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Studying the lifetime of orbits around Moons in elliptic motion

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Abstract The main goal of the present paper is to study the lifetime of orbits around moons that are in elliptic motion around their parent planet. The lifetime of the orbits is defined as the time the orbit stays in orbit around the moon without colliding with its surface. The mathematical model used to solve this problem is the second order expansion of the potential of the disturbing planet, assumed to be in an elliptical orbit. The results are presented in maps showing the lifetime of the orbit as a function of its initial inclination and eccentricity. The only perturbation acting on the orbit of the spacecraft is assumed to be the gravity of the planet, so the problem is solved by studying the orbital evolution of the spacecraft perturbed by a third body in an elliptical orbit. The region of inclination above the critical value of the third-body perturbation (around 63°) is studied, since below that value the orbits survive for a long time. The influence of the eccentricity of the primaries is also investigated, assuming a hypothetical system that has the same mass parameter and sizes of the Earth–Moon system, but the eccentricity can be in the range 0.0–0.2.

Keywords Astrodynamics · Third-body perturbation · Stability of orbits · Averaged methods

Mathematics Subject Classification 37N05 · 70F07 · 70F15

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1 Introduction

One of the most important part of the design of a space mission is the search for the most suitable orbits to place a spacecraft. The insertion into orbit and the necessary maneuvers to keep the spacecraft in the desired orbit spend fuel, which is an important resource and its use needs to be minimized. The problem of reducing fuel consumption in space maneuvers received much attention of several researches by many decades, with results appearing in the literature as early as Hohmann (1925), which described an important bi-impulsive maneuver to transfer a satellite between two circular coplanar orbits. After that, Hoelker and Silber (1959) and Shternfeld (1959), independently, generalized that transfer to a three-impulsive maneuver. Other examples of impulsive maneuvers can be found in Prussing and Chiu (1986), Prado and Broucke (1996), Sukhanov and Prado (2004) and Fernandes and Marinho (2012a, b). Other methods of space maneuvers use low thrust maneuvers (Casalino et al. 1999; Fernandes and Carvalho 2008; Fernandes 2009; Gomes and Prado 2012). Alternative solutions use natural forces to maneuver a spacecraft, like in Broucke (1988), D'Amario et al. (1982), Farquhar et al. (1985), Felipe and Prado (1999), Strange and Longuski (2002), Prado (2003), Formiga and Prado (2014).

The present paper has the goal of measuring the lifetimes of orbits that have inclination above the critical values, so searching for more stable natural trajectories. Those are the ones that stay a certain amount of time in orbit around a celestial body without the need to perform space maneuvers, so avoiding fuel expenditure for this phase of the maneuver.

Looking at the astronomy literature, a large number of planets outside the solar system have been found lately. Many of those planets, specially the larger ones, probably have moons around it. Some of those moons may be in elliptical orbits around those planets. Even in our Solar System it is possible to find systems with high eccentricities. The dwarf planet Haumea (Ragozzine and Brown 2009) is a good example of this case, because it has a moon with eccentricity near 0.25. Thus, it is interesting to study how the eccentricity of the orbit of the moon around the planet affects the stability of the orbits of spacecrafts orbiting those moon, in particular when thinking about future missions to systems that have these characteristics.

The study lifetimes of spacecrafts around celestial bodies are very common in the literature, always concentrated in bodies in the Solar System, due to the more immediate applications. Some examples are found in: Prado (2003), Scheeres et al. (2001), Lara and Russell (2006), Carvalho et al. (2010, 2012), Paskowitz and Scheeres (2006a, b).

In this way, the present research generalizes this type of study to consider an elliptic motion with high eccentricity for the moon around the planet. Similarly to what was done in Prado (2003), the time evolution of a satellite orbiting a moon that is perturbed by the planet is made, but this time under the third-body perturbation using a single-averaged model expanded up to the second order. This model is shown in detail next, but the main goal is to eliminate the dependence of the perturbations with the true anomaly of the spacecraft, so generating results that are independent of the initial conditions of the orbit. Domingos et al. (2014) showed that this type of approximation is very good for eccentricities in the range of 0.0–0.2, so this limit is used. The end of the lifetime is assumed to occur when there is a collision of the spacecraft with the surface of the moon.

2 Mathematical models

To develop the single-averaged model, it is assumed that the moon is in the center of the reference system $x - y$, with mass m_0. The perturbing body, the planet, is in an elliptic orbit with the orbital elements semi-major axis (a'), eccentricity (e'), and mean motion (n'). Its mass is m'. The satellite, which has a mass m, assumed to be negligible, is in a three-dimensional orbit with the orbital elements: a (semi-major axis), e (eccentricity), i (inclination), ω (argument of periapsis), Ω (right ascension of the ascending node). Its mean motion is called n. Those are the same hypothesis of the elliptic restricted three-body problem. An averaged model is interesting because it shows the behavior of the orbit from medium to longer times, removing short-term oscillations that are not important in the present case. It is also used because it generates analytical equations of motion that can be used to understand the evolution of the spacecraft. The averages are made with respect to the eccentric anomaly, which is the more standard way of doing and that generates results that can be verified in the literature.

The equations of motion used in the present research were studied in details in Domingos et al. (2014). The present paper summarizes the final results, in the form used here to perform the numerical integrations that evaluate the lifetime of the spacecraft. Different initial eccentricities (0.0, 0.05, 0.10, 0.15 and 0.20) are considered for the orbit of the moon, which means that, in a system fixed in the moon, those are the eccentricities of the disturbing body, the planet. Those choices are made based on Domingos et al. (2013, 2014), which showed that averaged models have good accuracy up to those values. The system used as an example for the present studies has the same physical data (masses and distances) of the Earth–Moon system, but considers possible eccentricities for the orbit of the secondary body around the primary. The literature has several examples of studies related to the third-body perturbation, like Cook (1962) and Smith (1962) which studied lunisolar perturbations in satellites around the Earth. Smith (1962) assumed an elliptical orbit for the perturbing body, as presented here, while Cook (1962) studied the circular problem. Liu et al. (2012) also considered the perturbing body in elliptic orbit.

Domingos et al. (2008) made a detailed development of the disturbing potential and the average process. The equations of motion used here are the same ones used in Domingos et al. (2008, 2014), which are the papers that developed the force models, but they did not consider the study of lifetimes of spacecrafts. The model is based in an expansion in Legendre's polynomials, considering that the semi-major axis of the orbit of the disturbing body is much larger than the one of the spacecraft. So, the second order term of the disturbing potential, making an average with respect to the eccentric anomaly of the satellite, to eliminate this variable, is given by Domingos et al. (2008):

$$\langle R_2\rangle = \frac{\mu' a^2 n'^2}{2}\left(\frac{a'}{r'}\right)^3\left\{\left(1+\frac{3}{2}e^2\right)\left[\frac{3}{2}\left(\alpha^2+\beta^2\right)-1\right]+\frac{15}{4}\left(\alpha^2-\beta^2\right)e^2\right\} \quad (1)$$

where $r' = \frac{a'(1-e'^2)}{1+e'\cos f'}$, $\cos f' = \cos M' + e'(\cos 2M' - 1) + \frac{9}{8}e'^2(\cos 3M' - \cos M') + \cdots$ and $\sin f' = \sin M' + e' \sin 2M' + e'^2(\frac{9}{8}\sin 3M' - \frac{7}{8}\sin M') + \cdots$

If the perturbing body is in an elliptic orbit, α and β are given by $\alpha = \cos\omega\cos D - \cos i\sin\omega\sin D$, $\beta = -\sin\omega\cos D - \cos i\cos\omega\sin D$ and $D = \Omega - f' - \omega'$. The term μ' represents the mass ratio of the perturbing body, which is its real mass divided by the total mass of the system, while μ represents the mass ratio of the central body. The mean anomaly of the perturbing body (M') can be written as $M' = M'_o + n't$. The equations of motion can now be derived from the Lagrange's planetary equations (Murray and Dermott 1999). After the algebraic manipulation and application of the equation for the perturbing term, it is possible to get the equations shown below (Domingos et al. 2014):

$$\frac{\mathrm{d}e}{\mathrm{d}t} = K\frac{15}{4}\mu' n'^2\frac{e\sqrt{1-e^2}}{n}\left[\sin 2\omega\left(\cos^2 D - \cos^2 i\sin^2 D\right) - \cos i\cos 2\omega\sin 2D\right] \quad (2)$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = K\frac{1}{\sin\,i\left[\mu a\left(1-e^2\right)\right]^{\frac{1}{2}}}\frac{3}{4}\mu' n'^2 a^2\left\{e^2\left[-5\cos\,2\omega\,\sin\,2D\,\cos^2 i + \frac{3}{2}\right.\right.$$
$$\left.\left(-\sin\,2D + \sin\,2D\,\cos^2 i\right) - \frac{5}{2}\sin\,D\,\cos\,2\omega\left(1+\cos^2 i\right)\right]$$
$$\left. + \left(\sin\,D - \sin\,2D\,\cos^2 i\right) - 5\,\cos\,2D\,\sin\,2\omega\,\cos\,i\right\} \quad (3)$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = K\frac{3}{4}\mu' n'^2\frac{a^2}{\sin\,i\left[\mu a\left(1-e^2\right)\right]^{\frac{1}{2}}}\left\{\left(1+\frac{3}{2}e^2\right)\left(-\sin^2 D\,\sin\,2i\right) + \left[\cos\,2i\,\cos\,2\omega\,\sin^2 D\right.\right.$$
$$\left.\left. + \sin\,i\,\sin\,2D\,\sin\,2\omega\right]\left(\frac{5}{2}e^2\right)\right\} \quad (4)$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = K\frac{3}{4}\mu' n'^2\left\{-\frac{a^2}{\sin\,i\left[\mu\,a\left(1-e^2\right)\right]^{1/2}}\left[\left(1+\frac{3}{2}e^2\right)\left(-\sin^2 D\,\sin\,2i\right)\right.\right.$$
$$\left. + \left(\frac{5}{2}e^2\right)\left(\cos\,2i\,\sin^2 D\,\cos\,2\omega\right) + \frac{1}{2}\sin\,2i\,\sin 2D\,\sin\,2\omega\right] + \frac{\sqrt{1-e^2}}{n}$$
$$\left.\left[5\left(\cos^2 D - \cos^2 i\,\sin^2 D\right)\cos\,2\omega - 5\cos\,i\,\sin\,2D\,\sin\,2\omega + \frac{6}{2}\left(\cos^2 D + \cos^2 i\,\sin^2 D\right)\right]\right\} \quad (5)$$

The semi-major axis is constant, since the mean anomaly M was eliminated from the perturbing function. The K value is given by:

$$K = \frac{1 + 3e'\cos\,f' + 3e'^2\cos^2 f'}{1 - 6e' + 15e'^2} \quad (6)$$

where the mean value of K represents increase and/or decrease of the velocity of the dynamics of the system, when compared with the circular disturbing problem.

3 Results

Using the mathematical model described above, some numerical simulations are now presented, always making numerical integrations of Eqs. (2)–(5). Figure 1 shows the system in more details, with the Moon, the planet and the spacecraft.

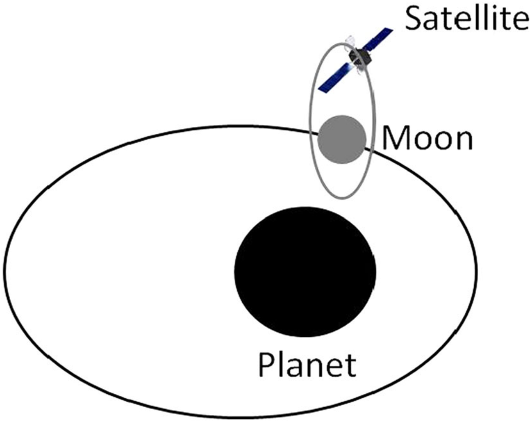

Fig. 1 A view of the complete system: the main planet, the Moon and the spacecraft

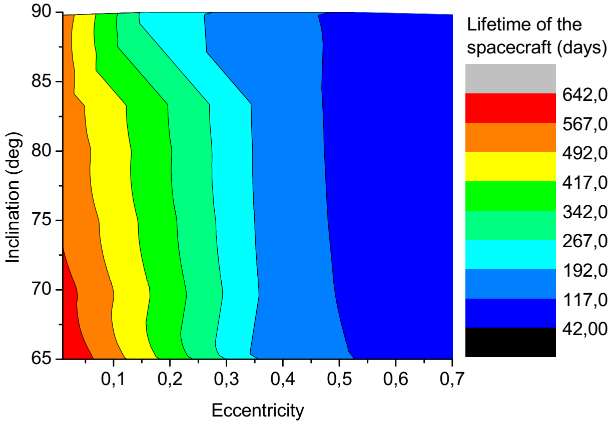

Fig. 2 Lifetime of the orbit of the spacecraft (days) as a function of the initial inclination (deg) and eccentricity of the initial orbit of the spacecraft assuming a perturbing body in circular orbit

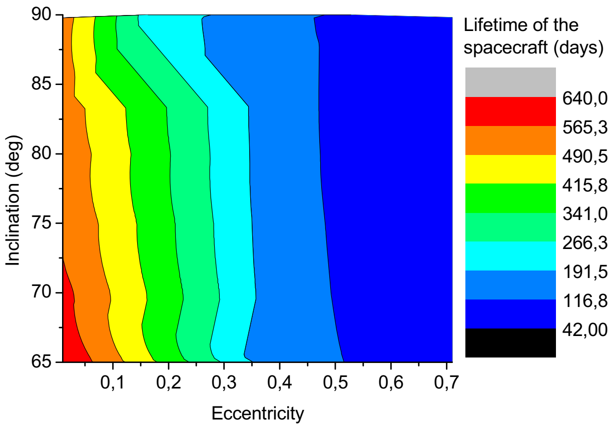

Fig. 3 Lifetime of the orbit of the spacecraft (days) as a function of the initial inclination (deg) and eccentricity of the initial orbit of the spacecraft assuming a perturbing body in an elliptical orbit with eccentricity 0.05

Figures 2, 3, 4, 5 and 6 show the results. The horizontal axis represents the initial eccentricity of the orbit of the spacecraft, in the range from 0.0 to 0.7. The vertical axis represents the initial inclination of the orbit of the spacecraft, in the range from 65° to 90°, since our goal is to study near-polar orbits above the critical inclination of the third-body perturbation. The reason to study this range of orbits is that they are better to make observations of the surface of the body, as well as to map their gravity fields. The problem is that they are the most perturbed orbits, because they are above the already mentioned critical inclination of the third-body perturbation (Prado 2003). The color code shows the lifetime, in days, that the spacecraft remains in orbit without any type of control. Those values are obtained by numerical integrations of the equations of motion, which are stopped when a collision occurs between the spacecraft and the Moon.

The canonical system of units is used, and the semi-major axis of the orbit of the Moon around the planet is used as unit of distances. The unit of time is defined such that the period of the Moon around the planet is 2π, but it is converted to days to plot the results. The unit

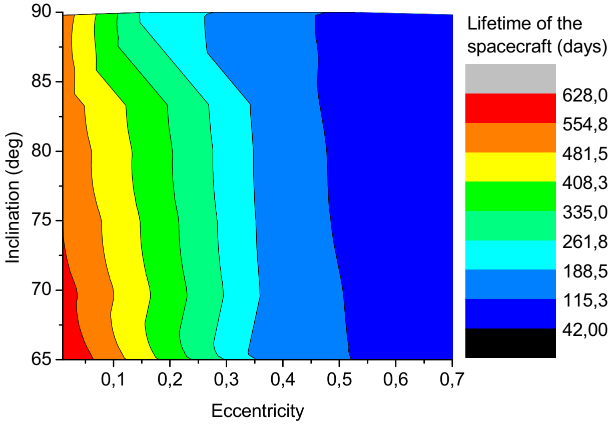

Fig. 4 Lifetime of the orbit of the spacecraft (days) as a function of the initial inclination (deg) and eccentricity of the initial orbit of the spacecraft assuming a perturbing body in an elliptical orbit with eccentricity 0.10

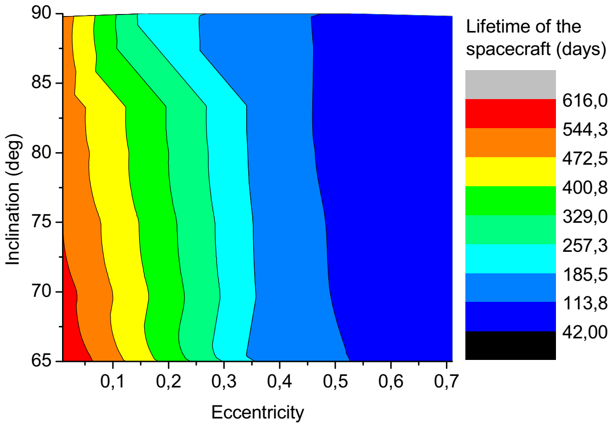

Fig. 5 Lifetime of the orbit of the spacecraft (days) as a function of the initial inclination (deg) and eccentricity of the initial orbit of the spacecraft assuming a perturbing body in an elliptical orbit with eccentricity 0.15

of mass is the total mass of the system, which means the mass of the planet added to the mass of the Moon, since the spacecraft has a negligible mass. The mass ratio is the same of the Earth–Moon system, equal to 0.0121. The reason for using this system of units is that it gives more generic results. A transformation back to the standard units can be made for any system of primaries, which means that the results presented here are valid for any system that has the mass parameter used in the present research. The radius of the Moon, in canonical units, is 0.004521, which is the equivalent to the radius of the Moon in the Earth–Moon system. All the orbits have a semi-major axis of four times the radius of the Moon, to follow the work done in Prado (2003), so a collision occurs when the eccentricity of these orbits reaches 0.75, giving a periapsis distance equal to the radius of the Moon. The effects of the eccentricity of the Moon around the planet are also studied, and there is one figure for each value used: 0.00, 0.05, 0.10, 0.15 and 0.20. So, all the trajectories have the following initial conditions: semi-major axis = 0.018084 canonical units, argument of periapsis = 0, argument

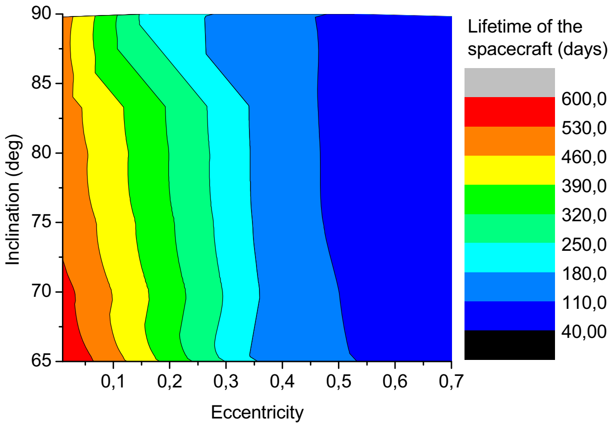

Fig. 6 Lifetime of the orbit of the spacecraft (days) as a function of the initial inclination (deg) and eccentricity of the initial orbit of the spacecraft assuming a perturbing body in an elliptical orbit with eccentricity 0.20

of ascending node = 0 and inclination and eccentricity given by the values of the axis of the figures.

The results show and quantify the effects of the orbital elements of the initial orbit of the spacecraft in their lifetimes. The semi-major axis of the orbit remains constant, since the study is made using an averaged model for the perturbing body. All the figures are similar, but the scales of the time are different, showing that the initial eccentricity has a very strong importance in the phenomenon. In general, initial circular orbits have much longer lifetimes, in the order of 600 days. Those lifetimes decrease very fast with the initial eccentricity, being of the order of 300 days for eccentricities of 0.25 and less than 100 days when the initial eccentricity reaches around 0.6.

The increase of the inclination also reduces the lifetime of the orbits, because the effects of the perturbation are larger. For example, using an initial circular orbit for the spacecraft and considering an elliptical orbit with eccentricity of 0.1 for the disturbing body, the lifetime reduces from 628 days, when the initial orbit is 65° inclined, to around 450 days, when the initial orbit is 90° inclined. It is clear that the effects of the inclination are more important for lower values of the initial eccentricity, as shown by the fact that the slopes of the lines present in the figures become closer to vertical when the eccentricities increase.

The effects of the eccentricity of the perturbing body go on the same line, and its increase reduces the lifetime of the orbit. The figures look very similar to each other, but note that the time scale is different. When the disturbing body is in a circular orbit, the maximum lifetime obtained is 642 days, which reduces to 600 days for a perturbing body in an eccentric orbit with eccentricity of 0.2.

4 Conclusions

The present paper studied the lifetimes of orbits of spacecrafts orbiting Moons of elliptical planetary systems. The effects of the eccentricity and inclination of the initial orbit of the spacecraft, as well as the eccentricity of the orbit of the Moon around the planet are investigated. An hypothetical system was used, which has the same masses and distances of the

Earth–Moon system, but a variable eccentricity, to study its influence in the lifetime of the orbits.

The results showed how the increase of the initial eccentricity and inclination of the initial orbit reduces the lifetime of the orbits, with the eccentricity playing the most important role in the phenomenon.

The increase of the eccentricity of the orbit of the Moon around the planet also reduces the lifetime of the orbits, in general in the order of 7–8 %, depending on the initial conditions.

In general, the results shown in the present paper can help mission designers in the task of making mission planning, in particular in deciding if it is necessary or not to perform orbital maneuvers for a specific mission.

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