On Switched Regulator Design of Uncertain Nonlinear Systems Using Takagi–Sugeno Fuzzy Models

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***Abstract*—This paper is concerned with the design of state-feedback switched controllers for a class of uncertain nonlinear plants described by Takagi–Sugeno (T–S) fuzzy models. The proposed methodology eliminates the need to find the membership function expressions to implement the control law, which is relevant in cases where the membership function depends on uncertain parameters. The design of the switched controllers is based on a minimum-type Lyapunov function and the minimization of the time derivative of this Lyapunov function. The conditions of the new stability criterion are represented by a kind of bilinear matrix inequalities (BMIs) that has been solved by the path-following method. A numerical example and the nonlinear control design of a magnetic levitator with uncertainties illustrate the procedure.**

***Index Terms*—Bilinear matrix inequalities (BMIs), control of uncertain nonlinear systems, linear matrix inequalities (LMIs), switched control, Takagi–Sugeno (T–S) fuzzy model.**

I. Introduction

Currently, there are efficient control system design techniques for nonlinear systems described by Takagi–Sugeno (T–S) fuzzy models, based on parallel distributed compensation, as can be seen in the considerable number of published papers in this well-established research field (see, for instance, [1]–[15]). In the last decade, many researchers have studied new relaxation methods for stability analysis and controller synthesis, based on linear matrix inequalities (LMIs) or bilinear matrix inequalities (BMIs), of T–S fuzzy models [3], [6]–[8], [10], [12], [16], [17].

Nowadays, according to the authors' knowledge, the conditions stated in [8], [12], and [17] offer three of the most important contributions of relaxation stability conditions for continuous T–S fuzzy models. In [17], a switching fuzzy controller, based on a minimum-type piecewise Lyapunov function, is proposed, and conditions more relaxed than the known conditions are presented. Some of these conditions are represented by a kind of BMIs, and the authors suggest the path-following method as an adequate solution for this problem [18].

The study of switched systems grown a lot in the past decade, initiating mainly with the control of linear systems [19]–[30], and then, it has also been broadly used in the control of nonlinear systems described by T–S fuzzy models, as can be seen in [4], [5], [13], [17], and [31]–[38].

This paper proposes a new method to design switched control for a class of uncertain nonlinear systems described by T–S fuzzy models. The proposed controller is defined by a switching law whose design consists of two stages. The first stage is based on [17] and [26] and selects a positive definite matrix using a minimum-type piecewise Lyapunov function, and the second stage chooses the controller gains that minimize the time derivative of the Lyapunov function. For the development of these control design methods, new stability conditions were established, and some of these conditions are a kind of BMI. These BMIs, which contain some bilinear terms as the product of a full matrix and a scalar, have been solved by the path-following method [18]. Furthermore, the proposed switched controller can also operate even with an uncertain reference control signal.

Due to the fact that the control law is switched, the main advantage of this new procedure is its practical application because it eliminates the need to find the explicit expressions of the membership functions, which can often have long and/or complex expressions, or may not be known, for instance, due to the plant uncertainties. Additionally, with the proposed methodology, the closed-loop systems usually present a settling time smaller than those obtained with fuzzy controllers without using switching, which are widely studied in the literature. Moreover, other constraints, for instance on the plant's input and output, can also be added to the control design specifications.

The proposed method was applied in the control design of a benchmark nonlinear T–S fuzzy system [17] that is used to compare relaxation stabilization criterions. Furthermore, simulation results of the application of the procedure in the control of a magnetic levitator with uncertainties is presented. The computational implementations were carried out using the modeling language YALMIP [39] with the solver SeDuMi [40].

The paper is organized as follows. Section II presents the preliminary results on T–S fuzzy model and switching fuzzy controller. Section III offers a new switching control method and new stability conditions for a class of uncertain nonlinear systems described by T–S fuzzy models. Examples illustrate the performance of the new proposed methods in Section IV. Finally, Section V draws our conclusions.

For convenience, in some places, the following notation is used:

$$\mathbb{K}_r = \{1, 2, \ldots, r\},\ r \in \mathbb{N}, \quad x(t) = x$$

$$\alpha_i(x(t)) = \alpha_i,\ V(x(t)) = V,\ \|x\|_2 = \sqrt{x^T x}$$

$$(A, B, C, K)(\alpha) = \sum_{i=1}^{r} \alpha_i (A_i, B_i, C_i, K_i)$$

$$\text{with } \alpha_i \geq 0,\ \sum_{i=1}^{r} \alpha_i = 1 \text{ and } \alpha^T = [\alpha_1, \alpha_2, \ldots, \alpha_r]. \quad (1)$$

II. Takagi–Sugeno Fuzzy Systems and Switching Fuzzy Controller

Consider the T–S fuzzy model as described in [41]:

$$\underline{\text{Rule}}\ i : \text{IF } z_1(t) \text{ is } M_1^i \text{ and } \ldots \text{ and } z_p(t) \text{ is } M_p^i$$

$$\text{THEN} \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \quad (2)$$

where $i \in \mathbb{K}_r$, M_j^i, $j \in \mathbb{K}_p$ is the fuzzy set j of rule i, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^q$ is the output vector, $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{q\times n}$, and $z_1(t), \ldots, z_p(t)$ are premise variables, which, in this paper, are the state variables.

From [1], $\dot{x}(t)$ given in (2) can be written as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} \alpha_i(x(t))(A_i x(t) + B_i u(t)) = A(\alpha)x(t) + B(\alpha)u(t) \quad (3)$$

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where $\alpha_i(x(t))$ is the normalized weight of each local model system $A_i x(t) + B_i u(t)$, $i \in \mathbb{K}_r$ that satisfies (1).

Assuming that the state vector x is available, from the T–S fuzzy model (2), the control input of fuzzy regulators via parallel distributed compensation has the following structure [1]:

$$\underline{\text{Rule}}\ j: \text{ IF } z_1(t) \text{ is } M_1^j \text{ and} \ldots \text{and } z_p(t) \text{ is } M_p^j$$
$$\text{THEN } u(t) = -K_j x(t). \tag{4}$$

Similar to (3), from (4), one can consider the control law [1]

$$u(t) = u_\alpha = -\sum_{j=1}^{r} \alpha_j(x(t)) K_j x(t) = -K(\alpha)x(t). \tag{5}$$

From (1), (3), and (5), one obtains

$$\begin{aligned}\dot{x}(t) &= (A(\alpha) - B(\alpha)K(\alpha))x(t)\\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} \alpha_i(x(t))\alpha_j(x(t))\Big[A_i - B_i K_j\Big]x(t).\end{aligned} \tag{6}$$

Now, a switching fuzzy controller based on a minimum-type piecewise Lyapunov function proposed in [17] is presented. Let the piecewise Lyapunov candidate function be as follows:

$$V(x) = \min_{k\in\mathbb{K}_N} (x^T(t) P_k x(t)) \tag{7}$$

where $P_k,\ k \in \mathbb{K}_N$ are symmetric positive definite matrices.

Definition 1: Consider the index set $\Omega_H(t)$ defined in the following:

$$\begin{aligned}\Omega_H(t) &= \arg \min_{i\in\mathbb{K}_N} x^T(t) H_i x(t)\\ &= \left\{ j \in \mathbb{K}_N : x^T(t) H_j x(t) = \min_{i\in\mathbb{K}_N} x^T(t) H_i x(t)\right\}\end{aligned}$$

where $H_i \in \mathbb{R}^{n\times n},\ i \in \mathbb{K}_N$, and $x(t) \in \mathbb{R}^n$. The smallest index $j \in \Omega_H(t)$ will be denoted by

$$\arg \min_{i\in\mathbb{K}_N}{}^{*}\{x(t)^T H_i x(t)\} = \min_{j\in\Omega_H(t)} j.$$

In [17], the switching index $\min_{j\in\Omega_{P(t)}} j$ was used to select, at each instant of time, a positive definite matrix $P_j,\ j \in \mathbb{K}_N$, such that the piecewise Lyapunov function (7) is equal to $x(t)^T P_j x(t)$. Thus, considering (7) and from Definition 1, the switching fuzzy controller proposed in [17] can be written as follows:

$$u(t) = u_\sigma(t) = -\sum_{i=1}^{r} \alpha_i(x(t)) K_{i\sigma} x(t)$$
$$\text{where} \qquad \sigma = \arg \min_{k\in\mathbb{K}_N}{}^{*}\{x(t)^T P_k x(t)\}. \tag{8}$$

Therefore, from (1), the controlled system (3) and (8) is given by

$$\begin{aligned}\dot{x}(t) &= A(\alpha)x(t) + B(\alpha)u_\sigma(t)\\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} \alpha_i\alpha_j \Big(A_i - B_i K_{j\sigma}\Big)x(t).\end{aligned} \tag{9}$$

In this context, considering the piecewise Lyapunov function (7) and the switching fuzzy controller (8), in [17], a theorem was proposed which established a relaxed stabilization criterion. However, some of the proposed conditions are BMIs which contain some bilinear terms as the product of a full matrix and a scalar. Fortunately, this kind of problem has been solved by the path-following method [18].

III. Main Result

A. Switched Regulator Design of Uncertain Nonlinear Systems Using Takagi–Sugeno Fuzzy Models

In this section, a new design method of switched controllers for the T–S fuzzy systems described in (3) is presented. To find the feedback gains the proposed controller uses two stages. The first stage is based on the procedures presented in [17] and [26], and chooses an index $\sigma = \arg\min_{k\in\mathbb{K}_N}{}^{*}\{x^T P_k x\}$, where $P_k,\ k \in \mathbb{K}_N$ are symmetric positive definite matrix. Note that the Lyapunov function given in (7) is equal to $V(x) = x^T P_\sigma x$. The basic idea of the second stage is the minimization of the time derivative of the Lyapunov function (7), through the selection of the controller gain, which belongs to the set of gains $\{K_{j\sigma},\ j \in \mathbb{K}_r\}$, where σ was obtained in the first stage. This stage uses auxiliary symmetric matrices $Q_{jk}, j \in \mathbb{K}_r, k \in \mathbb{K}_N$, the index σ, and selects an index $\nu = \arg\min_{j\in\mathbb{K}_r}{}^{*}\{x^T Q_{j\sigma} x\}$. Therefore, considering the indexes σ and ν cited above and Definition 1, the switched controller is defined as follows:

$$\begin{aligned}&u(t) = u_{\nu\sigma}(t) = -K_{\nu\sigma} x(t)\\ &\sigma = \arg \min_{k\in\mathbb{K}_N}{}^{*}\{x^T P_k x\}, \quad \nu = \arg \min_{j\in\mathbb{K}_r}{}^{*}\{x^T Q_{j\sigma} x\}.\end{aligned} \tag{10}$$

Therefore, from (1), the controlled systems (3) and (10) are given by

$$\begin{aligned}\dot{x}(t) &= A(\alpha)x(t) + B(\alpha)u_{\nu\sigma}(t)\\ &= \sum_{i=1}^{r} \alpha_i \Big(A_i - B_i K_{\nu\sigma}\Big)x(t).\end{aligned} \tag{11}$$

In this context, considering the piecewise Lyapunov function (7) and the switching controller (10), the following theorem is proposed.

Theorem 1: Assume that there exist symmetric positive definite matrices $X_k \in \mathbb{R}^{n\times n}$, symmetric matrices $Z_{ik},\ R_{ik}, Y_{ik} \in \mathbb{R}^{n\times n}$, matrices $M_{ik} \in \mathbb{R}^{m\times n}$, and scalars $\lambda_{isk} > 0$, $\beta < 0$ such that, for all $i, j \in \mathbb{K}_r$ and $k, s \in \mathbb{K}_N$:

$$-B_i M_{jk} - M_{jk}^T B_i^T - Z_{ik} - R_{jk} \prec 0 \tag{12}$$

$$Y_{ik} \prec 0 \tag{13}$$

$$\begin{bmatrix} O_{ik} & * & * & \cdots & * \\ \lambda_{i1k}X_k & -\lambda_{i1k}X_1 & 0 & \ldots & 0 \\ \lambda_{i2k}X_k & 0 & -\lambda_{i2k}X_2 & \ldots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \lambda_{iNk}X_k & 0 & \ldots & 0 & -\lambda_{iNk}X_N \end{bmatrix} \prec 0 \tag{14}$$

where $O_{ik} = X_k A_i^T + A_i X_k + Z_{ik} + R_{ik} - \beta X_k - \sum_{s=1}^{N} \lambda_{isk} X_k - Y_{ik}$.

Then, the switched control law (10), where $Q_{jk} = X_k^{-1} R_{jk} X_k^{-1}$, $P_k = X_k^{-1}$ and controller gains are given by $K_{jk} = M_{jk} X_k^{-1}$, $j \in \mathbb{K}_r$, $k \in \mathbb{K}_N$, makes the equilibrium point $x = 0$ of the system (3) asymptotically stable in the large.

Proof: Consider a piecewise Lyapunov candidate function given in (7). Suppose that $V(x(t)) = \min_{i\in\mathbb{K}_N}\{x(t)^T P_i x(t)\} = x(t)^T P_\sigma x(t)$, where σ is selected as described in (10) and Definition 1. From the analysis presented in [17], considering that $V(x(t^+)) \le V_\sigma(x(t^+))$, then $\dot{V}(x(t)) \le \dot{V}_\sigma(x(t))$. This fact follows from

$$\begin{aligned}\dot{V}(x(t)) &= \lim_{t^+\to t} \frac{V(x(t^+)) - V(x(t))}{t^+ - t}\\ &= \lim_{t^+\to t} \frac{V(x(t^+)) - V_\sigma(x(t))}{t^+ - t}\end{aligned}$$

and on the other hand

$$\dot{V}_\sigma(x(t)) = \lim_{t^+ \to t} \frac{V_\sigma(x(t^+)) - V_\sigma(x(t))}{t^+ - t}.$$

Consequently, $V(x(t^+)) \le V_\sigma(x(t^+))$ implies that

$$\begin{aligned}\dot{V}(x(t)) &= \lim_{t^+ \to t} \frac{V(x(t^+)) - V_\sigma(x(t))}{t^+ - t}\\ &\le \lim_{t^+ \to t} \frac{V_\sigma(x(t^+)) - V_\sigma(x(t))}{t^+ - t} = \dot{V}_\sigma(x(t)).\end{aligned}$$

Thus, recalling that $\beta < 0$, then from the analysis above and from (11), observe that

$$\begin{aligned}&\dot{V}(x) - \beta V(x)\\ &\le \dot{V}_\sigma(x) - \beta V_\sigma(x) = \dot{x}^T P_\sigma x + x^T P_\sigma \dot{x} - \beta x^T P_\sigma x\\ &\le \sum_{i=1}^r \alpha_i x^T (P_\sigma A_i + A_i^T P_\sigma - P_\sigma B_i K_{\nu\sigma} - K_{\nu\sigma}^T B_i^T P_\sigma - \beta P_\sigma)x.\end{aligned} \tag{15}$$

Considering the relaxing parameters $\lambda_{is\sigma} > 0,\ i \in \mathbb{K}_r,\ s, \sigma \in \mathbb{K}_N$, note that from (8)

$$\sum_{s=1}^N \lambda_{is\sigma} x^T (P_s - P_\sigma)x \ge 0.$$

Now, suppose that there exist symmetric matrices $\bar{Z}_{ik},\ Q_{jk} \in \mathbb{R}^{n\times n}$ such that

$$-\left(P_k B_i K_{jk} + K_{jk}^T B_i^T P_k\right) \prec \bar{Z}_{ik} + Q_{jk},\ \forall\, i,j \in \mathbb{K}_r,\ k \in \mathbb{K}_N. \tag{16}$$

Therefore, from (15) and (16) it follows that for $x \neq 0$

$$\begin{aligned}&\dot{V}(x) - \beta V(x)\\ &\le \sum_{i=1}^r \alpha_i x^T \Bigg[P_\sigma A_i + A_i^T P_\sigma - P_\sigma B_i K_{\nu\sigma} - K_{\nu\sigma}^T B_i^T P_\sigma\\ &\quad - \beta P_\sigma + \sum_{s=1}^N \lambda_{is\sigma}(P_s - P_\sigma)\Bigg]x\\ &< \sum_{i=1}^r \alpha_i x^T \Bigg[P_\sigma A_i + A_i^T P_\sigma + \bar{Z}_{i\sigma} + Q_{\nu\sigma}\\ &\quad - \beta P_\sigma + \sum_{s=1}^N \lambda_{is\sigma}(P_s - P_\sigma)\Bigg]x\\ &= \sum_{i=1}^r \alpha_i x^T \Bigg[P_\sigma A_i + A_i^T P_\sigma + \bar{Z}_{i\sigma}\\ &\quad - \beta P_\sigma + \sum_{s=1}^N \lambda_{is\sigma}(P_s - P_\sigma)\Bigg]x + x^T Q_{\nu\sigma} x.\end{aligned} \tag{17}$$

From (1) and (10), note that $Q_{\nu\sigma} = \min_{i\in\mathbb{K}_r}(x^T Q_{i\sigma} x) \le \sum_{i=1}^r \alpha_i x^T Q_{i\sigma} x$. Then, from (17)

$$\begin{aligned}\dot{V}(x) - \beta V(x) &\le \sum_{i=1}^r \alpha_i x^T \Bigg[P_\sigma A_i + A_i^T P_\sigma\\ &\quad + \bar{Z}_{i\sigma} + Q_{i\sigma} - \beta P_\sigma + \sum_{s=1}^N \lambda_{is\sigma}(P_s - P_\sigma)\Bigg]x.\end{aligned} \tag{18}$$

Now, suppose that there exist symmetric matrices $W_{ik},\ i \in \mathbb{K}_r,\ k \in \mathbb{K}_N$, such that

$$P_k A_i + A_i^T P_k + \bar{Z}_{ik} + Q_{ik} - \beta P_k + \sum_{s=1}^N \lambda_{isk}(P_s - P_k) - W_{ik} \preceq 0. \tag{19}$$

Therefore, from (18) and (19), assume that

$$\dot{V}(x) - \beta V(x) < \sum_{i=1}^r \alpha_i x^T (W_{i\sigma})x < 0, \quad x \neq 0. \tag{20}$$

Remembering that $\alpha_i \ge 0,\ i \in \mathbb{K}_r$ and $\sum_{i=1}^r \alpha_i = 1$, from (20), $\dot{V}(x) - \beta V(x) < 0$ (for $x \neq 0$) if for all $i \in \mathbb{K}_r$ and $k \in \mathbb{K}_r$, the following condition holds:

$$W_{ik} \prec 0. \tag{21}$$

Now, define $X_k = P_k^{-1}$, $Z_{ik} = X_k \bar{Z}_{ik} X_k$, $R_{ik} = X_k Q_{ik} X_k$, $M_{jk} = K_{jk} X_k$, and $Y_{ik} = X_k W_{ik} X_k$. Pre- and postmultiplying (16), (21), and (19) by X_k, it follows (12), (13), and

$$\begin{aligned}&A_i X_k + X_k A_i^T + Z_{ik} + R_{ik} - \beta X_k\\ &\quad + \sum_{s=1}^N \lambda_{isk}(X_k P_s X_k - X_k) - Y_{ik} \preceq 0.\end{aligned} \tag{22}$$

Applying the Schur complement in (22), this condition is equivalent to (14), and the proof is concluded. ■

Remark 1: Nowadays, to the best knowledge of the authors, there are no solvers that can find solutions for all kinds of BMIs. Note that the conditions of Theorem 1 are given by two LMIs [see (12) and (13)] and one BMI (14). However, this kind of problem has been solved by the path-following method [18], whose steps are presented with details in [17, App.]. In the examples of this paper, the path-following method was implemented using the modeling language YALMIP [18] with the solver SeDuMi [17].

Remark 2: The conditions of Theorem 1 can be rewritten when $N = 1$, which is described as follows:

$$-B_i M_{j1} - M_{j1}^T B_i^T - Z_{i1} - R_{j1} \prec 0, \quad Y_{i1} \prec 0$$

$$\begin{bmatrix} O_{i1} & * \\ \lambda_{i11} X_1 & -\lambda_{i11} X_1 \end{bmatrix} \prec 0, \qquad i,j \in \mathbb{K}_r \tag{23}$$

where $O_{i1} = X_1 A_i^T + A_i X_1 + Z_{i1} + R_{i1} - \beta X_1 - \lambda_{i11} X_1 - Y_{i1}$.

Now, applying the Schur complement in the third inequality of (23), one obtains the simplified conditions given by

$$\begin{aligned}&-B_i M_{j1} - M_{j1}^T B_i^T - Z_{i1} - R_{j1} \prec 0, \quad Y_{i1} \prec 0\\ &X_1 A_i^T + A_i X_1 + Z_{i1} + R_{i1} - \beta X_1 - Y_{i1} \prec 0,\ i,j \in \mathbb{K}_r.\end{aligned} \tag{24}$$

B. Switched Controller With Uncertainty in the Reference Control Signal

In this case, it is assumed that the plant given by $\dot{\bar{x}} = f(\bar{x}, \bar{u})$ has an equilibrium point $\bar{x} = x_0$ and that the respective control input is $\bar{u} = u_0$, such that $f(x_0, u_0) = 0$. Suppose that x_0 is known, $u_0 \in \mathbb{R}$ is uncertain because it depends on the plant uncertainties, but $0 < u_0 \in [u_{0_{\min}},\ u_{0_{\max}}]$, where $u_{0_{\min}}$ and $u_{0_{\max}}$ are constants that are known, and the plant can be described by the T–S fuzzy system (1)–(3)

$$\dot{x}(t) = A(\alpha)x(t) + B(\alpha)u \tag{25}$$

where
$x(t) = \bar{x}(t) - x_0$, $\bar{x}(t)$ is the state vector of the plant;
$u(t) = \bar{u}(t) - u_0$, $\bar{u}$ is the control input of the plant.
Now, consider that $B(\alpha)$ can be written as follows:

$$B(\alpha) = Bg(x(t)) \tag{26}$$

where B is a known constant matrix, and $g(x(t)) > 0$, for all x, is an uncertain bounded nonlinear function. Thus, the system (25) can be rewritten as follows:

$$\dot{x}(t) = A(\alpha)x(t) + B(\alpha)u = A(\alpha)x(t) + Bg(x(t))u. \tag{27}$$

Assume that the conditions (12)–(14) of Theorem 1 are feasible. Then, one can obtain the gains $K_{jk} = M_{jk}X_k^{-1}$, the matrices $P_k = X_k^{-1}$, and the matrices Q_{jk}, $j \in \mathbb{K}_r$, $k \in \mathbb{K}_N$. Now, given a constant $\xi > 0$, define the control law

$$u(t) = u_{(\nu,\sigma,\xi)} = \bar{u}_{(\nu,\sigma,\xi)}(t) - u_0$$

$$\text{with } \bar{u}_{(\nu,\sigma,\xi)}(t) = -K_{\nu\sigma}x + \gamma_\xi \tag{28}$$

where

$$K_{\nu\sigma} \in \{K_{11}, K_{21}, \ldots, K_{r1}, \ldots, K_{rN}\}$$

$$\sigma = \arg \min_{k\in\mathbb{K}_N}{}^{*}\{x^T P_k x\}$$

$$\nu = \arg \min_{j\in\mathbb{K}_r}{}^{*}\{x^T Q_{j\sigma} x\}$$

$$\gamma_\xi = \begin{cases} u_{0_{\max}}, & \text{if } x^T P_\sigma B \le 0 \\ u_{0_{\min}}, & \text{if } x^T P_\sigma B > 0. \end{cases} \tag{29}$$

Note that the control law (28) and (29) is only applicable to single input systems. Based on the considerations above the following theorem is proposed.

Theorem 2: Suppose that the conditions from Theorem 1 hold, from the system (25) with the control law (10), and obtain $K_{jk} = M_{jk}X_k^{-1}$, $P_k = X_k^{-1}$, and Q_{jk}, $j \in \mathbb{K}_r$, $k \in \mathbb{K}_N$. Then, the switched control law (28) and (29) makes the equilibrium point $x = 0$ of the system (25) asymptotically stable in the large.

Proof: Consider a piecewise Lyapunov candidate function $V = \min_{i\in\mathbb{K}_r}\{x^T P_i x\} = x^T P_\sigma x$. Define $\dot{V}_{u_{\nu\sigma}}$ and $\dot{V}_{u_{(\nu,\sigma,\xi)}}$ as the time derivatives of V for the system (25) and (26), with the control laws (10) and (28) and (29), respectively. Then

$$\begin{aligned}
\dot{V}_{u_{(\nu,\sigma,\xi)}} &= 2x^T P_\sigma \dot{x} = 2x^T P_\sigma [A(\alpha)x + B(\alpha)u_{(\nu,\sigma,\xi)}] \\
&= 2x^T P_\sigma [A(\alpha)x + B(\alpha)(\bar{u}_{(\nu,\sigma,\xi)}(t) - u_0)] \\
&= 2x^T P_\sigma [A(\alpha)x + B(\alpha)(-K_{\nu\sigma}x + \gamma_\xi - u_0)] \\
&= 2x^T P_\sigma A(\alpha)x - 2x^T P_\sigma B(\alpha)K_{\nu\sigma}x \\
&\quad + 2x^T P_\sigma B(\alpha)(\gamma_\xi - u_0) \\
&= 2\sum_{i=1}^{r} \alpha_i x^T (P_\sigma A_i - P_\sigma B_i K_{\nu\sigma})x \\
&\quad + 2g(x)x^T P_\sigma B(\gamma_\xi - u_0) \\
&= \dot{V}_{u_{\nu\sigma}} + 2g(x)x^T P_\sigma B(\gamma_\xi - u_0).
\end{aligned} \tag{30}$$

Now, note that from (29), $g(x)x^T P_\sigma B(\gamma_\xi - u_0) \le 0$. Thus, from (30), $\dot{V}_{u_{(\nu,\sigma,\xi)}} \le \dot{V}_{u_{\nu\sigma}} < 0$ for $x \ne 0$, since from Theorem 1, the equilibrium point $x = 0$ of the system (25) with the control law (10) is globally asymptotically stable, because $\dot{V}_{u_{\nu\sigma}} < 0$ for $x \ne 0$, and therefore, the proof is concluded. ■

Remark 3: Observe that the control input $\bar{u} = -K_{\nu\sigma} + \gamma_\xi$ defined in (28) can be discontinuous due to the term γ_ξ (29). Note also that in some practical implementations, this phenomenon of discontinuity may be inconvenient or cannot be allowed. Thus, as in [30], one can modify the function γ_ξ to make it continuous and ensure the uniform ultimate boundedness of the system and smoothness of the control input. Thus, for instance, consider a function γ_ξ as follows [30]:

$$\gamma_\xi = \begin{cases} u_{0_{\max}}, & \text{if } x^T P_\sigma B < -\xi \\ [(u_{0_{\min}} - u_{0_{\max}})x^T P_\sigma B \\ \quad +\xi(u_{0_{\max}} + u_{0_{\min}})]/2\xi, & \text{if } |x^T P_\sigma B| \le \xi \\ u_{0_{\min}}, & \text{if } x^T P_\sigma B > \xi. \end{cases} \tag{31}$$

IV. Numerical Examples

A. Example 1

The following example has been used in several papers and is considered as an index to compare the relaxation of the stabilization criterions for T–S fuzzy systems [8], [17].

Let the continuous T–S fuzzy be defined by the following rules:

$$\begin{aligned}
&\text{Rule } i: \text{ IF } x_1 \text{ is } M_i \\
&\qquad \text{Then } \dot{x}(t) = A_i x(t) + B_i u(t) \\
&\qquad\qquad x(t) = [x_1(t)\ \ x_2(t)]^T,\ i \in \mathbb{K}_3
\end{aligned} \tag{32}$$

where $[A_1|A_2|A_3|B_1|B_2|B_3] = \left[\begin{array}{cc|cc|cc|c|c|c} 1.59 & -7.29 & 0.02 & -4.64 & -a & -4.33 & 1 & 8 & -b+6 \\ 0.01 & 0 & 0.35 & 0.21 & 0 & 0.05 & 0 & 0 & -1 \end{array}\right]$.

Considering $a = 2$ and $b \in [0,\ 7]$, the maximum value of b in the discrete range $0 : 0.5 : 7$ such that a given stabilization condition holds is calculated, in order to compare it with other stabilization methods. From the conditions of Theorem 1, the maximum obtained value was $b = 6$. Thus, by fixing $a = 2$ and $b = 6$, from the inequalities (12)–(14) presented in the Theorem 1 for $N = 4$, by the path-following method [18] considering in the initial step $\beta_0 = 0.4429$ and $\lambda_{isk}(0)$, $i \in \mathbb{K}_r$, $s, k \in \mathbb{K}_N$ equal to random values between 0 and 1, a feasible solution was obtained for $\beta = -0.0071$. In this case, the controller gains, the symmetric positive definite matrices of the piecewise Lyapunov function (7), and the symmetric matrices Q_{jk} of this feasible solution were the following:

$$\begin{aligned}
&K_{11} = [3.1964\ \ 1.5996], \quad K_{21} = [3.1438\ \ 1.6354] \\
&K_{31} = [3.1957\ \ 1.5990], \quad K_{12} = [2.9822\ \ 1.4083] \\
&K_{22} = [2.9814\ \ 1.4087], \quad K_{32} = [2.9822\ \ 1.4072] \\
&K_{13} = [88.7641\ \ 73.1443], \quad K_{23} = [75.2306\ \ 63.7048] \\
&K_{33} = [75.2309\ \ 63.6697], \quad K_{14} = [1.9312\ \ -0.5863] \\
&K_{24} = [0.8267\ \ -0.0527], \quad K_{34} = [1.8368\ \ -0.5619]
\end{aligned} \tag{33}$$

$$\begin{aligned}
P_1 &= \begin{bmatrix} 169.1387 & 145.8907 \\ 145.8907 & 906.3213 \end{bmatrix} \\
P_2 &= \begin{bmatrix} 168.1260 & 141.6762 \\ 141.6762 & 900.0816 \end{bmatrix} \\
P_3 &= \begin{bmatrix} 171.0932 & 152.2472 \\ 152.2472 & 915.8180 \end{bmatrix} \\
P_4 &= \begin{bmatrix} 167.1593 & 131.3891 \\ 131.3891 & 892.2195 \end{bmatrix}
\end{aligned} \tag{34}$$

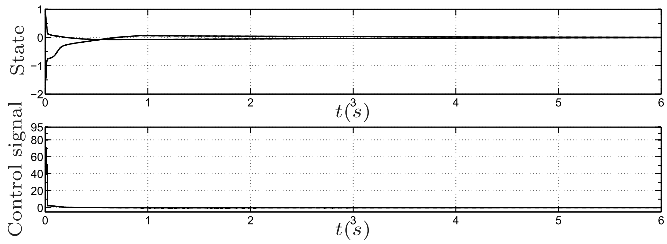

Fig. 1. State variables and control signal of the controlled system using the switched controller (10) (solid line) and the switching fuzzy controller (8) (dotted line), considering the initial condition $[-2 \quad 1]^T$.

$$Q_{11} = 10^3 \begin{bmatrix} 0.9411 & 3.1029 \\ 3.1029 & 3.0112 \end{bmatrix}, Q_{21} = 10^3 \begin{bmatrix} 2.9040 & 4.5544 \\ 4.5544 & 4.0824 \end{bmatrix}$$
$$Q_{31} = 10^3 \begin{bmatrix} 0.9444 & 3.0926 \\ 3.0926 & 3.0520 \end{bmatrix}, Q_{12} = 10^3 \begin{bmatrix} 0.8452 & 2.8836 \\ 2.8836 & 2.5406 \end{bmatrix}$$
$$Q_{22} = 10^3 \begin{bmatrix} 2.6570 & 4.1871 \\ 4.1871 & 3.4788 \end{bmatrix}, Q_{32} = 10^3 \begin{bmatrix} 0.8452 & 2.8835 \\ 2.8835 & 2.5453 \end{bmatrix}$$
$$Q_{13} = 10^5 \begin{bmatrix} 0.4798 & 1.0200 \\ 1.0200 & 1.3921 \end{bmatrix}, Q_{23} = 10^5 \begin{bmatrix} 0.7306 & 1.2216 \\ 1.2216 & 1.5526 \end{bmatrix}$$
$$Q_{33} = 10^5 \begin{bmatrix} 0.2294 & 0.7877 \\ 0.7877 & 1.1771 \end{bmatrix}, Q_{14} = 10^3 \begin{bmatrix} 0.6052 & 1.1802 \\ 1.1802 & 1.1775 \end{bmatrix}$$
$$Q_{24} = 10^3 \begin{bmatrix} 2.6153 & 1.6752 \\ 1.6752 & 0.2856 \end{bmatrix}, Q_{34} = \begin{bmatrix} 662.1756 & 967.9830 \\ 967.9830 & 988.6938 \end{bmatrix}. \tag{35}$$

For a simulation, the same conditions from the numerical example presented in [17] were considered, namely, the initial condition $x(0) = [-2 \;\; 1]^T$ and the membership functions

$$\alpha_1(t) = \frac{\cos(10x_1(t)) + 1}{4}, \quad \alpha_2(t) = \frac{\sin(10x_1(t)) + 1}{4}$$
$$\alpha_3(t) = \frac{\sin(10x_1(t)) + \cos(10x_1(t)) + 2}{4}.$$

The simulation results of the controlled systems (10), (32), (33)–(35), and (8), (32)–(34) are shown in Fig. 1.

The proposed methodology (see Theorem 1) presented a good result because it was feasible for the maximum value $b = 6$, which is the same value of the relaxation methods described in [6] and [7]. Note that, according to the comparative analysis described in [17], this value was only smaller than those presented in [8] ($b = 6.5$) and [17] ($b = 7.0$). However, it is worth remembering that the main goal of this paper is not to establish a new relaxed stabilization criterion but to propose a new switched control design method for uncertain nonlinear plants described by T–S fuzzy models that does not use the membership functions to implement the control law. Additionally, this new control design method can be mainly useful to control plants with uncertain parameters when the membership functions depend on these parameters and are unknown. For instance, the methods given in [6]–[8] and [17] cannot be directly used when the membership functions are unknown, because for their implementations, the membership functions are necessary.

B. Example 2

In this example, the proposed nonlinear control method is applied in a magnetic levitator, whose mathematical model [42, p. 24] is given by

$$m\ddot{y} = -k\dot{y} + mg - \frac{\lambda\mu i^2}{2(1+\mu y)^2} \tag{36}$$

where

$g = 9.8$ m/s^2 is the gravity acceleration;

$\lambda = 0.460$ H, $\mu = 2$ m^{-1} and $k = 0.001$ Ns/m are positive constants;

m is the mass of the ball that is an uncertain parameter;

i is the electric current, and y is the position of the ball.

Define the state variables $\bar{x}_1 = y$ and $\bar{x}_2 = \dot{y}$. Then, (36) can be written as follows [9]:

$$\dot{\bar{x}}_1 = \bar{x}_2, \qquad \dot{\bar{x}}_2 = g - \frac{k}{m}\bar{x}_2 - \frac{\lambda\mu i^2}{2m(1+\mu\bar{x}_1)^2}. \tag{37}$$

Consider that during the required operation, $[\bar{x}_1 \;\; \bar{x}_2]^T \in D_1$, where

$$D_1 = \{[\bar{x}_1 \;\; \bar{x}_2]^T \in \mathbb{R}^2 : 0 \le \bar{x}_1 \le 0.15\}. \tag{38}$$

The objective in this example is to design a controller that keeps the ball at a desired position $y = \bar{x}_1 = y_0$, after a transient response. Thus, the equilibrium point of the system (37) is $\bar{x}_e = [\bar{x}_{1_e} \quad \bar{x}_{2_e}]^T = [y_0 \quad 0]^T$.

From the second equation in (37), observe that in the equilibrium point, $\dot{\bar{x}}_2 = 0$ and $i = i_0$, where

$$i_0^2 = \frac{2mg}{\lambda\mu}(1+\mu y_0)^2. \tag{39}$$

Note that the equilibrium point is not at the origin $[\bar{x}_1 \quad \bar{x}_2]^T = [0 \quad 0]^T$. Thus, for the stability analysis, the following change of coordinates is necessary:

$$x_1 = \bar{x}_1 - y_0, \quad x_2 = \bar{x}_2, \quad u = i^2 - i_0^2, \qquad \text{i.e.,}$$
$$\bar{x}_1 = x_1 + y_0, \quad \bar{x}_2 = x_2, \quad i^2 = u + i_0^2.$$

Therefore, $\dot{x}_1 = \dot{\bar{x}}_1$, $\dot{x}_2 = \dot{\bar{x}}_2$ and, from (39), $i^2 = u + i_0^2 = u + \frac{2mg}{\lambda\mu}(1+\mu y_0)^2$.

Hence, the system (37) can be rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ f_{21}(z) & f_{22}(z) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ g_{21}(z) \end{bmatrix} u \tag{40}$$

where

$$f_{21}(z) = \frac{g\mu(\mu x_1 + 2\mu y_0 + 2)}{(1+\mu(x_1+y_0))^2}, \quad f_{22}(z) = -\frac{k}{m}$$
$$g_{21}(z) = \frac{-\lambda\mu}{2m(1+\mu(x_1+y_0))^2} \tag{41}$$

where $z = [x_1 \;\; x_2 \;\; y_0 \;\; m]^T \in \mathbb{R}^4$. Thus, to find the local models, the maximum and minimum values of the functions f_{21}, f_{22}, and g_{21} must be obtained. In this case, the methodology proposed in [9] will be used. Then, suppose that the desired position belongs to set $y_0 \in [0.05, \;\; 0.1]$, the mass $m \in [0.06, \quad 0.1]$, and the domain D_1 stated in (38), and consider y_0 and m as new variables for the specification of the domain D_2 of the nonlinear functions f_{21}, f_{22}, and g_{21}:

$$D_2 = \{z = [x_1 \; x_2 \; y_0 \; m]^T \in \mathbb{R}^4 : -0.1 \le x_1 \le 0.1$$
$$0.05 \le y_0 \le 0.1, \;\; 0.06 \le m \le 0.1\}. \tag{42}$$

Observe that the system (40) can be rewritten as in (27), i.e.,

$$\dot{x} = A(\alpha)x + Bg(x)u \tag{43}$$

where $B = [0 \quad -1]^T$ and $g(z) = -g_{21}(z) = \frac{\lambda\mu}{2m\,(1+\mu(x_1+y_0))^2}$. Note that $g(z) > 0$, for all $z \in D_2$.

After the calculations, the maximum and minimum values of the functions f_{21}, f_{22}, g_{21}, and $u_0 = i_0^2$, in the domain D_2, one obtains

$$\begin{aligned}
a_{21_1} &= \max_{z\in D_2}\{f_{21}(z)\} = 48.3951\\
a_{21_2} &= \min_{z\in D_2}\{f_{21}(z)\} = 26.0000\\
a_{22_1} &= \max_{z\in D_2}\{f_{22}(z)\} = -0.0100\\
a_{22_2} &= \min_{z\in D_2}\{f_{22}(z)\} = -0.0167\\
b_{21_1} &= \max_{z\in D_2}\{g_{21}(z)\} = -2.3469\\
b_{21_2} &= \min_{z\in D_2}\{g_{21}(z)\} = -9.4650\\
\max\, u_0 &= \max_{z\in D_2}\{i_0^2(z)\} = 3.0678\\
\min\, u_0 &= \max_{z\in D_2}\{i_0^2(z)\} = 1.5467. \qquad (44)
\end{aligned}$$

From (44), the local models of the plant (40) and (41) are the following:

$$\begin{aligned}
A_1 &= \begin{bmatrix} 0 & 1\\ a_{21_1} & a_{22_1}\end{bmatrix},\ A_3 = \begin{bmatrix} 0 & 1\\ a_{21_1} & a_{22_2}\end{bmatrix}\\
A_5 &= \begin{bmatrix} 0 & 1\\ a_{21_2} & a_{22_1}\end{bmatrix},\ A_7 = \begin{bmatrix} 0 & 1\\ a_{21_2} & a_{22_2}\end{bmatrix}\\
B_1 &= \begin{bmatrix} 0\\ b_{21_1}\end{bmatrix},\ B_2 = \begin{bmatrix} 0\\ b_{21_2}\end{bmatrix} \qquad (45)
\end{aligned}$$

where $A_1 = A_2$, $A_3 = A_4, A_5 = A_6, A_7 = A_8, B_1 = B_3 = B_5 = B_7$, and $B_2 = B_4 = B_6 = B_8$.

It is worth observing that this system presents no problems with feasibility. Thus, to solve the inequalities (12)–(14) presented in the Theorem 1 for $N = 2$, $\beta_0 = -2$, and $\lambda_{isk}(0)$, $i \in \mathbb{K}_8$, $s, k \in \mathbb{K}_2$ randomly chosen between 0 and 1, a feasible solution was obtained. In this case, the controller gains, the positive definite matrices of the piecewise Lyapunov function (7), and the matrices symmetric matrices Q_{jk}, $j \in \mathbb{K}_8$, $k \in \mathbb{K}_2$ were the following:

$$\begin{aligned}
K_{11} &= [-28.2676 \quad -3.3652],\ K_{12} = [-27.5830 \quad -3.5562]\\
K_{21} &= [-12.5336 \quad -1.2352],\ K_{22} = [-12.5957 \quad -1.3727]\\
K_{31} &= [-28.3030 \quad -3.3638],\ K_{32} = [-27.5806 \quad -3.5367]\\
K_{41} &= [-12.5093 \quad -1.2289],\ K_{42} = [-12.6320 \quad -1.3664]\\
K_{51} &= [-23.5195 \quad -2.9365],\ K_{52} = [-22.4997 \quad -3.0800]\\
K_{61} &= [-10.0612 \quad -1.2000],\ K_{62} = [-10.5085 \quad -1.3822]\\
K_{71} &= [-23.5313 \quad -2.9361],\ K_{72} = [-22.5533 \quad -3.0798]\\
K_{81} &= [-10.4454 \quad -1.2359],\ K_{82} = [-10.1927 \quad -1.3458]
\end{aligned} \qquad (46)$$

$$P_1 = 10^3\begin{bmatrix} 2.0288 & 0.2755\\ 0.2755 & 0.0672\end{bmatrix},\ P_2 = 10^3\begin{bmatrix} 1.6511 & 0.2549\\ 0.2549 & 0.0607\end{bmatrix} \qquad (47)$$

$$\begin{aligned}
Q_{11} &= \begin{bmatrix} 0.0053 & -0.1359\\ -0.1359 & 0.3891\end{bmatrix},\ Q_{12} = \begin{bmatrix} 0.0077 & -0.1876\\ -0.1876 & 0.6888\end{bmatrix}\\
Q_{21} &= \begin{bmatrix} 0.0033 & -0.0776\\ -0.0776 & 0.3173\end{bmatrix},\ Q_{22} = \begin{bmatrix} 0.0045 & -0.1098\\ -0.1098 & 0.5256\end{bmatrix}
\end{aligned}$$

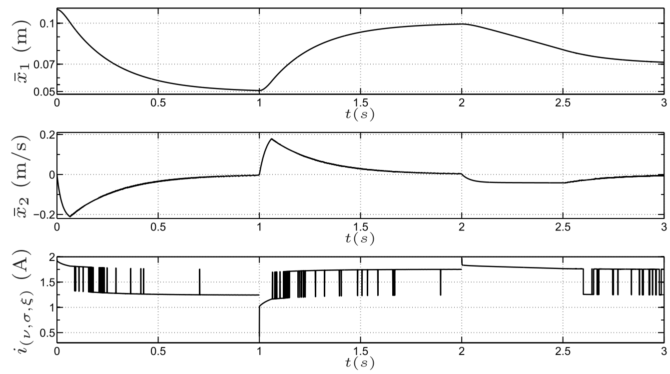

Fig. 2. Position $(y(t) = \bar{x}_1(t))$, velocity $(\bar{x}_2(t))$, and electric current $(i_{(\nu,\sigma,\xi)}(t))$ of the controlled system, considering $y_0 = 0.05$ m and $m = 0.06$ Kg, $y_0 = 0.1$ m, and $m = 0.1$ Kg, and $y_0 = 0.07$ m and $m = 0.1$ Kg, for $t \in [0\ 1)$, $t \in [1\ 2)$ and $t \geq 2$, respectively.

$$\begin{aligned}
Q_{31} &= \begin{bmatrix} 0.0054 & -0.1364\\ -0.1364 & 0.3923\end{bmatrix},\ Q_{32} = \begin{bmatrix} 0.0080 & -0.1894\\ -0.1894 & 0.7016\end{bmatrix}\\
Q_{41} &= \begin{bmatrix} 0.0033 & -0.0776\\ -0.0776 & 0.3193\end{bmatrix},\ Q_{42} = \begin{bmatrix} 0.0046 & -0.1108\\ -0.1108 & 0.5372\end{bmatrix}\\
Q_{51} &= \begin{bmatrix} 0.0040 & -0.1114\\ -0.1114 & 0.2148\end{bmatrix},\ Q_{52} = \begin{bmatrix} 0.0053 & -0.1467\\ -0.1467 & 0.3896\end{bmatrix}\\
Q_{61} &= \begin{bmatrix} 0.0044 & -0.0586\\ -0.0586 & 0.1472\end{bmatrix},\ Q_{62} = \begin{bmatrix} 0.0060 & -0.0802\\ -0.0802 & 0.2481\end{bmatrix}\\
Q_{71} &= \begin{bmatrix} 0.0040 & -0.1116\\ -0.1116 & 0.2160\end{bmatrix},\ Q_{72} = \begin{bmatrix} 0.0054 & -0.1477\\ -0.1477 & 0.3962\end{bmatrix}\\
Q_{81} &= \begin{bmatrix} 0.0043 & -0.0595\\ -0.0595 & 0.1517\end{bmatrix},\ Q_{82} = \begin{bmatrix} 0.0061 & -0.0793\\ -0.0793 & 0.2435\end{bmatrix}.
\end{aligned} \qquad (48)$$

Therefore, the control law (28) and (29) for the levitator is given by

$$\begin{aligned}
u(t) &= u_{(\nu,\sigma,\xi)}(t) = i^2_{(\nu,\sigma,\xi)}(t) - i_0^2\\
\text{with}\quad & i^2_{(\nu,\sigma,\xi)}(t) = -K_{\nu\sigma}x(t) + \gamma_\xi \qquad (49)\\
& K_{\nu\sigma} \in \{K_{11}, K_{21}, \ldots, K_{81}, \ldots, K_{82}\}\\
& \sigma = \arg\min_{k\in\mathbb{K}_N}{}^{*}\{x^T P_k x\},\quad \nu = \arg\min_{j\in\mathbb{K}_r}{}^{*}\{x^T Q_{j\sigma} x\},\\
& \gamma_\xi = \begin{cases} 3.0678, & \text{if } x^T P_\sigma B \leq 0\\ 1.5467, & \text{if } x^T P_\sigma B > 0\end{cases}
\end{aligned}$$

where K_{ik}, $i \in \mathbb{K}_8$, $k \in \mathbb{K}_2$ are given in (46).

For the simulation illustrated in Fig. 2, at $t = 0$ s, the initial condition $\bar{x}(0) = [0.11 \quad 0]^T$, $y_0 = 0.05$ m, and $m = 0.06$ Kg were adopted. In $t = 1$ s, from Fig. 2, the system is practically at the point $\bar{x}(1) = [0.05 \quad 0]^T$. After changing y_0 from 0.05 to 0.1 m and m from 0.06 to 0.1 Kg at $t = 2$ s, one can see that the system is practically at the point $\bar{x}(2) = [0.1 \quad 0]^T$, which will be the new initial condition. Finally, y_0 was changed from 0.1 to 0.07 m at $t = 2$ s and $m = 0.01$ Kg for $t \geq 2$ s. Thus, observe that in Fig. 2, $x(\infty) = [0.07 \quad 0]^T$.

Note that in this case, it is not possible to directly obtain the membership functions, since the mass is uncertain, but the proposed method overcomes this problem, because it does not depend on such functions. Observe also that even with uncertainty in the reference control signal (because $u = i^2 - i_0^2$ and i_0^2 given in (39) is uncertain considering that

m is uncertain), the proposed methodology was efficient and provided an appropriate transient response, as shown in Fig. 2.

V. Conclusion

This paper proposed a new switched control design method and establishes a new criterion of stability for uncertain nonlinear plants described by Takagi–Sugeno fuzzy models. The methodology eliminates the need to obtain the explicit expressions of the membership functions to implement the control law. This is relevant in cases where the membership functions depend on uncertain parameters or are difficult to implement. Some conditions of the stabilization criterion are represented by a class of BMIs which has been solved by the path-following method. The controller gain is chosen by a switching law that minimizes a piecewise Lyapunov function and its time derivative. Simulating this new procedure, the controlled system presented an appropriate transient response, as displayed in Figs. 1 and 2. Thus, it is considered that the proposed method may be an useful procedure in practical applications for the control design of uncertain nonlinear systems. Future research on the subject include the design of robust discrete-time switched T–S controllers [43] for plants with uncertainties or subjected to structural failures [44]–[46].

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