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Some results from a Mellin transform expansion for the heat kernel

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In the case of a differential operator containing a gauge field, coefficients of a new heat kernel expansion obtained in a preceding paper (A. P. C. Malbouisson, M. A. R. Monteiro, and F. R. A. Simão, CBPF-NF-024/88, to be published in J. Math. Phys.) are calculated. The prior expansion allows it to be shown that the meromorphic structure of the generalized zeta function is much richer than was known previously. Also, an application to anomalies is done, resulting in a general formula for the arbitrary dimension D. The special cases $D = 2$ and $D = 3$ are investigated.

I. INTRODUCTION

In a previous paper[1] an asymptotic expansion was obtained for the diagonal part of the heat kernel associated with a given elliptic operator H of order m, based on the connection, through a Mellin transform, between the heat kernel and the Seeley's kernel $K(s;x,y)$[2] of the complex sth power H^s of the operator H and the meromorphic properties of $K(s;x,x)$. We recall that "heat kernel" means the solution of the "heat equation"

$$\frac{\partial}{\partial t} F(t;x,y) = HF(t;x,y), \tag{1.1}$$

where t is a "time" or "temperature" parameter and x and y are, in the case we are interested in, points of a D-dimensional compact manifold M. The Seeley's kernel is defined for $\mathrm{Re}(s) < -D/m$ such that

$$H^s f(x) = \int_M dy\, K(s;x,y) f(y).$$

The expansion mentioned above is obtained by analytic continuation of K in the variable s and reads as

$$F(t;x,x) = -\sum_{l=0}^{\infty} t^l \left(\frac{d\phi}{ds}\right)_{s=l} - \sum_j t^{(j-D)/m} \Gamma\left(\frac{D-j}{m}\right) R_j(x). \tag{1.2}$$

The sum over j is such that we take $j = 0,1,2,\ldots$ *excluding the terms such that* $(j-D)/m = 0,1,2,\ldots$ and $R_j(x)$ is the residue of $K(s;x,x)$ at the pole $s = (j-D)/m$:

$$R_j(x) = \frac{1}{im(2\pi)^{D+1}} \int_{\|\xi\|=1} \int_\Gamma d\lambda \times \lambda^{(j-D)/m} b_{-m-j}(x,\xi,\lambda), \tag{1.3}$$

where Γ is a curve coming from ∞ along a ray of minimal growth, clockwise on a small circle around the origin, and then going back to ∞. The quantities b_{-m-j} are obtained from the coefficients of the symbol of H (see Sec. III) and $\|\xi\| = 1$ means that the set of variables $\{\xi\}$ is constrained to be at the surface of the unit sphere in D-dimensional space. The function $\phi(s)$ is introduced to account for the coincidence of the poles of the gamma function $\Gamma(-s)$ and those of $K(s;x,x)$ at the positive integers l and is defined by

$$\Gamma(-s) K(s;x,x) \approx \phi(s)/(s-l)^2 \tag{1.4}$$

for $s \approx l$.

As was remarked in Ref. 1, the expansion (1.2) is rather different from de Witt's ansatz currently used.[3] In particular (1.2) contains fractionary powers at even dimension and even operator order, coming from the second term in the expansion.

In the rest of the paper we explore some consequences of the new expansion (1.2). In Sec. II we show that the generalized zeta function $\zeta(s)$ has an infinity of poles at real values of s. In Sec. III we calculate the coefficients of the leading and next-to-leading terms in (1.2). In Sec. IV we obtain a general formula for the anomaly in arbitrary dimension D and particularize to the special cases $D = 2$ and $D = 3$.

II. MEROMORPHY OF THE GENERALIZED ZETA FUNCTION

One of the implications of the series (1.2) is of a mathematical character and concerns the meromorphic structure of the Hawking's generalized zeta function,[4] which is much richer than the structure known previously. This may be easily seen as follows.

The generalized zeta function is written as

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^1 dt\, t^{s-1} \int d^D x\, F(t;x,x) + Q(s), \tag{2.1}$$

where $Q(s)$ converges for all s.

Let us take $D = 4$ and consider an operator of order $m = 2$. Replacing $F(t;x,x)$ in (2.1) by the series (1.2) we see that the first term of the expansion gives no poles as a result of the factor $1/\Gamma(s)$ in front of the integral in (2.1). From the second term of the expansion we have the sum

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$$-\frac{1}{\Gamma(s)}\sum_j \Gamma\left(\frac{4-j}{2}\right)R_j(x)\int_0^1 dt\, t^{s+j/2-3}, \qquad (2.2)$$

which gives poles at $s = 2 - j/2$ for integer values of j and $(j-4)/2 \neq 0,1,2,...$.

Thus the poles of the generalized zeta function are not situated only at $s=1$ $(j=2)$ and $s=2$ $(j=0)$. We also have poles at $s=\frac{3}{2}$ $(j=1)$ and $s=\frac{1}{2}$ $(j=3)$; for $j=5,7,...$ we have an infinity of poles in s at the negative half-integers. There are no poles at negative integers, as a result of the vanishing of the residues of $K(s;x,x)$ at those values.[2] The residues at the poles are given by the corresponding coefficients $-[1/\Gamma(2-j/2)]\Gamma((4-j)/2)R_j(x)$ in (2.2).

III. APPLICATION TO A DIFFERENTIAL OPERATOR

Let us consider a differential operator H of order $m=2$,

$$H = -[g^{\mu\nu}(x)(\partial_\mu + B_\mu(x))(\partial_\nu + B_\nu(x)) + P(x)], \qquad (3.1)$$

acting on a D-dimensional compact manifold M and endowed with a metric $g_{\mu\nu}(x)$ $(\mu,\nu = 1,2,...,D)$. In (3.1) $P(x)$ is a nondifferential operator and

$$B_\mu(x) = gA_\mu(x) + \eta_\mu(x), \qquad (3.2)$$

where $A_\mu(x)$ and g are, respectively, the gauge field and a coupling constant (not to be confused with the metric tensor or its determinant). The quantity $\eta_\mu(x)$ contains information about curvature and torsion. The usual convention of summation over repeated indices will be adopted.

In Seeley's notation[2] the operator H must be written in the form

$$H = \sum_{\{\alpha\}|\alpha|\leq 2} (-i)^{|\alpha|} H^{|\alpha|}_{\alpha_1\cdots\alpha_D}(x)\frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1}\cdots\partial x_D^{\alpha_D}}, \qquad (3.3)$$

where $|\alpha| = \alpha_1 + \cdots + \alpha_D$.

Expanding (3.1) and comparing with (3.3) we obtain the set of coefficients $H^{|\alpha|}_{\alpha_1\cdots\alpha_D}(x)$:

$$H^{(2)}_{\alpha_1\cdots\alpha_D}(x) = H^{(2)}_{0\cdots 01(\mu)0\cdots 01(\nu)0\cdots 0}(x) = g_{\mu\nu}(x), \qquad (3.4a)$$

$$H^{(1)}_{\alpha_1\cdots\alpha_D}(x) = H^{(1)}_{0\cdots 01(\nu)\cdots 0} = -2ig_{\mu\nu}(x)B^\mu(x), \qquad (3.4b)$$

$$H^{(0)}_{\alpha_1\cdots\alpha_D}(x) = H^{(0)}_{0\cdots 0\cdots 0}(x) = -g_{\mu\nu}(x)(2\partial^\mu B^\nu + B^\mu B^\nu) - P(x). \qquad (3.4c)$$

Now, to calculate the coefficients of the second term of expansion (1.2) we need the quantities b_{-2-j} [see Eq. (1.3)], which are expressed in terms of the coefficients $a_{2-k}(x,\xi)$ of the symbol of H^2,

$$a_{2-k}(x,\xi) = \sum_{|\alpha|=2-k} H^{|2-k|}_{\alpha_1\cdots\alpha_D}\xi_1^{\alpha_1}\cdots\xi_D^{\alpha_D}, \qquad (3.5)$$

by the following set of equations:

$l=0$:

$$b_{-2}[a_2(x,\xi) - \lambda] = 1, \qquad (3.6a)$$

$l>0$:

$$b_{-2-l}(a_2-\lambda) + \sum_{j,k}(-i)^{|\alpha|}\sum_{\{\alpha\}}\frac{\partial^{|\alpha|}b_{-2-j}}{\partial\xi_1^{\alpha_1}\cdots\partial\xi_D^{\alpha_D}} \times \frac{\partial^{|\alpha|}a_{2-k}}{\alpha_1!\cdots\alpha_D!\partial_{x_1}^{\alpha_1}\cdots\partial_{x_D}^{\alpha_D}} = 0,$$

$$\text{with } j<l,\quad j+k+|\alpha| = l. \qquad (3.6b)$$

The coefficients a_{2-k} are easily obtained from Eqs. (3.4):

$$a_2(x,\xi) = g_{\mu\nu}(x)\xi^\mu\xi^\nu \equiv \|\xi\|^2, \qquad (3.7a)$$

$$a_1(x,\xi) = -2ig_{\mu\nu}(x)B^\mu(x)\xi^\nu, \qquad (3.7b)$$

$$a_0(x,\xi) = -g_{\mu\nu}(x)(\partial^\mu B^\nu - B^\mu B^\nu) - P(x). \qquad (3.7c)$$

Then the first two quantities b_{-2-j} that we need for calculating the leading and next-to-leading contributions in the second term of expansion (1.2) are given by

$$b_{-2}(x,\xi,\lambda) = (\|\xi\|^2 - \lambda)^{-1}, \qquad (3.8)$$

$$b_{-3}(x,\xi,\lambda) = \frac{2iB\cdot\xi}{(\|\xi\|^2-\lambda)^2} - \frac{2i\xi\cdot\partial\|\xi\|^2}{(\|\xi\|^2-\lambda)^3}, \qquad (3.9)$$

where the scalar product is defined with the metric $g_{\mu\nu}(x)$.

From (1.2), (1.3), (3.8), and (3.9), the contributions that are coefficients of the powers $t^{-D/2}$ and $t^{(1-D)/2}$, are given, respectively, by

$$-\Gamma\left(\frac{D}{2}\right)R_0(x) = -\Gamma\left(\frac{D}{2}\right)\frac{1}{2i(2\pi)^{D+1}}\int_{\|\xi\|=1}d\xi \times\int_\Gamma d\lambda\,\lambda^{-D/2}(\|\xi\|^2-\lambda)^{-1}, \qquad (3.10)$$

$$-\Gamma\left(\frac{D-1}{2}\right)R_1(x) = -\Gamma\left(\frac{D-1}{2}\right)\frac{1}{(2\pi)^{D+1}}\left[\int_{\|\xi\|=1}d\xi\,B\cdot\xi \times\int_\Gamma\frac{\lambda^{(1-D)/2}d\lambda}{(\|\xi\|^2-\lambda)^2} - \int_{\|\xi\|=1}d\xi\,\xi\cdot\partial\|\xi\|^2 \times\int_\Gamma\frac{\lambda^{(1-D)/2}d\lambda}{(\|\xi\|^2-\lambda)^3}\right], \qquad (3.11)$$

where we take the integration path Γ as the curve coming from $-\infty$ along the negative real axis, then clockwise along the unit circle around the origin, and then backward to $-\infty$ along the negative real axis. Since we must restrict the ξ's to the surface of the unit D-dimensional sphere, the last integral in (3.11) vanishes; to avoid the singularity at $\lambda = 1$, we introduce a regulator $p>1$.[2] Then (3.10) and (3.11) become

$$-\Gamma\left(\frac{D}{2}\right)R_0(x) = \Gamma\left(\frac{D}{2}\right)\frac{1}{2\cdot(2\pi)^{D+1}}\int d\xi \times\left[2\sin\left(\frac{\pi D}{2}\right)\int_{-1}^{-\infty}\frac{d\lambda\,|\lambda|^{-D/2}}{p-\lambda} - i\int_\pi^{-\pi}\frac{d\theta\,e^{i(1-D/2)\theta}}{p-e^{i\theta}}\right], \qquad (3.12)$$

$$-\Gamma\left(\frac{D-1}{2}\right)R_1(x)$$
$$= -\Gamma\left(\frac{D-1}{2}\right)\frac{1}{(2\pi)^{D+1}}\int d\xi$$
$$\times B\cdot\xi\left[-2i\sin\left(\frac{\pi(1-D)}{2}\right)\int_{-1}^{-\infty}\frac{d\lambda\,|\lambda|^{(1-D)/2}}{(p-\lambda)^2}\right.$$
$$\left.+\,i\int_{\pi}^{-\pi}\frac{d\theta\, e^{(i/2)(3-D)\theta}}{(p-e^{i\theta})^2}\right]. \qquad (3.13)$$

In (3.12), (3.13), and the subsequent formulas, the integrations over the ξ's are constrained to the unit sphere $\|\xi\|\equiv\sqrt{g_{\mu\nu}(x)\xi^\mu\xi^\nu}=1$.

In dimension $D=4$, making the change of variables $p^{-1/2}e^{i\theta/2}=e^{i\phi}$, the integrations over λ and θ may be performed. The results, after suppression of the regularization, are

$$-\Gamma(2)R_0(x)=\frac{1}{2(2\pi)^4}\int d\xi \qquad (3.14)$$

and

$$-\Gamma\left(\frac{3}{2}\right)R_1(x) = -\Gamma\left(\frac{3}{2}\right)\frac{3i}{2(2\pi)^4}\int d\xi\, B\cdot\xi. \qquad (3.15)$$

Analogously, in dimension $D=2$, the coefficients of the two first powers of the second term in (1.2) (powers t^{-1} and $t^{-1/2}$, respectively) are obtained from (3.12) and (3.13):

$$-\Gamma(1)R_0(x)=\frac{1}{2(2\pi)^2}\int d\xi, \qquad (3.16)$$

$$-\Gamma\left(\frac{1}{2}\right)R_1(x) = -\Gamma\left(\frac{1}{2}\right)\frac{i}{2(2\pi)^2}\int d\xi\, B\cdot\xi. \qquad (3.17)$$

As an example, we calculate the coefficients (3.16) and (3.17) in the Penrose compactified two-dimensional Minkowski space,[5] which has the metric

$$\bar{g}_{\mu\nu}=\tfrac{1}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.$$

In this case the unit sphere $\|\xi\|=1$ is the section of hyperbola depicted in Fig. 1. Using polar coordinates (r,θ) and the well-known formula for the induced metric on a $(D-1)$-dimensional surface embedded in D-dimensional metric space, it is easy to see that the integration on the "surface" $\|\xi\|=1$ reduces simply to integration over θ between the limits θ_1, θ_2 and $\theta_1+\pi$, $\theta_2+\pi$:

$$\int_{\|\xi\|=1}d\xi=\int_{\theta_1}^{\theta_2}d\theta+\int_{\theta_1+\pi}^{\theta_2+\pi}d\theta,$$

with

$$\theta_1=\arctan(1/\pi^2), \qquad (3.18a)$$
$$\theta_2=\arctan\pi^2. \qquad (3.18b)$$

We obtain

$$-\Gamma(1)R_0(x)=[1/(2\pi)^2](\theta_2-\theta_1), \qquad (3.19)$$

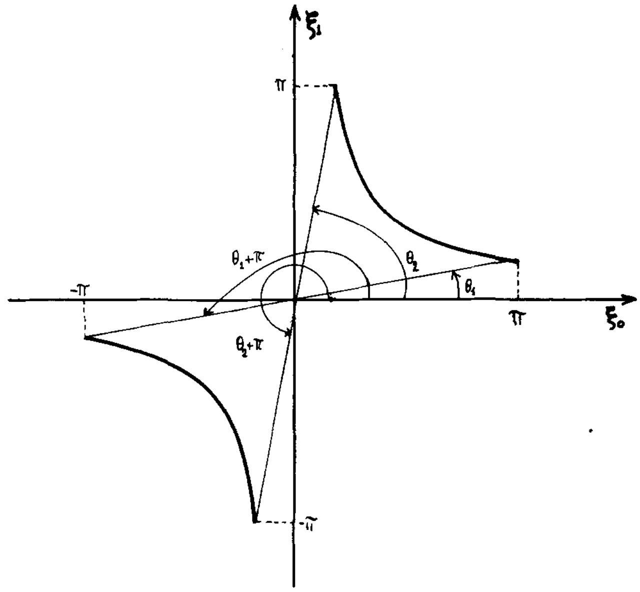

FIG. 1. We show ξ variables submitted to the constraint $g_{\mu\nu}\xi^\mu\xi^\nu=1$ in the Penrose compactified two-dimensional Minkowski space.[5]

$$-\Gamma\left(\frac{1}{2}\right)R_1(x)$$
$$= -\Gamma\left(\frac{1}{2}\right)\frac{i}{(2\pi)^2}\sqrt{2}\left\{\left(\frac{1+i}{2}\right)\Pi\left(\alpha,\frac{1+2}{2},\frac{1}{\sqrt{2}}\right)\right.$$
$$\left.+F\left(\alpha,\frac{1}{\sqrt{2}}\right)-2E\left(\alpha,\frac{1}{\sqrt{2}}\right)\right\}\Bigg|_{\theta_1}^{\theta_2}$$
$$+(\theta_1\to\theta_1+\pi,\theta_2\to\theta_2+\pi), \qquad (3.20)$$

where Π and E are the elliptic integrals of the third and second kinds, respectively; F is the generalized hypergeometric series.

IV. ANOMALIES

In this section we apply expansion (1.2) to study anomalies using the heat kernel method.[6] We borrow some of the notation and methods employed in a recent work by Cognola and Zerbini[7] since they are suitable for our purposes. Using the generalized zeta-function regularization, the anomaly may be written in the form

$$\mathrm{A}=-q\lim_{s\to 0}\mathrm{Tr}\left\{(X+Y)\frac{1}{\Gamma(s)}\int_0^1 dt\,t^{s-1}\right.$$
$$\left.\times\Big[F(t;x,x)-P_0(x,x)\Big]\right\}, \qquad (4.1)$$

where $q=-1,\frac{1}{2}$, or 1 for fermions, neutral or charged bosons, respectively; $X=X_1+X_2$ and $Y=Y_1+Y_2$ are operators satisfying the relation $\delta K(J)=(\delta JX_1+Y_1\delta J)K+K(Y_2\delta J+\delta JX_2)$ and $K(J)$ is such that $H(J)\propto K(J)$ for bosons and $H(J)=K^2(J)$ for fermions, where J is a classical source. Here P_0 is the projector onto the zero modes. For the axial anomaly, $X=Y=i\gamma_5$.

In (4.1) we replace $F(t;x,x)$ by (1.2); after some simple manipulations we see that the sole contribution to the anomaly comes from the coefficient of the power t^0, giving, for arbitrary dimension D,

$$\mathrm{A} = -q\,\mathrm{Tr}\left\{(X+Y)\left[-\left(\frac{d\phi}{ds}\right)\bigg|_{s=0} - P_0(x,x)\right]\right\}. \tag{4.2}$$

Now, from (1.4) and the formula

$$\Gamma(z) = \frac{\Gamma(z+l+1)}{z+l}\prod_{n=1}^{l}\frac{1}{z+l-n},$$

we have, for integer $l \geqslant 0$,

$$\frac{d\phi}{ds}\bigg|_{s=l} = -2\frac{(-1)^l}{l!}K(l;x,x), \tag{4.3}$$

where the Seeley's kernel for integer l is[2]

$$K(l;x,x) = \frac{1}{(-1)^l 2(2\pi)^D}\int d\xi \int_0^\infty dt \times t^l b_{-2-2l-D}(x,\xi,te^{i\theta}). \tag{4.4}$$

Thus taking $\arg\lambda = \theta = \pi$ in (4.4) the anomaly may be obtained for *arbitrary* dimension D from (4.2), with

$$\frac{d\phi}{ds}\bigg|_{s=0} = \frac{-1}{(2\pi)^D}\int d\xi \int_0^\infty dt\, b_{-2-D}(x,\xi,-t). \tag{4.5}$$

Next we apply (4.2) to the cases $D=2$ and $D=3$. The case $D=3$ is particularly interesting since, in spite of the well-known difficulties in defining the matrix γ_5 in odd dimension,[8] certain aspects of even-dimensional axial anomaly could appear in odd-dimensional field theories (see Niemi and Semenoff[9] and the references therein). This results from the fact that the connection between zero modes of Dirac operators and nontriviality of the background field topology is valid for any value of D, as shown by Callias.[10]

Moreover, there is a technical difficulty to (formally) calculating anomalies in odd dimension using the de Witt ansatz in the heat kernel method which is not present with our expansion: When one uses the de Witt ansatz for expanding $F(t)$, the anomaly depends on the coefficient of the power $t^{D/2}$, which does not exist for odd values of D, while with our expansion the anomaly depends directly on the coefficient of the zeroth power of t, given by (4.5), for any even or odd dimension.

Calculations for a general coordinate-dependent metric are extremely involved. Here, we restrict ourselves to the simpler situation of a symmetric, coordinate-independent metric tensor $g_{\mu\nu}$. In this case we obtain

for $D=2$:

$$\mathrm{A}_2 = \frac{q}{(2\pi)^2}\mathrm{Tr}\left\{(X+Y)\int d\xi\left[4\xi^\mu(\partial_\mu B_\nu)\xi^\nu + ig_{\mu\nu}(\partial^\mu B^\nu - B^\mu B^\nu) + P(x) + 2(B\cdot\xi)^2\right]\right\}, \tag{4.6}$$

for $D=3$:

$$\mathrm{A}_3 = \frac{q}{(2\pi)^3}\mathrm{Tr}\left\{(X+Y)\int d\xi\left[-g^{\mu\nu}(\partial_\mu\partial_\nu B^\sigma)\xi_\sigma + 2i(B\cdot\xi)(g_{\mu\nu}(\partial^\mu B^\nu - B^\mu B^\nu) + P(x)) - 2iB^\mu(\partial_\mu B^\nu)\xi_\nu + i\xi^\mu g_{\rho\sigma}(\partial_\mu\partial^\rho B^\sigma + B^\rho\partial_\mu B^\sigma + (\partial_\mu B^\rho)B^\sigma) + i\xi^\mu\partial_\mu P(x) - \frac{8}{3}(B\cdot\xi)^3 - \frac{4}{3}\xi^\mu\xi_\sigma(B\cdot\xi)(\partial_\mu B^\sigma) - \frac{16}{3}i\xi^\mu\xi^\nu\xi_\sigma(\partial_\mu\partial_\nu B^\sigma)\right]\right\}. \tag{4.7}$$

In the Penrose compactified two-dimensional Minkowski space,[5] (4.6) gives the result

$$\mathrm{A}_2 = \frac{q}{(2\pi)^2}\mathrm{Tr}\left\{(X+Y)\left[(\theta_2-\theta_1)\left((1/2+i)\times\left(\frac{\partial B_1}{\partial x^0} - \frac{\partial B_0}{\partial x^1}\right) + (2-i)B_0B_1 + P(x)\right) + \tfrac{1}{2}\ln\frac{\sin\theta_2}{\sin\theta_1}\left(\frac{B_1^2}{2} + \frac{\partial B_1}{\partial x^1}\right) - \tfrac{1}{2}\ln\frac{\cos\theta_2}{\cos\theta_1}\left(\frac{B_0^2}{2} + \frac{\partial B_0}{\partial x^0}\right)\right]\right\},$$

where the angles θ_1, θ_2 are given by (3.18a) and (3.18b) and B_0, B_1 are the components of $B_\mu(x)$ given by (3.2).

[1] A. P. C. Malbouisson, M. A. R. Monteiro, and F. R. A. Simão, to be published in J. Math Phys.
[2] R. T. Seeley, Am. Math. Soc. Proc. Symp. Pure Math. **10**, 288 (1967).
[3] See B. de Witt, *The Dynamical Theory of Groups and Fields* (Gordon and Breach, New York, 1965); see, also, Birrel and Davies in Ref. 5.
[4] S. W. Hawking, Commun. Math. Phys. **55**, 133 (1977).
[5] R. Penrose, in *Relativity, Groups and Topology*, edited by B. de Witt and C. de Witt (Gordon and Breach, New York, 1964); N. D. Birrel and P. C. W. Davies, *Quantum Fields in Curved Spaces* (Cambridge U.P., Cambridge, 1984).
[6] K. Fujikawa, Phys. Rev. Lett. **42**, 1195 (1979); Phys. Rev. D **21**, 2848 (1980); A. M. Polyakov and P. B. Wilyman, Phys. Lett. B **131**, 121 (1983); O. Alvarez; Nucl. Phys. B **238**, 61 (1984); R. E. Gamboa Saravi, F. A. Schaposnik, and J. E. Solomin, Phys. Rev. D **30**, 1353 (1984); L. C. L. Botelho and M. A. do Rego Monteiro, *ibid.* **30**, 2242 (1984); A. P. Balachandran, G. Marmo, Y. P. Nair, and C. G. Traherm, *ibid.* **25**, 2713 (1982).
[7] G. Cognola and S. Zerbini, Phys. Lett. B **195**, 435 (1988); Mod. Phys. Lett. A **3**, 599 (1988).
[8] T. L. Trueman, Phys. Lett. B **88**, 331 (1979); G. 't Hooft and M. Veltman, Nucl. Phys. B **44**, 189 (1972); G. Bonneau, *ibid.* **171**, 477 (1980).
[9] A. J. Niemi and G. W. Semenoff, Phys. Rev. Lett. **54**, (9), 873 (1985).
[10] C. Callias, Commun. Math. Phys. **62**, 213 (1978).