the Carnival, though these do not appear in the Canadian material. The bridging process emerges once more, prompting educators to compare or supplement the foreign examples with local ones. The question that arises is whether a teacher should skip references to foreign religious festivities altogether or use them as an opportunity to explore global cultural diversity, thus addressing specific objective (b), which focuses on investigating how cultural and linguistic aspects in Canadian materials can be reinterpreted for Brazilian contexts. The engagement with foreign cultural items can be constructive, yet it calls for thoughtful guidance that clarifies the historical, spiritual, or festive dimension behind each event.

The exploration of games in Category 3 makes clear that the term game is not monolithic. A single deck of playing cards might be accepted in one community and contested in another, depending on religious or cultural beliefs. In the Canadian context, referencing What’s My Product? might seem a neutral or benign approach to multiplying integers, but in other areas, the presence of playing cards can raise objections if parents or administrators associate them with gambling or other stigmatized activities. Meanwhile, an indigenous game like All the Sticks, drawn from Blackfoot Nation cultural traditions, offers a rich avenue to discuss probability, heritage, and sociocultural identity. However, its meaning may not be transparent to students unfamiliar with the Blackfoot people’s history or to teachers who have little background knowledge about indigenous practices in Canada. The bridging role here is evident when educators choose to treat these references either as windows into a different world or as tasks requiring adaptation, possibly substituting or comparing the Blackfoot game with a Brazilian indigenous game or a widely known local game.

This multiplicity of examples - sports, art, religious festivities, and games - illustrates how mathematics transcends numbers and procedures, mingling with cultural symbols, community beliefs, and personal experiences. The bridging necessary in each scenario often relies on teacher mediation. In a multilingual Brazilian context, teachers face the additional dimension of language. The textbooks, originally produced for Canadian students, may use English in short instructions, key terms, or as an underlying structure. Teachers have to determine whether to translate, reinterpret, or incorporate such language references, always mindful that some terms may have no direct equivalent in Portuguese or might carry connotations that Brazilian students cannot immediately grasp.

Connecting these findings to the main research question reveals why cultural elements in mathematics textbooks cannot be reduced to decorative examples. They form part of a larger socio-educational dynamic. The process of bridging the foreign content to local realities means recognizing that sociocultural differences, while occasionally challenging, can also enrich the learning experience. Such enrichment occurs when teachers harness students’ curiosity about unfamiliar sports, rituals, or architectural designs, and relate them back to local knowledge, thus fostering both mathematical understanding and cultural awareness. This outcome has ramifications for the general objective of the thesis, which is to explore how these differences affect the Brazilian educational setting. The bridging metaphor suggests that textbooks act as a pathway connecting distinct points. They bring references from a different part of the world and invite local educators and students to walk across to discover new landscapes or to reflect on their own.

However, crossing that bridge is not without tension or negotiation. The sports references might feature hockey and freeze out the possibility of including futsal or capoeira-related tasks. The references to winter festivals might seem uninteresting for learners who have never experienced sub-zero temperatures or do not own clothes suited for a Canadian winter. Yet, as teachers adapt or recontextualize these references, they engage in a transformation that goes beyond a simple translation of content. They are essentially reauthoring the textbooks, drawing upon their professional judgment and creativity. This process resonates with the potential sociopolitical implications mentioned in the specific objective (c), since adopting foreign materials involves policy decisions and cultural negotiations about what knowledge is deemed valid and what might need to be reworked.

Seen through this lens, bridging is not merely about making the foreign friendly. It is also about leveraging the tension between different references to illuminate how mathematics is shaped by and shapes culture. For instance, a teacher who notices repeated references to a predominantly European or North American viewpoint might decide to interject local experiences, or indigenous contexts from Brazil, to ensure that the classroom is not echoing a single worldview. That choice, while pedagogical on the surface, is also sociopolitical, reflecting beliefs about inclusivity, representation, and equity in education.

Another bridging point arises when we return to the nature of the categories themselves. Category 1 centers on the interplay of concepts and linguistic representations, Category 2 explores sociocultural contexts such as sports and community events, and Category 3 examines games as sociocultural artifacts. When the analysis from these categories is woven together, it demonstrates how multiple aspects - ranging from fraction models to indigenous playful traditions - are part of one unified tapestry of teaching and learning experiences. This tapestry is precisely what the guiding question highlights: the Canadian context, embedded in textual and visual materials, interacts with a Brazilian environment that is multilingual, multifaceted, and shaped by its own traditions. By bridging them, teachers become architects of new forms of classroom practice, blending borrowed strategies with local knowledge, and deciding which foreign references can remain as is and which must be molded to fit local conditions.

This entire process underscores the role of teacher agency, a theme that emerges implicitly across all categories. Teachers are not passive conduits of the textbook's content. Instead, they orchestrate the bridging, deciding when to clarify concepts, how to rephrase an example, or whether to supplement an activity with details from local contexts. Their choices have repercussions for how students see the relevance of mathematics. If a teacher acknowledges the Canadian origin of a task about negative temperatures, but invites students to explore temperature fluctuations in their own region, the mathematics of integers becomes anchored in a comparative perspective. Students learn that negative numbers appear in certain contexts (a Canadian winter) while also recognizing that these ideas can be extended to other domains, whether altitude below sea level, financial debts, or even daily temperature ranges in mountainous or southern areas of Brazil.

Reflections on bridging also highlight how sociocultural diversity can be integrated into lessons about geometry, rational numbers, and probability. A quilt pattern might spark a broader discussion about cultural patchwork traditions across continents, or a chance to compare how geometry emerges in the symmetrical designs of local crafts. A religious reference to Hanukkah might prompt an exploration of festivals of light or end-of-year celebrations in various cultures, revealing the essential math behind event planning, probability of random draws in holiday activities, or the distribution of resources in local feasts. The question is whether teachers, guided by institutional structures and personal beliefs, feel empowered to expand upon or adjust these references.

The possibility of bridging worlds also underscores the complexities that remain open, as noted at the end of the text. Adapting or replacing foreign references can be time-consuming. Teachers must weigh practical factors such as how thoroughly they are able to rework tasks, how supportive school leadership is in encouraging these changes, and whether parents and administrators support the reinterpretation of the original textbook content. Some may prefer the direct adoption of a foreign resource because it is believed to reflect global standards or superior quality. Others may fear that too much adaptation risks losing the perceived rigor or authenticity of the imported approach. These tensions lie at the intersection of the sociopolitical dimension and the cultural-linguistic dimension that the study addresses.

It becomes clear, then, that bridging is not a linear process. It involves reflection, negotiation, occasional disagreement, and compromise. As the teacher or curriculum designer moves between the Canadian contexts expressed in the materials and the Brazilian context where teaching occurs, multiple viewpoints must be considered. The more a teacher grapples with this bridging, the more it becomes evident that mathematics is not a simple universal domain, but a discipline that can either suppress or celebrate cultural differences, depending on how it is framed, contextualized, and discussed. That, in essence, is a recurring discovery across Categories 1, 2, and 3. Each category, despite focusing on different content - fractions and language, sports and festivities, or games - reveals that the differences are not minor details but are embedded in the entire structure of how mathematics is introduced and used.

By bringing these reflections together in this concluding section of the chapter, we see how they build toward the central objective of this thesis. In many ways, the bridging described here mirrors the process that the research itself undergoes: analyzing a foreign textbook, observing how it is organized and what cultural assumptions it carries, and subsequently exploring the interplay between that material and the local classroom realities. This bridging logic also illuminates how teacher training could benefit from preparing educators to navigate these cultural references. Teachers might be encouraged to study background information about Canadian traditions if they plan to use the textbook as is, or to collect parallel local traditions if they wish to adapt the tasks. In either case, teacher education programs play a role in enabling educators to become skillful mediators. By fostering an understanding of the necessity for bridging, these programs address the guiding question from a professional development standpoint, preparing teachers to interpret textbook material in nuanced ways.

In concluding these reflections, it is worth noting that bridging worlds does not imply a complete fusion of foreign content and local context. Sometimes, an educator might decide that a reference is too removed from students' reality to be useful, or that certain tasks are too richly intertwined with the Canadian environment to be worth adapting. Nevertheless, the bridging perspective can illuminate the broader reason behind such decisions. Rather than simply dismissing the foreign context, the teacher acknowledges it, weighs its potential, and either modifies it or sets it aside. This reflective act upholds the central principles of cultural respect, contextual awareness, and inclusive mathematics teaching that the research aims to highlight. As a result, the bridging approach enriches the local classroom with new insights while maintaining sensitivity to the cultural identity of the students and the local community.

Thus, each dimension explored - whether the representation of negative numbers, the geometry of totem poles, the probability embedded in a Blackfoot game, or the fraction operations illustrated by a Canadian winter scenario - contributes to a more comprehensive vision of mathematics as culturally and linguistically situated. As teachers integrate or rework these resources, they move closer to answering the guiding question, fulfilling the general objective, and addressing the specific objectives of the study. The synergy among Category 1, Category 2, and Category 3 becomes clear, as each category contributes a piece of the mosaic that underscores the central claim of this thesis: foreign textbooks carry cultural traces that do not vanish when translated or adapted, prompting users, teachers, students, curriculum planners - to become agents of recontextualization.

In resume, the bridging that emerges here is as much a methodological phenomenon as it is a pedagogical one, echoing the dual focus on sociocultural understanding and local adaptation. The presence of diverse references, from sports to festivities, from indigenous games to fraction models, compels educators to see mathematics as part of a wider tapestry of human activity, inviting them to engage with the complexities of implementing these resources in multilingual Brazilian contexts.

As a result, mathematics instruction becomes a site where the global and the local intersect, forming a learning environment that aims to be not only rigorous but also culturally affirming and contextually relevant. The remaining chapters of this dissertation will delve further into how these insights feed into the final considerations, including the broader implications for policy, teacher training, and curriculum design, thus continuing to address the thesis's objectives and main inquiry about the potential of Canadian textbooks in a distinct cultural and linguistic setting.

7 FINAL CONSIDERATIONS

The reflections presented here arise from a trajectory that began with a personal concern about how mathematics textbooks developed in one sociocultural context might resonate with students and teachers in another. Early classroom experiences showed that certain examples – often involving cold climates, sports played on ice, or cultural references from other regions – do not always connect with the local realities of multilingual Brazilian students. Inspired by the lyrics of “Duas Cidades,” this work offers a perspective to understand how teachers, students, and administrators can navigate the distance between a foreign textbook’s original setting and the everyday experiences of those who learn from it. This distance is not limited solely to geography or language, nor is it simply a matter of transferring individual concepts from one country to another, as curriculum choices are intertwined with broader social and political dimensions. Instead, it involves a continuous process of bridging, in which mathematics is presented as knowledge shaped by local practices, climates, languages, and beliefs.

In the early chapters of this thesis, the aim was to explore what cultural aspects of the Canadian context are embedded and made explicit in mathematics textbooks and how these elements might be interpreted in the context of multilingual schools in Brazil. Under this guiding question, the general objective became to understand and examine how the sociocultural differences in foreign textbooks – particularly those from Canada – can influence Brazilian educational settings. The specific objectives included analyzing sociocultural differences, investigating cultural and linguistic factors, and discussing the sociopolitical implications related to the use of these materials. The analysis revealed that the textbooks include several references to local festivals, sports, artistic traditions, or spiritual rites that are more familiar in their country of origin but may appear remote or puzzling for students who have not experienced, for example, very low temperatures or ice-based sports.

A key element introduced in this work is the concept of bridging. Bridging refers to a practical approach where educators connect cultural references found in foreign textbooks with the everyday experiences of local students. This process involves thoughtful reflection and an iterative practice where teachers decide which elements to keep, adjust, or replace in order to better align the content with the students’ cultural context.

Throughout the research, mathematics appears as a discipline linked to culture, language, and social structures. For instance, fraction bars might illustrate measurements in recipes from a festive meal, while local Brazilian realities may require that tasks be recontextualized to reflect dishes typical of regional celebrations. Similarly, negative integers could be demonstrated by depicting significant drops in temperature, a common occurrence in many Canadian towns but less so in many parts of Brazil. In this context, bridging occurs when a teacher actively identifies parallels in the local daily life, preserving the mathematical content while adjusting the cultural frame to be more recognizable for the students. This process calls for careful planning and reflection, as well as support from educational policies and school leadership.

As the analysis moved through various categories, it became evident that sports such as hockey or baseball are elements associated with Canadian identity but may not be as relevant in communities where soccer, capoeira, or other regional practices prevail. Instead of dismissing these foreign references, a teacher who is aware of the local context can demonstrate that the mathematical ideas behind calculating statistics in hockey can be applied, for example, to counting soccer goals or analyzing data from local dance competitions. A similar approach applies to artistic or architectural references; symbols like totems, which are used to discuss geometric patterns in Canadian textbooks, may be compared with local indigenous artifacts or decorative elements found in regional festivals.

Regarding religious or cultural festivities, the textbooks often mention winter events such as building ice castles or present tasks involving data from events like powwows or Hanukkah. At the same time, many Brazilian students might relate more easily to experiences such as June Festivals or Carnival, which could similarly illustrate concepts like proportions, arithmetic progressions, or geometric transformations. This situation again calls for bridging, where teachers design or select comparable tasks that incorporate local cultural expressions, addressing the tension between distant references and elements that are familiar to the learners.

The findings address the specific objectives by showing, first, how the sociocultural differences in Canadian textbooks can influence the interpretation of tasks in Brazilian classrooms, particularly when some references appear distant; second, by indicating that teachers may adjust or clarify these cultural references while honoring the multilingual context and local knowledge; and third, by noting that decisions concerning the use or modification of such materials are linked to broader discussions about educational policies and curriculum practices.

This dissertation also echoes the themes found in “Duas Cidades,” where the imagery of moving through distinct neighborhoods suggests what teachers do when they adjust a discussion about cold temperatures to a tropical setting or relate a winter festival to a local holiday. Such bridging shows that mathematics is not presented in isolation but emerges in various cultural, climatic, and historical forms. The teacher listens to the foreign rhythm, adapts it, and then merges it with local elements.

In response to the question of how cultural aspects from the Canadian context might be reinterpreted in multilingual Brazilian settings, the thesis demonstrates that the process depends on the willingness and knowledge of teachers, as well as the institutional flexibility available to make adjustments. Bridging does not occur automatically through a simple translation of terms; rather, it requires that educators compare elements such as climates, sports, festivals, or indigenous traditions, highlighting similarities and differences.

By addressing these reflections, the work explores how the cultural differences in foreign textbooks shape educational practices in Brazil while also discussing aspects related to linguistic adaptation and sociopolitical considerations. The outcomes suggest that bridging can be more successful when supported by teacher education programs that prepare educators to interpret foreign references and adjust content for a diverse classroom. In settings where such preparation is limited, the bridging process might be uneven, resulting in some teachers reproducing foreign scenarios with minimal contextual information.

Returning to the metaphor of two cities, the lyrics suggest that moving from one environment to another involves both challenges and opportunities. This dissertation shows that bridging these environments involves weaving together elements from cold landscapes and tropical heat, from ice rinks and samba rhythms, from freezing winters and Carnival nights. In this way, the teacher acts as a traveler between these worlds, listening to the stories of each, deciding what to retain and what to modify. Though tension may persist, this interaction can lead to new forms of understanding, transforming mathematics from a series of disconnected tasks into a discipline that is integrated with the life of the students.

In essence, the central thesis is that the potential and challenges of using Canadian textbooks in Brazilian multilingual classrooms depend on the teacher’s ability to interpret, adjust, or transform foreign cultural references into shared learning experiences that connect with local realities. The presence of these foreign elements can prompt curiosity about unfamiliar sports, festivals, or indigenous cultures, expanding perspectives, or they can create a sense of distance if presented without cultural mediation. The bridging approach emphasizes that mathematics can be taught as a universal language only when it is also recognized as inherently local, influenced by everyday signs and diverse communities. Each teacher, along with their students, determines how much of the foreign context to incorporate, how much to translate, and how much to reshape using local language and examples. For some classes, the foreign reference may broaden horizons; for others, rich local examples may fulfill the same mathematical objectives. In both cases, this strategy demonstrates the interconnection between culture, language, and mathematics.

The answers provided here, which address how to interpret Canadian textbooks in Brazilian contexts, highlight the coexistence of two distinct environments that, when combined through bridging, form a comprehensive whole. Each environment retains its identity, yet the teacher's process of bridging intertwines elements from both to reflect the complexities of each setting. This reflection shows not only the challenges of using a foreign textbook but also the promise of mathematics education that embraces the interaction of multiple cultural worlds.

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