Flexible Solution Approach for Multistage Transmission Network Expansion Planning with Multiple Generation Scenarios

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Abstract One of the approaches to represent competitive market rules, uncertainties in the generation, and different seasonal climatic conditions in the multistage transmission network expansion planning (TNEP) problem is through scenario-based modeling, by considering several generation scenarios. However, the investment costs of the solutions obtained by this type of approach are much higher when compared to the costs of the expansion plans provided by a single-scenario formulation. To overcome this issue, this work presents a flexible solution approach based on a mixed-integer linear programming formulation, which generates a pool of high-quality solutions with reasonable investment costs and small operating infeasibilities for the problem. The proposed method is evaluated using the IEEE 24-bus system and the southern Brazilian 46-bus system. The results indicate that the proposed approach can provide solutions with acceptable investment costs and small infeasibilities in the operation of the system, which can be easily corrected in the short- and medium-term expansion planning.

Keywords Mixed-integer linear programming, multiple generation scenarios, multistage planning, optimization, transmission network expansion planning.

Nomenclature

Functions:

v	Investment cost
v'	Penalized investment cost

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Indices:

i^*	Index for the reference bus
i, j	Indices for buses
ij, ji	Indices for corridors
s	Index for the scenarios
t	Index for the stages
y	Index for a line

Sets:

Ω_b	Set of buses
Ω_c	Set of corridors
Ω_s	Set of scenarios
Ω_t	Set of stages

Constants:

$\overline{f}_{ij}$	Maximum power flow of a line on corridor ij
$\underline{g}_{i,t,s}$	Minimum generation at bus i, stage t, scenario s
$\overline{g}_{i,t,s}$	Maximum generation at bus i, stage t, scenario s
$\overline{n}_{ij}$	Number of lines that can be constructed on corridor ij
c_{ij}	Construction cost of a line on corridor ij
$d_{i,t}$	Demand at bus i, stage t
$g^*_{i,t,s}$	Scheduled generation at bus i, stage t, scenario s
n^o_{ij}	Number of existing lines on corridor ij, equal to n^o_{ji}
x_{ij}	Reactance of a line on corridor ij
$\overline{\theta}$	Maximum voltage phase angle
$\overline{\sigma}$	Maximum overload allowed for a line
τ_t	First year of stage t
I	Annual interest rate
M	Big-M factor
α	Parameter for adjusting the maximum load shedding in the system at stage t, scenario s
β	Penalization for generation displacements
γ	Penalization for lines overloads
δ	Penalization for load shedding

Continuous Variables:

$f^{o}_{ij,t,s}$ Total power flow on the existing lines of corridor ij, at stage t, scenario s

$f_{ij,y,t,s}$ Power flow on corridor ij, candidate line y, at stage t, scenario s

$g_{i,t,s}$ Generation at bus i, stage t, scenario s

$n_{ij,t}$ Number of lines constructed on corridor ij, at stage t

$r_{i,t,s}$ Artificial generation at bus i, stage t, scenario s

$\theta_{i,t,s}$ Voltage phase angle at bus i, stage t, scenario s

$\sigma_{ij,t,s}$ Overload on corridor ij, stage t, scenario s

Binary Variable:

$w_{ij,y,t}$ Binary variable that indicates if the line y is present on corridor ij at stage t

1 Introduction

The main objective of the long-term transmission network expansion planning (TNEP) problem is to find the minimal cost investment in transmission lines and transformers for a network to operate adequately in a planning horizon (Romero et al. 2002). The data available for the problem is the current topology of the network that must be expanded and reinforced, the candidate options for construction, and the future generation and demand for the planning horizon. In the multistage TNEP problem, it must also be defined the adequate time for each investment in the network (Escobar et al. 2004; Sum-Im et al. 2009).

One of the factors that determine the quality and significance of a solution obtained for the TNEP problem in the real world is the type of formulation used to represent the network operation. Several models have been proposed in the literature, with different levels of approximation, such as the transportation model, hybrid models, the DC model, and the AC model (Romero et al. 2002). More relaxed formulations, such as the transportation model, can be easily solved, even for large instances; however, the obtained solutions may be of no practical interest. On the other hand, more precise formulations, such as the AC model, lead to nonconvex mixed-integer nonlinear programming problems, which cannot be efficiently solved by the existing exact optimization methods. In this work, we use the DC model represented by the disjunctive (big-M) formulation for the TNEP problem (Bahiense et al. 2001), which is a mixed-integer linear programming problem that has a good compromise between accuracy and complexity.

Traditionally, the TNEP problem is solved considering a single generation scenario: the most probable one or the worst-case scenario. The necessity of considering several generation scenarios in the TNEP problem appears when the planned system must be robust enough to operate under different generation dispatch patterns, either due to environmental factors, such as the hydro regime for hydroelectric plants, or the intermittence of nondispatchable renewable energy sources, such as for solar and wind power plants, or due to competitive market reasons, in which the power production control is not centralized (Hemmati et al. 2013; Lumbreras and Ramos 2016; Oree et al. 2017). An alternative to dealing with uncertain parameters in the TNEP problem is by using robust optimization (Baringo and Baringo 2018; García-Bertrand and Mínguez 2017; Jabr 2013; Mínguez and García-Bertrand 2016) or fuzzy sets (da Rocha and Saraiva 2013). Two methods to estimate future scenarios for demand and wind power generation are presented in (Baringo and Conejo 2013), based on forecasted data and historical measurements, together with strategies to construct a reduced set of representative scenarios.

Besides being harder to solve than the traditional TNEP problem with a single generation scenario, due to its increased size, the main drawback of the TNEP problem with multiple generation scenarios is the high investment costs of its expansion plans. The increased costs are due to two reasons: (i) for each generation scenario, a different pattern for the power flows distribution is imposed on the network, which must have enough transmission capacity to supply the demands, and (ii) the construction of additional lines usually implies the creation of additional loops in the system, which may negatively influence the power flow distribution on other loops, due to the necessity for the solution to comply with Kirchhoff's voltage law.

In this paper, we study the multistage TNEP problem with multiple generation scenarios, proposing to find a pool of low-cost and high-quality expansion plans that operate adequately in each generation scenario.

In the literature, some works have solved the TNEP problem considering competitive market aspects and renewable generation, which are modeled in this paper, considering multiple generation scenarios. The main solution approaches consist of (i) exact methods, (ii) heuristics, and (iii) meta-heuristics. One of the first publications that solve the TNEP problem considering multiple generation scenarios is (Fang and Hill 2003). In this paper, the authors propose an approach based on the minimization of the maximum regret to select an investment plan from a pool of solutions for the TNEP problem considering competitive market operation requirements and considering each generation scenario separately.

Formulations using scenarios for representing future generation and demand values have been broadly studied in the literature. A specialized genetic algorithm is presented in (Escobar et al. 2008) to solve the single-stage TNEP problem with multiple generation scenarios to obtain a network that is robust enough to supply the demand considering every possible generation pattern, which is called *open access* by the authors. An algorithm that combines linear programming and genetic algorithms was presented in (Leou 2011) to solve the multistage TNEP problem in a deregulated market environment, in which the load levels were represented through scenarios. The authors (Rahmani et al. 2013) proposed three models for the single-stage TNEP problem considering multiple future generation and demand scenarios and a risk evaluation approach, using the Pareto front of the solutions. In (Correa et al. 2014), the authors proposed a hybrid nondominated sorting genetic algorithm II (NSGA-II)/Chu-Beasley algorithm to solve the single-stage multiobjective TNEP problem with multiple generation scenarios. The objective functions of the problem accounted for the investment and load shedding costs. A particle swarm optimization algorithm is proposed in (Kamyab et al. 2014) to solve the multistage TNEP problem in electricity markets considering future demand scenarios. The authors of (Lumbreras et al. 2017) use scenarios to formulate a model for the TNEP problem with high renewable penetration. The solution approach uses a modified version of the Benders decomposition algorithm and is applied to large systems. In (Abbasi et al. 2018), the authors present an NSGA-II algorithm to solve the TNEP problem, with load correlation as a multiobjective optimization problem with three objective functions, which include investment, congestion, and risk costs. In (Freitas et al. 2018), we have presented a model and three different strategies to solve the multistage TNEP problem, considering multiple generation scenarios with reduced investment costs, by allowing small modifications in the data of the problem. Finally, in (Freitas et al. 2019), we have proposed a method for obtaining a set of good-quality solutions for the single-stage TNEP problem with multiple generation scenarios by allowing small values of load shedding and generation displacements in the solutions of the problem.

Scenario-based stochastic optimization has also been widely used to represent the uncertainty of the parameters in the TNEP problem. In (López et al. 2007), the authors present a stochastic mixed-integer nonlinear programming model for generation and transmission expansion considering the demand, the equivalent availability of the generation plants, and the transmission capacity factor of the transmission lines as random events. The authors of (Maghouli et al. 2011) considered a scenario-based formulation to model and an NSGA-II to solve, the multistage TNEP problem considering three criteria that included the social cost, a robustness criterion, and a flexibility criterion.

The main contribution of this paper is a new flexible solution method based on a mixed-integer linear programming formulation, which generates a pool of high-quality solutions with reasonable investment costs and small operating infeasibilities for the multistage TNEP problem. The main innovations of this work in relation to the conference paper (Freitas et al. 2018) are (i) the strategies for flexibility considered in this paper are more suitable for the problem, *e.g.*, the line overloads are minimized in the problem in order to obtain better-operating conditions for the solutions; and (ii) the tests are conducted on larger instances, with more stages, and the results are discussed more deeply.

It should be highlighted that the solutions for the multistage TNEP problem must be evaluated and reinforced with other tools, *e.g.*, reactive power planning, power flow studies, short circuit analysis, and stability analyses. Thus, even solutions that are feasible to the DC model, with very high investment costs, may be infeasible when the AC operation of the network is considered.

A set of low-cost, high-quality solutions is of great interest from the practical perspective of a network planner. Since the solutions for the TNEP problem must always be analyzed and reinforced using other methods, when the planner has a pool of high-quality solutions for the problem, some low-cost, good-quality solutions may perform better overall than the optimal solution for the problem (without any infeasibility in the operation) and may be a better option for construction.

The proposed approach was implemented in the mathematical modeling language AMPL (Fourer et al. 2003) and solved with the commercial solver CPLEX (IBM 2018). The IEEE 24-bus system and the southern Brazilian 46-bus system are used to demonstrate the effectiveness of the method. In each case, it was possible to obtain a set of high-quality solutions with lower investment costs when compared to the results for the traditional multistage TNEP model with multiple generation scenarios.

The rest of the paper is structured as follows. Section 2 presents the traditional mathematical formulation for multistage TNEP with multiple generation scenarios. In Section 3, we present the flexible formulation for the problem; the tests and results, as well as an analysis of the results, are presented in Section 4. The main conclusions are presented in Section 5.

2 Traditional Model for Multistage TNEP with Multiple Generation Scenarios

The traditional linear disjunctive (big-M) model for the multistage DC TNEP problem with multiple generation scenarios is presented in (1)–(12), based on the formulations presented in (Bahiense et al. 2001; Escobar et al. 2004; Freitas et al. 2018).

$$\begin{aligned} \text{minimize}\, v = &\sum_{ij\in\Omega_c}\sum_{y=1}^{\overline{n}_{ij}} c_{ij} w_{ij,y,} \\ &+ \sum_{ij\in\Omega_c}\sum_{y=1}^{\overline{n}_{ij}}\sum_{t\in\Omega_t|t>1} \frac{1}{(1+I)^{\tau_t}} c_{ij}(w_{ij,y,t} - w_{ij,y,t-1}) \end{aligned} \tag{1}$$

subject to

$$\begin{aligned} &\sum_{ji\in\Omega_c}\left(f^o_{ji,t,s} + \sum_{y=1}^{\overline{n}_{ji}} f_{ji,y,t,s}\right) \\ &\qquad - \sum_{ij\in\Omega_c}\left(f^o_{ij,t,s} + \sum_{y=1}^{\overline{n}_{ij}} f_{ij,y,t,s}\right) + \ g^*_{i,t,s} = d_{i,t} \\ &\qquad \forall i \in \Omega_b, t\in\Omega_t, s\in\Omega_s \end{aligned} \tag{2}$$

$$f^o_{ij,t,s} = n^o_{ij}\frac{(\theta_{i,t,s} - \theta_{j,t,s})}{x_{ij}} \qquad \forall ij\in\Omega_c, t\in\Omega_t, s\in\Omega_s \tag{3}$$

$$|f^o_{ij,t,s}| \le n^o_{ij}\overline{f}_{ij} \qquad \forall ij\in\Omega_c, t\in\Omega_t, s\in\Omega_s \tag{4}$$

$$\begin{aligned} &|x_{ij} f_{ij,y,t,s} - (\theta_{i,t,s} - \theta_{j,t,s})| \le x_{ij}\overline{f}_{ij}(1 - w_{ij,y,t}) \\ &\qquad \forall ij\in\Omega_c | n^o_{ij} > 0, y\in\{1,\dots,\overline{n}_{ij}\}, t\in\Omega_t, s\in\Omega_s \end{aligned} \tag{5}$$

$$|x_{ij}f_{ij,y,t,s} - (\theta_{i,t,s} - \theta_{j,t,s})| \le M(1 - w_{ij,y,t}) \quad (6)$$
$$\forall ij \in \Omega_c | n^o_{ij} = 0, y \in \{1, \dots, \overline{n}_{ij}\}, t \in \Omega_t, s \in \Omega_s$$

$$|f_{ij,y,t,s}| \le w_{ij,y,t}\overline{f}_{ij} \quad (7)$$
$$\forall ij \in \Omega_c, y \in \{1, \dots, \overline{n}_{ij}\}, t \in \Omega_t, s \in \Omega_s$$

$$|\theta_{i,t,s}| \le \overline{\theta} \qquad \forall i \in \Omega_b, t \in \Omega_t, s \in \Omega_s \quad (8)$$

$$w_{ij,y,t} \le w_{ij,y-1,t} \qquad \forall ij \in \Omega_c, y \in \{2, \dots, \overline{n}_{ij}\}, t \in \Omega_t \quad (9)$$

$$w_{ij,y,t-1} \le w_{ij,y,t} \; \forall ij \in \Omega_c, y \in \{1, \dots, \overline{n}_{ij}\}, t \in \Omega_t | t > 1 \quad (10)$$

$$\theta_{i^*,t,s} = 0 \qquad \forall t \in \Omega_t, s \in \Omega_s \quad (11)$$

$$w_{ij,y,t} \in \{0,1\} \qquad \forall ij \in \Omega_c, y \in \{1, \dots, \overline{n}_{ij}\}, t \in \Omega_t \quad (12)$$

The sets, parameters, and variables of the model (1)–(12) are defined in the Nomenclature. The binary decision variable $w_{ij,y,t}$ is related to the presence of line y on corridor ij at stage t (if $w_{ij,y,} = 1$, the line y is present on corridor ij at stage t; otherwise, the line is not present). Note that $w_{ij,y,t}$ does not indicate when a line was constructed, but whether the line is operating at a certain stage on a corridor or not. The number of lines constructed on corridor ij, at stage t, can be calculated as $n_{ij,1} = \sum_{y=1}^{\overline{n}_{ij}} w_{ij,y,1}$ and $n_{ij,t} = \sum_{y=1}^{\overline{n}_{ij}} (w_{ij,y,t} - w_{ij,y,t-1})$, for $t > 1$.

The objective function v in (1) minimizes the investment cost in the system. Its first term is related to the investment in the first stage, while the second term is the present value of the investments performed from the second stage on, in which $(1 + I)^{-\tau_t}$ is the present value factor.

Constraint (2) is the power balance at each bus i, stage t, scenario s, and it represents the application of Kirchhoff's current law to the problem. Constraint (3) represents the application of Kirchhoff's voltage law to the base topology of the network for each stage t and scenario s. Constraint (4) is the power flow limit for the existing lines of the base topology for each stage t and scenario s.

Constraints (5)–(7) represent the application of Kirchhoff's voltage law to the candidate network for each stage t and scenario s, in which (5) is defined for the corridors ij with at least one existing line ($n^o_{ij} > 0$) and (6) is defined for the corridors ij with no existing line ($n^o_{ij} = 0$). Note that when $w_{ij,y,} = 0$, in (7) $f_{ij,y,t,s} = 0$, and in (5) and (6) $\theta_{i,t,s}$ and $\theta_{j,t,s}$ are independent. Moreover, for corridors with existing lines, the maximum value for $\theta_{i,t,s} - \theta_{j,t,s}$ is $x_{ij}\overline{f}_{ij}$, as presented in (5), while for corridors without any line, M should be sufficiently large to ensure that $\theta_{i,t,s}$ and $\theta_{j,t,s}$ are not related. On the other hand, when $w_{ij,y,t} = 1$, the power flow is limited in (7), $|f_{ij,y,t,s}| \le \overline{f}_{ij}$, and in (5) and (6), $f_{ij,y,t,s} = (\theta_{i,t,s} - \theta_{j,t,s})/x_{ij}$.

Constraint (8) imposes a maximum voltage phase angle to each bus i of the system at stage t, scenario s. Constraint (9) is used to break the symmetry of the model and requires the sequential construction of new lines in each corridor ij at each stage t, *i.e.*, at stage t, line y can be constructed only if line $y - 1$ was already constructed. Constraint (10) ensures that if line y is present on corridor ij at stage $t - 1$, it must be present on the system on stage t, *i.e.*, a line that was installed cannot be removed from one stage to the subsequent one. Constraint (11) imposes the angular reference to the system at each stage t and scenario s. Constraint (12) indicates that the decision variable related to the existence of line y in corridor ij at stage t must be binary.

The main advantage of using the big-M formulation for the DC TNEP problem is that existing MILP solvers can ensure the optimality of the solution. Also, the optimal solution for the big-M formulation is also the optimal solution for the mixed-integer nonlinear programming DC model for TNEP, which is very hard to solve (Romero et al. 2002). However, the big-M formulation has a higher number of variables and constraints in comparison to the mixed-integer nonlinear programming DC model. Also, the parameter M needs to be adequately calculated to improve the convergence and stability of the model.

The transportation model can be obtained by not considering (3), (5), and (6) in (1)–(12), while the hybrid linear model can be obtained by not considering (5) and (6) in (1)–(12) (Romero et al. 2002). The investment costs of the solutions provided by these models are lower bounds for the investment cost of the solution for the DC model.

The next section introduces a new formulation for multistage TNEP with multiple generation scenarios, considering several strategies to reduce the high investment costs that appear in this problem. The approach will provide a pool of good-quality solutions with small infeasibilities, which can be corrected in the medium and short-term planning of the system, with, for example, flexible alternating current transmission systems (FACTS).

3 Flexible Formulation for Multistage TNEP with Multiple Generation Scenarios

The new mathematical model for flexible multistage DC TNEP with multiple generation scenarios, based on the previous disjunctive (big-M) formulation, is presented in (13)–(29).

$$\text{minimize } v' = v + \delta \sum_{i \in \Omega_b} \sum_{t \in \Omega_t} \sum_{s \in \Omega_s} r_{i,t,s} + \beta \sum_{i \in \Omega_b} \sum_{t \in \Omega_t} \sum_{s \in \Omega_s} |g_{i,t,s} - g^*_{i,t,s}| + \gamma \sum_{ij \in \Omega_c} \sum_{t \in \Omega_t} \sum_{s \in \Omega_s} \sigma_{ij,t,s} \quad (13)$$

subject to

$$\sum_{ji\in\Omega_c}\left(f^o_{ji,t,s}+\sum_{y=1}^{\overline{n}_{ji}}f_{ji,y,t,s}\right) - \sum_{ij\in\Omega_c}\left(f^o_{ij,t,s}+\sum_{y=1}^{\overline{n}_{ij}}f_{ij,y,t,s}\right)+g_{i,t,s}+r_{i,t,s}=d_{i,t} \quad \forall i\in\Omega_b, t\in\Omega_t, s\in\Omega_s \tag{14}$$

$$f^o_{ij,t,s}=n^o_{ij}\frac{(\theta_{i,t,s}-\theta_{j,t,s})}{x_{ij}} \qquad \forall ij\in\Omega_c, t\in\Omega_t, s\in\Omega_s \tag{15}$$

$$|f^o_{ij,t,s}|\le n^o_{ij}\overline{f}_{ij}(1+\sigma_{ij,t,s}) \qquad \forall ij\in\Omega_c, t\in\Omega_t, s\in\Omega_s \tag{16}$$

$$|x_{ij}f_{ij,y,t,s}-(\theta_{i,t,s}-\theta_{j,t,s})|\le x_{ij}(1+\overline{\sigma})\overline{f}_{ij}(1-w_{ij,y,t}) \quad \forall ij\in\Omega_c|n^o_{ij}>0, y\in\{1,\dots,\overline{n}_{ij}\}, t\in\Omega_t, s\in\Omega_s \tag{17}$$

$$|x_{ij}f_{ij,y,t,s}-(\theta_{i,t,s}-\theta_{j,t,s})|\le M(1-w_{ij,y,t}) \quad \forall ij\in\Omega_c|n^o_{ij}=0, y\in\{1,\dots,\overline{n}_{ij}\}, t\in\Omega_t, s\in\Omega_s \tag{18}$$

$$|f_{ij,y,t,s}|\le \overline{f}_{ij}(1+\sigma_{ij,t,s}) \tag{19}$$

$$|f_{ij,y,t,s}|\le w_{ij,y,t}(1+\overline{\sigma})\overline{f}_{ij} \tag{20}$$

$$\forall ij\in\Omega_c, y\in\{1,\dots,\overline{n}_{ij}\}, t\in\Omega_t, s\in\Omega_s$$

$$|\theta_{i,t,s}|\le\overline{\theta} \qquad \forall i\in\Omega_b, t\in\Omega_t, s\in\Omega_s \tag{21}$$

$$w_{ij,y,t}\le w_{ij,y-1,t} \qquad \forall ij\in\Omega_c, y\in\{2,\dots,\overline{n}_{ij}\}, t\in\Omega_t \tag{22}$$

$$w_{ij,y,t-1}\le w_{ij,y,t} \quad \forall ij\in\Omega_c, y\in\{1,\dots,\overline{n}_{ij}\}, t\in\Omega_t|t>1 \tag{23}$$

$$0\le r_{i,t,s}\le d_{i,t} \qquad \forall i\in\Omega_b, t\in\Omega_t, s\in\Omega_s \tag{24}$$

$$\underline{g}_{i,t,s}\le g_{i,t,s}\le\overline{g}_{i,t,s} \qquad \forall i\in\Omega_b, t\in\Omega_t, s\in\Omega_s \tag{25}$$

$$0\le\sigma_{ij,t,s}\le\overline{\sigma} \qquad \forall ij\in\Omega_c, t\in\Omega_t, s\in\Omega_s \tag{26}$$

$$\sum_{i\in\Omega_b}r_{i,t,s}\le(1-\alpha)\sum_{i\in\Omega_b}d_{i,t} \qquad \forall t\in\Omega_t, s\in\Omega_s \tag{27}$$

$$\theta_{i^*,t,s}=0 \qquad \forall t\in\Omega_t, s\in\Omega_s \tag{28}$$

$$w_{ij,y,t}\in\{0,1\} \qquad \forall ij\in\Omega_c, y\in\{1,\dots,\overline{n}_{ij}\}, t\in\Omega_t \tag{29}$$

The sets, parameters, and variables of the model (13)–(29) are defined in the Nomenclature. Besides the investment cost v, the objective function v' in (13) minimizes the total artificial generation in the system (second term), the generation displacements (third term), and the lines overloads (fourth term). Note that each of these terms is weighted by a factor, and by changing these factors, it is possible to obtain different solutions for the TNEP problem. Also, it is possible to fix these parameters with small values and change the values of α and $\overline{\sigma}$. Some of these solutions will present small infeasibilities with respect to the original formulation (1)–(12), but the investment cost will be drastically reduced in relation to the optimal solution of model (1)–(12). These infeasibilities can be fixed in the medium- and short-term planning of the network.

Constraint (14) is the application of Kirchhoff's current law to the problem. It is similar to (2), but instead of the scheduled generation $g^*_{i,t,s}$, it includes the dispatchable generation $g_{i,t,s}$, besides the artificial generation $r_{i,t,s}$. Constraint (15) is similar to (3), which represents the application of Kirchhoff's voltage law to the base topology of the network. Constraint (16) represents the power flow limit for the existing network shown in (4), modified to allow small overloads in the system. Constraints (17) and (18) represent the application of Kirchhoff's voltage law to the candidate network, similar to (5) and (6), modified to consider the lines overloads. Constraints (19) and (20) impose the power flow limit to the candidate lines as constraint (7), but considering the possibility of allowing a small overload in each line. Note that when $w_{ij,y,t}=0$, in (20), $f_{ij,y,t,s}=0$, and when $w_{ij,y,t}=1$, the capacity of line y in corridor ij, at stage t and scenario s is limited by (19). Constraints (21)–(23) are the same as (8)–(10).

The new constraints (24)–(27) impose the limits on the flexibility of the solutions provided by the model. Constraint (24) limits the artificial generation at node i, stage t, and scenario s to the corresponding demand of the node. Constraint (25) limits the generation at node i, stage t, and scenario s to the interval $\left[\underline{g}_{i,t,s},\overline{g}_{i,t,s}\right]$, around the scheduled value $g^*_{i,t,s}$, *i.e.*, $\underline{g}_{i,t,s}\le g^*_{i,t,s}\le\overline{g}_{i,t,s}$. Constraint (26) limits the overload on corridor ij, stage t, scenario s, $\sigma_{ij,t,s}$, to the maximum value $\overline{\sigma}$. Constraint (27) limits the total artificial generation (load shedding) in the system at each stage t and scenario s. Note that when $\alpha=1$, no load shedding is allowed, while when $\alpha=0$, all the load can be curtailed. Finally, constraints (28) and (29) are the same as (11) and (12).

As discussed, the proposed approach has three different strategies to allow flexibility in the multistage TNEP with multiple generation scenarios: (i) to allow a small artificial generation (that represents a load shedding) on the load nodes, which is included in the power balance equation (14) and limited at each node by (24), while the total artificial generation in the system, at each stage and each scenario, is limited in (27); (ii) to allow small generation displacements in relation to the scheduled values in (14), limited by (25); and (iii) to allow small overloads on the lines, considered in the power flow limit constraints (16), (19), and (20). Therefore, by fixing $\alpha=1$ in (27), $\underline{g}_{i,t,s}=\overline{g}_{i,t,s}=g^*_{i,t,s}$ in (25) $\forall i\in\Omega_b, t\in\Omega_t, s\in\Omega_s$, and $\overline{\sigma}=0$ in (17), (20), and (26), the solution of model (13)–(29) will be the same as the solution of the traditional model (1)–(12).

It is possible to obtain a pool of high-quality low-investment cost solutions by fixing δ, β, and γ in small positive values so that they do not compete with the investment cost in the system, and adjusting α close to one. The values for $\underline{g}_{i,t,s}$ and $\overline{g}_{i,t,s}$ must be close to $g^*_{i,t,s}$, while $\overline{\sigma}$ must be close to zero.

Table 1: Generation data for the IEEE 24-bus system

Bus i	Stage 1					Stage 2					Stage 3				
	$g^*_{i,1,1}$ [MW]	$g^*_{i,1,2}$ [MW]	$g^*_{i,1,3}$ [MW]	$g^*_{i,1,4}$ [MW]	Capacity [MW]	$g^*_{i,2,1}$ [MW]	$g^*_{i,2,2}$ [MW]	$g^*_{i,2,3}$ [MW]	$g^*_{i,2,4}$ [MW]	Capacity [MW]	$g^*_{i,3,1}$ [MW]	$g^*_{i,3,2}$ [MW]	$g^*_{i,3,3}$ [MW]	$g^*_{i,3,4}$ [MW]	Capacity [MW]
1	219	113	236	87	300	576	465	576	520	600	576	465	576	520	600
2	272	312	116	210	400	576	576	576	520	600	576	576	576	520	600
7	522	288	634	623	700	900	722	900	812	900	900	722	900	812	900
13	817	718	731	634	900	1210	1100	1097	1134	1300	1773	1424	1457	1599	1800
15	317	275	118	321	400	317	275	118	321	400	645	645	325	581	700
16	465	465	282	419	500	465	465	282	419	500	465	465	282	419	500
18	610	435	219	413	700	610	435	219	413	700	1200	1200	603	718	1200
21	831	809	951	1077	1200	831	809	951	1077	1200	1200	1200	951	1077	1200
22	900	900	900	900	900	900	900	900	900	900	900	900	900	900	900
23	315	953	1081	584	1100	315	953	1081	584	1100	315	953	1980	1404	2000

Table 2: Demand data for the IEEE 24-bus system

Bus i	$d_{i,1}$ [MW]	$d_{i,2}$ [MW]	$d_{i,3}$ [MW]
1	200	254	324
2	179	228	291
3	333	423	540
4	137	174	222
5	131	167	213
6	251	320	408
7	231	294	375
8	316	402	513
9	323	411	525
10	360	458	585
13	490	623	795
14	359	456	582
15	586	745	951
16	185	235	300
18	616	783	999
19	334	425	543
20	237	302	384

Table 3: Solution of the DC model for the IEEE 24-bus system

Stage	Constructed Lines	Cost [MUS$]	Total Cost [MUS$]
1	(3-24)×1, (6-10)×1, (7-8)×2, (14-16)×1, (16-17)×1, (16-19)×1	220.0000	355.4026
2	(1-5)×1, (7-8)×1	23.5950	
3	(10-11)×1, (15-24)×1, (16-17)×1, (17-18)×2, (20-23)×1, (13-14)×1	111.8076	

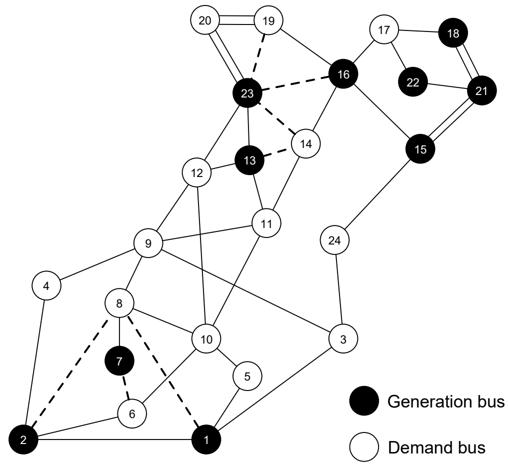

Figure 1: Topology of the IEEE 24-bus system

4 Tests and Results

To evaluate the effectiveness of the proposed approach, the IEEE 24-bus system (with 41 corridors) and the southern Brazilian 46-bus system (with 79 corridors) were used, considering three investment stages with four generation scenarios in each stage. The proposed approach was implemented in AMPL (Fourer et al. 2003) and solved using the CPLEX (IBM 2018) solver version 12.9 with default settings. A computer with a 3.2 GHz Intel® Core™ i7-8700 processor with 32 GB of RAM was used in the tests.

4.1 IEEE 24-bus system

The topology of the IEEE 24-bus system is shown in Figure 1. The branch data for this system is presented in (Freitas et al. 2019), while Table 1 presents the generation data and Table 2 presents the demand data for the three stages of the planning. In Table 2, the demands at buses 11, 12, 17, 21, 22, 23, and 24 are zero in all three stages. Complete data for this system is also available in ("LaPSEE Power System Test Cases Repository" 2019). The total demand of the system is 5268 MW on stage 1, 6700 MW on stage 2, and 8550 MW on stage 3. The installed generation capacity is 7100 MW on stage 1, 8200 MW on stage 2, and 10400 MW on stage 3. The maximum number of new lines that can be constructed on a corridor is three. The annual interest rate is $I = 10\%$. The first year for stage 1 is $\tau_1 = 0$, for stage 2 is $\tau_2 = 5$, and for stage 3 is $\tau_3 = 10$.

The first test is conducted considering the traditional formulation, (1)–(12), for solving the multistage DC TNEP problem with multiple generation scenarios. Table 3 shows the plan obtained. The computational time to solve the model is 102.38 s.

Five cases are considered for the IEEE 24-bus system using the proposed formulation, (13)–(29), to solve the multistage TNEP problem with multiple generation scenarios with flexibility for reduced investment costs:

i. *Case A1*: In this case, only load shedding is considered. The maximum value for the total load shedding in the system at any stage and any scenario is 1%. This is obtained by setting $\alpha = 0.99$ in the model (13)–(29). The cost for the load shedding is small, $\delta = 0.001$ MUS$/MW, in order for the load shedding to be minimized, but does not compete with the investment cost. The parameters β and γ are set to zero, as well as $\overline{\sigma} = 0$. Since load shedding is allowed in the system, the generation values $g_{i,t,s}$ will deviate from the ideal scheduled values $g^*_{i,t,s}$; however, they are only allowed to deviate to lower values. This is achieved by setting $\overline{g}_{i,t,s} = g^*_{i,t,s}$ and $\underline{g}_{i,t,s} = 0$ MW.

Table 4: Flexible solutions for the DC model for the IEEE 24-bus system

Plan	Stage	Constructed Lines	Cost [MUS$]	Total Cost [MUS$]
A1	1	(6-10)×1, (7-8)×2, (14-16)×1, (16-17)×1, (16-19)×1	170.0000	
	2	(7-8)×1, (10-12)×1	40.9808	291.1738
	3	(3-24)×1, (15-24)×1, (16-17)×1, (17-18)×1, (20-23)×1	80.1930	
A2	1	(3-24)×1, (6-10)×1, (7-8)×2, (14-16)×1, (16-17)×1	188.0000	
	2	(1-5)×1, (7-8)×1	23.5950	304.1254
	3	(10-12)×1, (15-24)×1, (16-17)×1, (16-19)×1, (17-18)×1, (20-23)×1	92.5304	
A3	1	(3-24)×1, (6-10)×1, (7-8)×2, (14-16)×1, (16-17)×1, (16-19)×1	220.0000	
	2	(1-5)×1, (7-8)×1	23.5950	323.7880
	3	(10-12)×1, (15-24)×1, (16-17)×1, (17-18)×1, (20-23)×1	80.1930	
A4	1	(6-10)×1, (7-8)×2, (14-16)×1, (16-17)×1	138.0000	
	2	(7-8)×1, (10-12)×1	40.9808	271.5112
	3	(3-24)×1, (15-24)×1, (16-17)×1, (16-19)×1, (17-18)×1, (20-23)×1	92.5304	
A5	1	(6-10)×1, (7-8)×2, (14-16)×1, (16-17)×1	138.0000	
	2	(1-5)×1, (7-8)×1	23.5950	254.1254
	3	(3-24)×1, (15-24)×1, (16-17)×1, (16-19)×1, (17-18)×1, (20-23)×1	92.5304	

ii. *Case A2*: Only generation displacements are considered in this case. The parameter $\beta = 0.001$ MUS$/MW, and 5% of the generation displacement is allowed. Therefore, $\underline{g}_{i,t,s} = 0.95 \times g^*_{i,t,s}$ and $\overline{g}_{i,t,s} = 1.05 \times g^*_{i,t,s}$, limited by the installed capacity at each bus in each stage. The other cost parameters, δ and γ, are set to zero, as well $\overline{\sigma}$, which is also zero, while $\alpha = 1$.

iii. *Case A3*: For this case, only lines overloads, up to a maximum of 5% ($\overline{\sigma} = 0.05$), are allowed. The cost parameters are set as $\gamma = 0.001$ MUS$/MW, while δ and β are set at zero. Also, no load shedding or generation displacement is allowed, and therefore, $\alpha = 1$, and $\underline{g}_{i,t,s} = \overline{g}_{i,t,s} = g^*_{i,t,s}$.

iv. *Case A4*: Both load shedding and generation displacements are allowed in this case. Thus, the cost parameters are set as $\delta = 0.001$ MUS$/MW and $\beta = 0.001$ MUS$/MW, with $\alpha = 0.99$ and 5% of the generation displacement limited up by the installed capacity at each bus in each stage. The cost parameter γ is zero, as well as $\overline{\sigma}$, which is also zero.

v. *Case A5*: In this case, load shedding, generation displacements, and lines overloads are allowed. The cost parameters are $\delta = 0.001$ MUS$/MW, $\beta = 0.001$ MUS$/MW, and $\gamma = 0.001$ MUS$/MW, with $\alpha = 0.99$, and 5% of the generation displacement limited up by the installed capacity at each bus in each stage. Besides that, $\overline{\sigma} = 0.05$.

Table 5: Infeasibilities of the proposed plans for the IEEE 24-bus system

Plan	Stage	Maximum Total Load Shedding in all Scenarios		Maximum Generation Displacement (Bus)		Maximum Line Overload (Branch)	
		[MW]	[%]	[MW]	[%]	[MW]	[%]
A1	1	49.1058	0.9322	–	–	–	–
	2	61.3334	0.9154	–	–	–	–
	3	79.5377	0.9303	–	–	–	–
A2	1	–	–	40.8500 (13)	5.0000	–	–
	2	–	–	0.0000	0.0000	–	–
	3	–	–	47.9610 (13)	2.7051	–	–
A3	1	–	–	–	–	0.0000	0.0000
	2	–	–	–	–	0.0000	0.0000
	3	–	–	–	–	20.5591 (12-13)	4.1118
A4	1	52.6800	1.0000	53.8500 (21)	5.0000	–	–
	2	67.0000	1.0000	54.2385 (13)	4.4825	–	–
	3	79.5488	0.9304	50.5343 (13)	2.8502	–	–
A5	1	38.7537	0.7356	23.2500 (16)	5.0000	25.0000 (16-19)	5.0000
	2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	85.5000	1.0000	79.9500 (13)	5.0000	20.0000 (10-11)	5.0000

The results for the investments in the IEEE 24-bus system using the proposed formulation, for Cases A1–A5, are shown in Table 4. All these plans have lower costs than the plan presented in Table 3. The reductions in the investment costs for plans A1–A5 are, respectively, 18.07%, 14.43%, 8.90%, 23.60%, and 28.50%.

Table 5 presents the infeasibilities of the investment plans presented in Table 4. The three metrics for infeasibility presented in Table 5 include the maximum total load shedding in the system for all the scenarios at each stage ($\max_s \sum_{i \in \Omega_b} r_{i,t,s}\,, \forall t \in \Omega_t$), the maximum generation displacement among all generation buses and all scenarios at each scenario ($\max_{i,s} |g_{i,t,s} - g^*_{i,t,s}|\,, \forall t \in \Omega_t$), and the maximum line overload considering all lines in the system and all the scenarios at each stage ($\max_{ij,} \sigma_{ij,t,s}\,, \forall t \in \Omega_t$). The values presented in Table 5 are reasonable since they can be fixed in the short- and medium-term planning. Regarding the lines overloads, for plan A3, only one line is overloaded on stage 3, scenarios 1, 2, and 4, while for plan A5, only one line is overloaded on stage 1, scenarios 1, 2, and 4, and stage 3, scenarios 1 and 4, while two lines are overloaded on stage 3, scenarios 2 and 3. It should be noted that it is possible to obtain several other solutions by changing the values of the flexibility parameters in the model.

The computational times to solve Cases A1–A5 are, respectively: 20.56 s, 21.78 s, 129.98 s, 47.67 s, and 54.55 s. It can be noted that, although the proposed formulation (13)–(29) has a larger number of constraints and variables than the traditional model (1)–(12), the computational times to solve Cases A1–A5 are lower than the times to solve the traditional model (102.38 s). This is explained by the effort of the branch and cut algorithm used by the CPLEX solver, which needs to go deeper, exploring more levels of the search tree of the problem, when more lines need to be installed in the system. An example to illustrate this argument is to solve Case A1 with $\alpha = 0$: the model is solved in 0.17 s at the root node of the search tree, no line is installed, and all the load that cannot be supplied by the base network is shedded. By increasing the value of δ, so that the load shedding costs start to compete with the investment costs, the total load shedding in the system decreases and the investment costs increase, resulting in more nodes being analyzed in the search tree and requiring higher computational times to solve the problem.

Table 6: Generation data for the southern Brazilian 46-bus system

Bus i	Stage 1					Stage 2					Stage 3				
	$g^{*}_{i,1,1}$ [MW]	$g^{*}_{i,1,2}$ [MW]	$g^{*}_{i,1,3}$ [MW]	$g^{*}_{i,1,4}$ [MW]	Capacity [MW]	$g^{*}_{i,2,1}$ [MW]	$g^{*}_{i,2,2}$ [MW]	$g^{*}_{i,2,3}$ [MW]	$g^{*}_{i,2,4}$ [MW]	Capacity [MW]	$g^{*}_{i,3,1}$ [MW]	$g^{*}_{i,3,2}$ [MW]	$g^{*}_{i,3,3}$ [MW]	$g^{*}_{i,3,4}$ [MW]	Capacity [MW]
14	884	50	600	1119	1257	884	50	600	1119	1257	935	715	1035	1200	1257
16	885	1189	339	700	1300	885	1189	339	700	1300	1685	1505	935	1420	2000
17	400	620	880	335	900	400	620	880	335	900	400	620	880	335	1050
19	471	701	181	451	950	950	810	425	580	950	1370	1200	600	900	1670
27	0	0	0	0	0	50	200	140	180	220	50	200	140	180	220
28	670	390	750	150	800	670	390	750	150	800	670	390	750	150	800
31	390	475	650	540	700	390	475	650	540	700	390	475	650	540	700
32	0	0	0	0	0	130	400	375	450	500	130	400	375	450	500
34	350	475	700	525	748	350	475	700	525	748	350	475	700	525	748
37	0	0	0	0	0	50	100	150	250	300	50	100	150	250	300
39	200	350	150	430	600	200	350	150	430	600	200	350	150	580	600
46	0	0	0	0	0	450	350	250	150	700	650	450	515	350	700

Table 7: Demand data for the southern Brazilian 46-bus system

Bus i	$d_{i,1}$ [MW]	$d_{i,2}$ [MW]	$d_{i,3}$ [MW]
2	274.0	348.0	443.1
4	185.0	236.0	300.7
5	147.0	187.0	238.0
8	44.0	56.0	72.2
12	317.0	403.0	511.9
13	114.0	146.0	185.8
20	676.0	859.0	1091.2
22	50.0	64.0	81.9
23	283.0	360.0	458.1
24	296.0	376.0	478.2
26	143.0	182.0	231.9
33	141.0	180.0	229.1
35	133.0	170.0	216.0
36	55.0	70.0	90.1
38	133.0	170.0	216.0
40	162.0	206.0	262.1
42	996.0	1266.0	1607.9
44	48.0	62.0	79.1
45	53.0	68.0	86.7

Table 8: Solution of the DC model for the southern Brazilian system

Stage	Constructed Lines	Cost [MUS$]	Total Cost [MUS$]
1	(20-23)×2, (20-21)×1, (42-43)×1, (46-6)×1, (31-32)×1, (28-31)×1, (5-6)×2	76.1240	
2	(14-22)×1, (18-19)×1, (20-21)×1, (42-43)×1	18.7804	121.7378
3	(18-20)×2, (32-41)×1, (40-41)×1	26.8334	

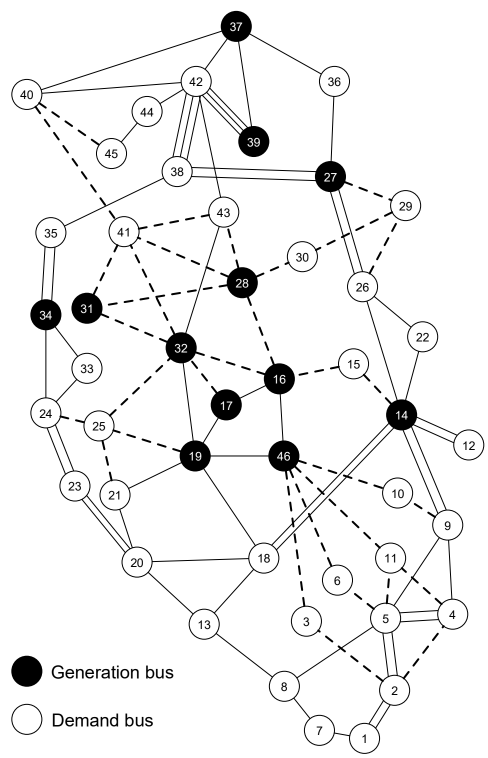

Figure 2: Topology of the southern Brazilian 46-bus system

4.2 Southern Brazilian 46-bus system

The topology of the southern Brazilian 46-bus system is shown in Figure 2. The branch data for this system is presented in (Haffner et al. 2000), while Table 6 presents the generation data and Table 7 presents the demand data for the three stages of the planning. In Table 7, the demands at buses 1, 3, 6, 7, 9, 10, 11, 14, 15, 16, 17, 18, 19, 21, 25, 27, 28, 29, 30, 31, 32, 34, 37, 39, 41, 43, and 46 are zero in all the three stages. Complete data for this system is also available in ("LaPSEE Power System Test Cases Repository" 2019).

The total demand in the system is 4250 MW on stage 1, 5409 MW on stage 2, and 6880 MW on stage 3. The installed generation capacity is 7255 MW on stage 1, 8975 MW on stage 2, and 10545 MW on stage 3. The maximum number of new lines that can be constructed on a corridor is three. The annual interest rate is $I = 10\%$. The first year for stage 1 is $\tau_1 = 0$, for stage 2 is $\tau_2 = 5$, and for stage 3 is $\tau_3 = 10$.

Table 8 shows the plan obtained for the DC model considering the traditional formulation, (1)–(12), for the multistage TNEP problem with multiple generation scenarios. The computational time to solve the model is 9973.70 s.

Again, five cases are considered for the southern Brazilian 46-bus system using the proposed formulation, (13)–(29), to solve the multistage TNEP problem with multiple generation scenarios, with flexibility for reduced investment costs:

i. *Case B1*: Only load shedding is considered in this case, with $\alpha = 0.98$ and $\delta = 0.001$ MUS$/MW;
ii. *Case B2*: Only generation displacements are considered, with $\beta = 0.001$ MUS$/MW and 10% of the generation displacement, limited up to the installed capacity at each bus in each stage;
iii. *Case B3*: Only lines overloads is considered, with $\gamma = 0.001$ MUS$/MW and $\overline{\sigma} = 0.05$;

Table 9: Flexible solutions for the DC model for the southern Brazilian 46-bus system

Plan	Stage	Constructed Lines	Cost [MUS$]	Total Cost [MUS$]
B1	1	(20-21)×1, (42-43)×1, (46-6)×1, (31-32)×1, (28-31)×1, (5-6)×2	63.5880	105.8664
	2	(20-23)×1, (18-19)×1, (42-43)×1	14.0477	
	3	(18-20)×1, (32-43)×1, (20-21)×2, (42-43)×1	28.2306	
B2	1	(20-21)×1, (42-43)×1, (46-6)×1, (31-32)×1, (28-31)×1, (5-6)×2	63.5880	110.6817
	2	(20-23)×1, (18-19)×1, (20-21)×1, (42-43)×1	19.1256	
	3	(18-20)×2, (20-23)×1, (31-41)×1, (40-41)×1	27.9681	
B3	1	(20-21)×1, (42-43)×1, (46-6)×1, (31-32)×1, (28-31)×1, (5-6)×2	63.5880	108.7568
	2	(20-23)×1, (18-19)×1, (42-43)×1	14.0477	
	3	(18-20)×2, (20-23)×1, (20-21)×1, (31-41)×1, (40-41)×1	31.1211	
B4	1	(20-21)×1, (42-43)×1, (46-10)×1, (31-32)×1, (28-31)×1, (9-10)×2	58.4720	100.0038
	2	(4-9)×1, (20-23)×1, (20-21)×1, (42-43)×1	17.9080	
	3	(5-9)×1, (18-20)×1, (31-41)×1, (40-41)×1	23.6238	
B5	1	(20-21)×1, (42-43)×1, (46-6)×1, (31-32)×1, (28-31)×1, (5-6)×2	63.5880	93.0072
	2	(42-43)×1	5.0779	
	3	(18-20)×1, (20-23)×1, (32-43)×1, (20-21)×1	24.3413	

iv. *Case B4*: Load shedding and generation displacements are considered, with $\alpha = 0.98$, $\delta = 0.001$ MUS$/MW, $\beta = 0.001$ MUS$/MW, and 10% of the generation displacement, limited up to the installed capacity at each bus in each stage;
v. *Case B5*: Load shedding, generation displacements, and lines overloads are considered, with $\alpha = 0.98$, $\delta = 0.001$ MUS$/MW, $\beta = 0.001$ MUS$/MW, 10% of the generation displacement limited up to the installed capacity at each bus in each stage, $\gamma = 0.001$ MUS$/MW, and $\overline{\sigma} = 0.05$.

The results for the investments in the southern Brazilian 46-bus system using the proposed formulation, for Cases B1–B5, are shown in Table 9.

Again, in this case, all the investment plans presented in Table 9 have lower costs than the plan presented in Table 8 for multi-stage DC TNEP considering multiple generation scenarios. The reductions in the investment costs for plans B1–B5 are, respectively, 13.04%, 9.08%, 10.66%, 17.85%, and 23.60%.

It can also be verified, as in the previous system, that load shedding is more efficient than generation displacements to reduce the investment costs. This is expected due to the fact that while generation displacements only rearrange the power flows in the system, maintaining the total generation and demand, load shedding reduces the power flows on the lines by reducing the total demand.

Table 10: Infeasibilities of the proposed plans for the southern Brazilian 46-bus system

Plan	Stage	Maximum Total Load Shedding in all Scenarios		Maximum Generation Displacement (Bus)		Maximum Line Overload (Branch)	
		[MW]	[%]	[MW]	[%]	[MW]	[%]
A1	1	15.8635	0.3733	–	–	–	–
	2	97.2915	1.7987	–	–	–	–
	3	124.5771	1.8107	–	–	–	–
A2	1	–	–	21.9535 (34)	3.1362	–	–
	2	–	–	25.5714 (28)	3.8166	–	–
	3	–	–	59.1698 (28)	8.8313	–	–
A3	1	–	–	–	–	7.8556 (24-34)	3.5707
	2	–	–	–	–	28.5622 (20-21)	4.7604
	3	–	–	–	–	13.6684 (40-41)	2.2781
A4	1	85.0000	2.0000	88.4000 (14)	10.0000	–	–
	2	108.1800	2.0000	94.0328 (16)	7.9086	–	–
	3	137.6000	2.0000	137.0000 (19)	10.0000	–	–
A5	1	0.0000	0.0000	0.0000	0.0000	7.8556 (24-34)	3.5707
	2	108.1800	2.0000	95.0000 (19)	10.0000	30.0000 (18-19)	5.0000
	3	137.6000	2.0000	137.0000 (19)	10.0000	30.0000 (42-43)	5.0000

Table 10 presents the infeasibilities of the investment plans presented in Table 9. Once again, the values presented in Table 10 are reasonable, since they can be corrected in the short- and medium-term planning. Regarding the lines overloads, for plan B3, at maximum, only one line is overloaded on stages 1 and 3 and three lines on stage 2, while for plan B5, at maximum, one line is overloaded on stage 1 and two lines are overloaded on stages 2 and 4.

The computational times to solve Cases B1–B5 are, respectively, 2498.05 s, 1767.31 s, 4631.30 s, 1333.02 s, and 1786.20 s. Again, these times are lower than the time required to solve the traditional model (1)–(12), due to the same reason discussed in the previous case.

5 Conclusions

This paper presents a flexible solution approach for the multi-stage transmission network expansion planning problem with multiple generation scenarios. The method provides a pool of high-quality solutions with acceptable investment costs, operating with small infeasibilities for some generation scenarios. The infeasibilities in the operation, however, can be easily corrected in the short- and medium-term expansion planning.

The tests were performed with the IEEE 24-bus system and the southern Brazilian 46-bus system. The results indicate that the proposed flexible solution approach leads to solutions with reasonable investment costs, with small infeasibilities in the operation. Besides that, several expansion plans can be obtained by adjusting the parameters of the model. The pool of expansion plans found by the proposed approach is of great practical interest, and the best plan for construction can be identified when the needs of the planning entity are considered.

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