Nonpertubative Solutions of Massless Gauged Thirring Model

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Abstract. We present a nonperturbative quantization of the two-dimensional massless gauged Thirring model by using the path-integral approach. First, we will study the constraint structure of model via the Dirac's formalism and by using the Faddeev-Senjanovic method we calculate the vacuum–vacuum transition amplitude in a R_ξ–gauge, then we compute the Green's functions in a nonperturbative framework.

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INTRODUCTION

The Thirring model (TM) [1], which was the first soluble field theory proposed, describes a self-interaction of non-gauge invariant massless Dirac's fermion fields in (1+1)−dimensions. The possibility of a gauge invariant TM was explored by Itoh et al. and Kondo [2]. Such reformulation was performed by using the *Hidden Local Symmetry technique* which gives a gauge field character, with coupling constant e, to the auxiliary vectorial field A_μ. However, it was not presented such works the solutions of gauged Thirring model (GTM), therefore, the main aim of the present work is to give the nonpertubative functional quantization of two-dimensional GTM.

This paper is organized as follow: In the second section, we study the Hamiltonian structure of model. In the third section, we construct correctly the generating functional from which all Green's functions are obtained by functional differentiation. And, in the last section, we present our final remarks and perspectives.

THE MASSLESS GAUGED THIRRING MODEL: HAMILTONIAN STRUCTURE

Describing the massless gauged Thirring model by the following Lagrangian density

$$\mathscr{L} = \frac{i}{2}\bar{\psi}\overleftrightarrow{\partial\!\!\!/}\psi + \bar{\psi}A\!\!\!/\psi + \frac{1}{2g}\left(A_\mu - \partial_\mu\theta\right)^2 - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}, \tag{1}$$

where the field-strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $e > 0$. The θ field was introduced following the Stückelberg procedure.

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We have the following canonical pairs $(\bar{p}_a, \psi_a)$, $(p_a, \bar{\psi}_a)$, (π^μ, A_μ) and (π_θ, θ), from the Hamiltonian analysis we find three primary constraints [3]

$$\phi_a = p_a + \frac{i}{2}\left(\gamma^0\right)_{ab}\psi_b \approx 0, \quad \bar{\phi}_a = \bar{p}_a + \frac{i}{2}\bar{\psi}_b\left(\gamma^0\right)_{ba} \approx 0, \quad \varphi_1 = \pi^0 \approx 0. \tag{2}$$

Now we write out the canonical Hamiltonian density as

$$\begin{aligned} \mathscr{H}_C &= -i\bar{\psi}\gamma^1\partial_1\psi - \bar{\psi}\not{A}\psi + \frac{g}{2}(\pi_\theta)^2 + \frac{e^2}{2}\left(\pi^1\right)^2 + \pi^1(\partial_1 A_0) + A_0\pi_\theta + \\ &+ \frac{1}{2g}(A_1)^2 + \frac{1}{2g}(\partial_1\theta)^2 - \frac{1}{g}A_1(\partial_1\theta). \end{aligned} \tag{3}$$

The full constraint analysis [3] yields a set $\{\varphi_1, \varphi_2\}$ of first–class constraints, where the constraint φ_2 is given by

$$\varphi_2 = \partial_1\pi^1 - \pi_\theta + i(\bar{p}\psi + \bar{\psi}p) \approx 0, \tag{4}$$

which is Gauss's law: $\partial_1\pi^1 + \bar{\psi}\gamma^0\psi - \pi_\theta = 0$. Also, we obtain from such analysis the second–class set given by $\left\{\phi_\alpha, \bar{\phi}_\alpha\right\}$.

At last, it is necessary to introduce two additional conditions to accomplish the constraint analysis. In that way, we choose the radiation gauge

$$\chi_1 = A_0 \approx 0, \quad \chi_2 = \partial_1 A_1 \approx 0. \tag{5}$$

THE VACUUM–VACUUM TRANSITION AMPLITUDE

Once the Hamiltonian structure, we can now quantize the model following the Faddeev-Senjanovic procedure [4] to implement the functional quantization of the gauged Thirring model in a correct way. Therefore, following the procedure cited above and using the Faddeev-Popov-De Witt ansatz [5] as well, in the R_ξ −gauge

$$R_\xi[A, \theta] = \partial_\mu A^\mu + \frac{\xi}{g}\theta, \tag{6}$$

we are able to find the following decoupled expression to the vacuum–vacuum transition amplitude

$$Z = \int \mathscr{D}\theta\mathscr{D}A_\mu\mathscr{D}\bar{\psi}\mathscr{D}\psi \det\left|\Box + \frac{\xi}{g}\right| \exp\left(i\int d^2x\left[\mathscr{L}_{\psi,A} + \mathscr{L}_\theta\right]\right), \tag{7}$$

where the Lagrangian densities $\mathscr{L}_{\psi,A}$ and $\mathscr{L}_\theta$ are given by

$$\mathscr{L}_{\psi,A} = \bar{\psi}(i\not{\partial} + \not{A})\psi - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2g}A_\mu A^\mu - \frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2, \tag{8}$$

and

$$\mathscr{L}_\theta = \frac{1}{2g}\left(\partial_\mu\theta\right)\left(\partial^\mu\theta\right) - \frac{\xi}{2g^2}\theta^2, \tag{9}$$

we see through the Eq.(7) that Faddeev-Popov-De Witt's ghosts are decoupled from the fields, then the determinant could be absorbed into a normalization constant.

The generating functional

In the $R_\xi-$gauge, the full generating functional can be factorized in the product of two generating functionals as

$$\mathscr{Z}\left[\eta,\bar{\eta},J^{\mu},K\right]=\tilde{\mathscr{Z}}\left[K\right]\times\bar{\mathscr{Z}}\left[\eta,\bar{\eta},J^{\mu}\right]. \tag{10}$$

where $\tilde{\mathscr{Z}}\left[K\right]$ is a free scalar generating functional.

The interacting generating functional $\bar{\mathscr{Z}}[\eta,\bar{\eta},J^{\mu}]$, after the fermionic integration, is written in the following form in terms of the gauge field

$$\bar{\mathscr{Z}}\left[\eta,\bar{\eta},J^{\mu}\right]=\int\mathscr{D}A_{\mu}\exp\left(i\int d^{2}x\left[\frac{1}{2}A_{\mu}B_{\xi}^{\mu\nu}A_{\nu}+J_{\mu}A^{\mu}-\int d^{2}y\;\bar{\eta}G\left(x,y;A\right)\eta\right]\right), \tag{11}$$

where $G\left(x,y;A\right)$ is the Green's function of Dirac equation, and we have defined the differential operator $B_{\xi}^{\mu\nu}$ as

$$B_{\xi}^{\mu\nu}=\left(\frac{1}{\pi}+\frac{1}{g}+\frac{\Box}{e^{2}}\right)\eta^{\mu\nu}+\left(\frac{1}{\xi}-\frac{1}{\pi\Box}-\frac{1}{e^{2}}\right)\partial^{\mu}\partial^{\nu}. \tag{12}$$

The Green's functions

The gauge field propagator

The gauge field propagator in momentum space has the following expression

$$i\tilde{\mathscr{D}}_{\mu\nu}^{\xi}\left(k\right)=\frac{e^{2}}{k^{2}-\frac{e^{2}}{g}-\frac{e^{2}}{\pi}}\eta_{\mu\nu}+\frac{1}{k^{2}}\left(\frac{\xi}{k^{2}-\frac{\xi}{g}}-\frac{e^{2}}{k^{2}-\frac{e^{2}}{g}-\frac{e^{2}}{\pi}}\right)k_{\mu}k_{\nu}. \tag{13}$$

From expression Eq.(13) to $\tilde{\mathscr{D}}_{\mu\nu}^{\xi}(k)$, we see clearly that the gauge field propagator is finite. Also, the transverse component of the gauge field propagator has a squared mass given by $\dfrac{e^{2}}{g}+\dfrac{e^{2}}{\pi}$.

The fermionic propagator

We find that the full fermionic propagator reads

$$\mathscr{S}^{\xi}\left(x\right)=i\exp\left(i\int\frac{d^{2}k}{\left(2\pi\right)^{2}}\frac{1}{k^{2}}\left(\frac{\xi}{k^{2}-\frac{\xi}{g}}-\frac{e^{2}}{k^{2}-\frac{e^{2}}{g}-\frac{e^{2}}{\pi}}\right)\left[1-e^{-ikx}\right]\right)G_{F}\left(x\right). \tag{14}$$

It is easy to verify that it is free of ultraviolet divergences.

The vertex function

The complete vertex function written conveniently in Fourier space

$$\tilde{\mathscr{G}}_{\mu}^{\xi}(p,q;k) = i(2\pi)^2 h_{\mu}(k)\left[\tilde{\mathscr{S}}^{\xi}(p+k) - \tilde{\mathscr{S}}^{\xi}(p)\right]\delta(p+k+q), \tag{15}$$

to show that, as fermionic propagator, the vertex function is a finite function as well. $\tilde{\mathscr{S}}^{\xi}(p)$ is the fermionic propagator (14) and $h_{\mu}(k)$ is defined as

$$h_{\mu}(k) = -\frac{\xi}{k^2\left(k^2 - \frac{\xi}{g}\right)}k_{\mu} - \frac{e^2}{k^2\left(k^2 - \frac{e^2}{g} - \frac{e^2}{\pi}\right)}\gamma_5\tilde{k}_{\mu}. \tag{16}$$

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REMARKS AND CONCLUSIONS

We give here a quantization of massless Gauged Thirring model in a nonperturbative way, this was the biggest advantage of studying a $(1+1)$-dimensions model. After a Hamiltonian analysis we derived the covariant transition amplitude with aid of Faddeev-Popov-De Witt ansatz in R_{ξ}–gauge. The R_{ξ}–gauge condition enables decouple the A_{μ} and θ fields. The study of the massless GTM leads us to obtain nonperturbative information about the model, the obtained expressions to the Green's functions are complete.

As last comment, the complete functional analysis of GTM and the isomorphism of it with the Schwinger and Thirring models at quantum level as well, are under development and will be reported elsewhere.

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