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f($\mathcal{R}$)-Einstein-Palatini formalism and smooth branes

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Abstract. In this work, we present the $f(\mathcal{R})$-Einstein-Palatini formalism in arbitrary dimensions and the study of consistency applied to brane models, the so-called braneworld sum rules. We show that a scenario of thick branes in five dimensions with compact extra dimension in the framework of the $f(\mathcal{R})$-Einstein-Palatini theory is possible by the accomplishment of an assertive criteria. We finalize investigating the outputs of the resulting sum rules with respect to a stabilization mechanism.

1 Introduction

Braneworld models have received a large amount of attention by the high-energy community, since the outstanding Randall-Sundrum model [1], which provides a precise relation between a warped geometry and the mass scale of an effective TeV universe. Soon after the establishment of warped models, a plethora of models, generalizations and applications were developed [2]. Most importantly to our purposes was the smooth extension of warped branes, first introduced by Gremm [3]. From the perspective that there must exist a typical length scale below which our understanding of the physical laws should be, at least, superseded by a full quantum gravity theory, the idea of infinitely thin branes, as used in the Randall-Sundrum model, is only an approximation, though highly non-trivial and powerful.

A crucial point concerning smooth extensions of braneworlds (see [4] for a comprehensive review) in the General Relativity theory is that it is always necessary to preclude the extra dimension orbifold topology used in the original Randall-Sundrum model. After all, $S^1/\mathcal{Z}_2$ is also important to make contact to Hořava-Witten theory [5,6]. With effect, there is an exhaustive theorem forbidding smooth generalizations of the usual Randall-Sundrum model [7] (see also [8]). By "usual" we mean a five-dimensional braneworld endowed with non-separable geometry, whose extra dimensions are compact, within the context of General Relativity. In a gravitational theory different from General Relativity, however, the situation may be different. In fact, by applying the so-called braneworld sum rules (a set of consistency conditions obtained from the gravitational equations of motion) it is possible to see that smooth generalizations of the Randall-Sundrum framework are indeed possible.

Many extensions based upon the ideas delineated in the previous paragraph were done. The investigation of braneworld sum rules applied to smooth branes generalizations in the context of Brans-Dicke and $f(\mathcal{R})$ gravity has been carried out in some detail in [9]. In these cases, it is always possible to show that the sum rules can be relaxed in the presence of additional terms coming from the gravitational theory in question (other than standard General Relativity). The $f(\mathcal{R})$ theory analyzed in [9] was worked out in the light of the metric formalism. As is well known, however, the metric and Palatini formalisms are not equivalent in the approach to $f(\mathcal{R})$ gravity [10,11]. One of the main differences between the two approaches is given by the fact that, in the metric formalism, the trace of fields equations gives rise to a dynamical degree of freedom, whilst in the Palatini formalism this procedure leads to an algebraic constraint. Concerning the problem we are interested in here, we shall see that the necessary condition leading to a smooth brane extension is considerably modified, presenting a more clear criteria to allow for a smooth five-dimensional braneworld extension, namely, a $f(\mathcal{R})$ theory with negative first derivative with respect to R.

The usual, four-dimensional motivation to deal with the $f(\mathcal{R})$-Einstein-Palatini formalism is twofold: on the one hand, the late time universe acceleration may be described by such a theory, through effective pressure and energy density coming from the additional curvature terms [12]. In this vein, such a stage of evolution may be addressed

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without any regard to dark energy, for instance. On the other hand, differently from the metric formalism, the Palatini formalism allows second-order field equations for the metric, facilitating the set up of calculations. In a certain sense, these aspects may also be reflected in a five-dimensional formulation. Some applications of the Palatini theory and affine-metric formalism can be found in [13,14] and [15], respectively. In this vein, though not pursuing a complete five-dimensional example here, the aforementioned reasons certainly perform a tempting scenario for braneworld investigation.

One of the difficulties that may arise when performing the sum rules concern managing the Einstein tensor. Here, we use a simple conform transformation in the metric to accomplish the sum rules program and extend the consistency conditions in the scenario of $f(\mathcal{R})$ theories adopting the Palatini formalism. This work is organized as follows. In sect. 2, we briefly present the sum rules idea for braneworld scenarios. In the following sect. 3 we construct the field equations in a $f(\mathcal{R})$-Einstein-Palatini formalism in arbitrary dimensions. In sect. 4, we apply the sum rules to the $f(\mathcal{R})$-Einstein-Palatini case, investigating the relevant condition which leads to smooth braneworlds. In the last section we conclude.

2 Sum rules for braneworld scenarios

Much of the necessary formalism to the implementation of sum rules in the $f(\mathcal{R})$-Einstein-Palatini context was developed elsewhere [8,9]. Therefore, we shall pinpoint some important aspects in this section. By considering the spacetime as a D-dimensional manifold endowed with a non-factorisable geometry, we write the line element as

$$\mathrm{d}s^2 = g_{AB}(X)\mathrm{d}X^A\mathrm{d}X^B = W^2(r)g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + g_{ab}(r)\mathrm{d}r^a\mathrm{d}r^b, \tag{1}$$

where $W^2(r)$ is the warp factor, X^A denotes the coordinates of the full D-dimensional spacetime, x^μ stands for the $(p+1)$ coordinates of the non-compact spacetime (brane), and r^a labels the (D-p-1) directions in the internal compact space. The classical action takes into account the spacetime dynamics coupled to a scalar field, namely

$$S = S_{\mathrm{gravity}} + \int \mathrm{d}^D X\,\sqrt{-g}\left(-\frac{1}{2}\partial_A\Phi\partial^A\Phi - V(\Phi)\right), \tag{2}$$

where we assume that the scalar field has only dependence on the internal space coordinates $\Phi = \Phi(r^m)$. The scalar field above shall be understood as the one responsible of generating the brane. We leave the potential unspecified, since it will not be relevant in our case. The energy-momentum tensor gives

$$T_{\mu\nu} = -W^2 g_{\mu\nu}\left(\frac{1}{2}\nabla\Phi\cdot\nabla\Phi + V(\Phi)\right) \tag{3}$$

and

$$T_{ab} = \nabla_a\Phi\nabla_b\Phi - g_{ab}\left(\frac{1}{2}\nabla\Phi\cdot\nabla\Phi + V(\Phi)\right). \tag{4}$$

It is possible to show [7,8] that the following expression holds:

$$\nabla\cdot(W^\alpha\nabla W) = \frac{W^{\alpha+1}}{p(p+1)}\left[\alpha\left(W^{-2}\bar{R} - R^\mu_\mu\right) + (p-\alpha)\left(\tilde{R} - R^a_a\right)\right], \tag{5}$$

where $R^\mu_\mu = W^{-2}g^{\mu\nu}R_{\mu\nu}$ and $R^a_a = g^{ab}R_{ab}$ are the partial traces, such that $R = R^\mu_\mu + R^a_a$ and α is an arbitrary parameter. Moreover, $\bar{R}$ is the scalar of curvature derived from $g_{\mu\nu}$ and $\tilde{R}$ the scalar of curvature associated to the internal space.

The braneworld sum rules can be obtained from two considerations: one physical and one mathematical. From the physical point of view, it is necessary to specify the gravitational theory in question, *i.e.* write S_{gravity} explicitly. Once this is done (notice that the dynamics is specified accordingly), one is able to use the fact that, as long as the internal space is periodic without boundary, the left-hand side of (5) vanishes under integration.

3 The f($\mathcal{R}$)-Einstein-Palatini formalism in arbitrary dimensions

In the so-called Palatini formalism the metric and the connection are assumed to be independent variables. The field equations are derived from the variation of the Einstein-Hilbert action with respect to metric and connection independently. Thus, the Ricci and Riemann tensors are objects constructed from a general affine connection.

It is well known that the definition $T_{AB} \equiv 2/\sqrt{-g}\delta S_M/\delta g^{AB}$, when implemented along with the principle of least action for the $f(\mathcal{R})$-Einstein-Palatini gravity, leads to the following field equations:

$$f'(\mathcal{R})\mathcal{R}_{AB} - \frac{1}{2}f(\mathcal{R})g_{AB} = 8\pi G_D T_{AB} \tag{6}$$

and

$$-\overline{\nabla}_C\left(\sqrt{-g}f'(\mathcal{R})g^{AB}\right) + \overline{\nabla}_D\left(\sqrt{-g}f'(\mathcal{R})g^{D(A}\right)\delta_C^{B)} = 0, \tag{7}$$

such that, when $f(R) = R$, the Palatini formalism restores General Relativity. Rewriting eq. (7) we get

$$-\overline{\nabla}_C\left(\sqrt{-g}f'(\mathcal{R})g^{AB}\right) + \frac{1}{2}\left[\overline{\nabla}_D\left(\sqrt{-g}f'(\mathcal{R})g^{DA}\right)\delta_C^B + \overline{\nabla}_D\left(\sqrt{-g}f'(\mathcal{R})g^{DB}\right)\delta_C^A\right] = 0 \tag{8}$$

and, by contracting C and B, we are left with

$$\overline{\nabla}_D\left(\sqrt{-g}f'(\mathcal{R})g^{DA}\right) = 0. \tag{9}$$

Therefore, eq. (7) simply reads

$$\overline{\nabla}_C\left(\sqrt{-g}f'(\mathcal{R})g^{AB}\right) = 0. \tag{10}$$

In this vein, by defining a metric h_{AB} as

$$h_{AB} \equiv f'(\mathcal{R})^{\frac{2}{D-2}}g_{AB}, \qquad h^{AB} \equiv f'(\mathcal{R})^{\frac{2}{2-D}}g^{AB}, \tag{11}$$

we formally have the connection equation

$$\overline{\nabla}_C\left(\sqrt{-h}h^{AB}\right) = 0. \tag{12}$$

Following this clue, it is possible to write the connection as

$$\overline{\Gamma}_{AB}{}^{C} = \left\{{}_{AB}{}^{C}\right\} + \frac{1}{2f'(\mathcal{R})^{\frac{2}{D-2}}}\Delta_{AB}{}^{C}, \tag{13}$$

where

$$\Delta_{AB}{}^{C} = \left\{\delta_B^C\partial_A f'(\mathcal{R})^{\frac{2}{D-2}} + \delta_A^C\partial_B f'(\mathcal{R})^{\frac{2}{D-2}} - g_{AB}g^{CD}\partial_D f'(\mathcal{R})^{\frac{2}{D-2}}\right\} \tag{14}$$

and $\left\{{}_{AB}{}^{C}\right\}$ are the usual Christoffel symbols.

The Ricci tensor, generalized via the conformal relation (11), is given by $\mathcal{R}_{AB} = \partial_C\overline{\Gamma}_{AB}{}^{C} - \partial_B\overline{\Gamma}_{AC}{}^{C} + \overline{\Gamma}_{CE}{}^{C}\,\overline{\Gamma}_{AB}{}^{E} - \overline{\Gamma}_{BE}{}^{C}\,\overline{\Gamma}_{AC}{}^{E}$, and can be recast as

$$\mathcal{R}_{AB} = R_{AB} + \frac{[D-1]}{2}\frac{(\nabla_A f'(\mathcal{R})^{\frac{2}{D-2}})(\nabla_B f'(\mathcal{R})^{\frac{2}{D-2}})}{f'(\mathcal{R})^{\frac{4}{D-2}}} - \frac{1}{f'(\mathcal{R})^{\frac{2}{D-2}}}\left(\nabla_A\nabla_B + \frac{1}{2}g_{AB}\Box\right)f'(\mathcal{R})^{\frac{2}{D-2}}, \tag{15}$$

and thus the generalized scalar of curvature reads

$$\mathcal{R} = R + \frac{[D-1]}{2}\frac{1}{f'(\mathcal{R})^{\frac{4}{D-2}}}\left(\nabla_A f'(\mathcal{R})^{\frac{2}{D-2}}\right)\left(\nabla^A f'(\mathcal{R})^{\frac{2}{D-2}}\right) - \frac{1}{f'(\mathcal{R})^{\frac{2}{D-2}}}\left(\frac{D}{2}+1\right)\Box f'(\mathcal{R})^{\frac{2}{D-2}}. \tag{16}$$

In the Palatini formalism the field equations are given by

$$\mathcal{R}_{AB} - \frac{f}{2f'(\mathcal{R})}g_{AB} = \frac{8\pi G_D T_{AB}}{f'(\mathcal{R})}. \tag{17}$$

Hence, inserting eq. (15) in (17) and adding on both sides of the term $-g_{AB}R/2$, we obtain, after some manipulation, the Einstein-Palatini field equations in arbitrary dimensions

$$\begin{aligned} R_{AB} - \frac{1}{2}Rg_{AB} = {} & \frac{8\pi G_D T_{AB}}{F(f'(\mathcal{R}))} - \frac{g_{AB}}{2}\left(f'(\mathcal{R}) - \frac{f(\mathcal{R})}{F(\mathcal{R})}\right) + \frac{1}{F(\mathcal{R})^{\frac{2}{D-2}}}\left(\nabla_A\nabla_B - g_{AB}\Box\right)F(\mathcal{R})^{\frac{2}{D-2}} \\ & - \frac{[D-1]}{2F(\mathcal{R})^{\frac{4}{D-2}}}\left[\left(\nabla_A F(\mathcal{R})^{\frac{2}{D-2}}\right)\left(\nabla_B F(\mathcal{R})^{\frac{2}{D-2}}\right) - \frac{g_{AB}}{2}\nabla_C F(\mathcal{R})^{\frac{2}{D-2}}\nabla^C F(\mathcal{R})^{\frac{2}{D-2}}\right], \end{aligned} \tag{18}$$

where $F(\mathcal{R}) = \mathrm{d}f(\mathcal{R})/\mathrm{d}\mathcal{R}$ and $\mathcal{R}$ is the Ricci scalar constructed out from $\mathcal{R}_{AB}$. Now we are able to implement the relevant partial traces, derived from (18), into eq. (5).

4 Braneworld sum rules in f($\mathcal{R}$)-Einstein-Palatini

Taking advantage of eq. (18), we see that the scalar of curvature reads

$$R=\frac{2}{(2-D)}\left\{\frac{8\pi G_D}{F(\mathcal{R})}T-\frac{D}{2}\left(\mathcal{R}-\frac{f(\mathcal{R})}{F(\mathcal{R})}\right)-(D-1)\frac{1}{F(\mathcal{R})^{\frac{2}{D-2}}}\Box F(\mathcal{R})^{\frac{2}{D-2}}\right.$$
$$\left.-\frac{[D-1]}{2F(\mathcal{R})^{\frac{4}{D-2}}}\left[\left(\nabla_A F(\mathcal{R})^{\frac{2}{D-2}}\right)\left(\nabla^A F(\mathcal{R})^{\frac{2}{D-2}}\right)-\frac{D}{2}\nabla_C F(\mathcal{R})^{\frac{2}{D-2}}\nabla^C F(\mathcal{R})^{\frac{2}{D-2}}\right]\right\}, \tag{19}$$

from which, reinserting it back in (18), we have

$$R_{AB}=\frac{1}{F(\mathcal{R})}\left[8\pi G_D\left(T_{AB}-\frac{g_{AB}}{(D-2)}T\right)\right]+\frac{\nabla_A\nabla_B F(\mathcal{R})^{\frac{2}{D-2}}}{F(\mathcal{R})^{\frac{2}{D-2}}}+\frac{g_{AB}}{(D-2)}\left\{\left(\mathcal{R}-\frac{f(\mathcal{R})}{F(\mathcal{R})}\right)+\frac{\Box F(\mathcal{R})^{\frac{2}{D-2}}}{F(\mathcal{R})^{\frac{2}{D-2}}}\right.$$
$$\left.-\frac{\nabla_C F(\mathcal{R})^{\frac{2}{D-2}}\nabla^C F(\mathcal{R})}{2F(\mathcal{R})^{\frac{4}{D-2}}}\right\}-\frac{(D-1)(D-3)}{2(D-2)F(\mathcal{R})^{\frac{4}{D-2}}}\left(\nabla_A F(\mathcal{R})^{\frac{2}{D-2}}\right)\left(\nabla_B F(\mathcal{R})^{\frac{2}{D-2}}\right). \tag{20}$$

The partial trace of the above equation with respect to the brane, non-compact, dimensions is given by

$$R^\mu_\mu=\frac{1}{(D-2)F(\mathcal{R})}\left[8\pi G_D\left((D-p-3)T^\mu_\mu-(p+1)T^a_a\right)\right]+\frac{(D+p-1)}{(D-2)F(\mathcal{R})^{\frac{2}{D-2}}}\left(W^{-2}\nabla_\mu\nabla^\mu F(\mathcal{R})^{\frac{2}{D-2}}\right)$$
$$+\frac{(p+1)}{(D-2)}\left[\left(\mathcal{R}-\frac{f(\mathcal{R})}{F(\mathcal{R})}\right)+\frac{\nabla_a\nabla^a F(R)^{\frac{2}{D-2}}}{F(\mathcal{R})^{\frac{2}{D-2}}}-\frac{1}{2F(R)^{\frac{4}{D-2}}}\left(\left(W^{-2}\nabla_\lambda F(R)^{\frac{2}{D-2}}\nabla^\lambda F(R)^{\frac{2}{D-2}}\right.\right.\right.$$
$$\left.\left.\left.+\ \nabla_c F(\mathcal{R})^{\frac{2}{D-2}}\nabla^c F(\mathcal{R})^{\frac{2}{D-2}}\right)\right)\right]-\frac{(D-1)(D-3)}{2(D-2)F(\mathcal{R})^{\frac{4}{D-2}}}W^{-2}\left(\nabla_\mu F(\mathcal{R})^{\frac{2}{D-2}}\right)\left(\nabla^\mu F(\mathcal{R})^{\frac{2}{D-2}}\right), \tag{21}$$

while its internal space counterpart reads

$$R^a_a=\frac{1}{(D-2)F(\mathcal{R})}\left[8\pi G_D\left((p-1)T^a_a-(D-p-1)T^\mu_\mu\right)\right]+\frac{(2D-p-3)}{F(\mathcal{R})^{\frac{2}{D-2}}(D-2)}\left(W^{-2}\nabla_\mu\nabla^\mu F(\mathcal{R})^{\frac{2}{D-2}}\right)$$
$$+\frac{(D-p-1)}{(D-2)}\left[\left(\mathcal{R}-\frac{f(\mathcal{R})}{F(\mathcal{R})}\right)+\frac{(\nabla_a\nabla^a F(\mathcal{R})^{\frac{2}{D-2}})}{F(\mathcal{R})^{\frac{2}{D-2}}}-\frac{1}{2F(\mathcal{R})^{\frac{4}{D-2}}}\left(W^{-2}\nabla_\lambda F(\mathcal{R})^{\frac{2}{D-2}}\nabla^\lambda F(\mathcal{R})^{\frac{2}{D-2}}\right.\right.$$
$$\left.\left.+\ \nabla_c F(\mathcal{R})^{\frac{2}{D-2}}\nabla^c F(\mathcal{R})^{\frac{2}{D-2}}\right)\right]-\frac{(D-1)(D-3)}{2(D-2)F(\mathcal{R})^{\frac{4}{D-2}}}\left(\nabla_a F(\mathcal{R})^{\frac{2}{D-2}}\right)\left(\nabla^a F(\mathcal{R})^{\frac{2}{D-2}}\right). \tag{22}$$

Now, by inserting (21) and (22) into eq. (5), one arrives at

$$\nabla\cdot(W^\alpha\nabla W)=\frac{W^{\alpha+1}}{p(p+1)(D-2)F(\mathcal{R})}\left\{8\pi G_D\left((p-\alpha)(D-p-1)-\alpha(D-p-3)\right)T^\mu_\mu\right.$$
$$\left.+\ 8\pi G_D\left(\alpha(p+1)-(p-\alpha)(p-1)\right)T^a_a+(D-2)\left(\alpha W^{-2}\bar{R}+(p-\alpha)\tilde{R}\right)F(\mathcal{R})\right\}$$
$$-\frac{W^{\alpha+1}}{p(p+1)(D-2)}\left\{\left[\frac{W^{-2}\nabla_\mu\nabla^\mu F(\mathcal{R})^{\frac{2}{D-2}}}{F(\mathcal{R})^{\frac{2}{D-2}}}\right][\alpha(D+p-1)+(p-\alpha)(2D-p-3)]\right.$$
$$-[\alpha(p+1)+(p-\alpha)(D-p-1)]\left[\left(\mathcal{R}-\frac{f(\mathcal{R})}{F(\mathcal{R})}\right)+\frac{(\nabla_a\nabla^a F(\mathcal{R})^{\frac{2}{D-2}})}{F(\mathcal{R})^{\frac{2}{D-2}}}\right.$$
$$\left.-\frac{1}{F(\mathcal{R})^{\frac{4}{D-2}}}\left(W^{-2}\nabla_\lambda F(\mathcal{R})^{\frac{2}{D-2}}\nabla^\lambda F(\mathcal{R})^{\frac{2}{D-2}}+\nabla_c F(\mathcal{R})^{\frac{2}{D-2}}\nabla^c F(\mathcal{R})^{\frac{2}{D-2}}\right)\right]$$
$$+\left[\frac{(D-1)(D-3)}{2F(\mathcal{R})^{\frac{4}{D-2}}}\right]\left[\alpha\left(W^{-2}\nabla_\lambda F(\mathcal{R})^{\frac{2}{D-2}}\nabla^\lambda F(\mathcal{R})^{\frac{2}{D-2}}\right)+(p-\alpha)\right.$$
$$\left.\left.\times\left(\nabla_a F(\mathcal{R})^{\frac{2}{D-2}}\right)\left(\nabla^a F(\mathcal{R})^{\frac{2}{D-2}}\right)\right]\right\}. \tag{23}$$

As a last step, by assuming the internal space compact (as in the standard cases) the left-hand side of eq. (23) vanishes upon integration. Following the standard presentation we denote these integrations by $\oint \nabla \cdot (W^\alpha \nabla W) = 0$. Hence, by inserting the energy-momentum partial traces and integrating over the internal space, it is possible to obtain the sum rules to the very general case in the scope of $f(\mathcal{R})$-Eintein-Palatini theory. The result is quite large, and its generality contributes to overshadow its physical content.

In order to extract physical information it is convenient to particularize the analysis to the $D = 5$ and $p = 3$ case. Thus we shall investigate a five-dimensional bulk with a unique extra dimension (implying $\tilde{R} = 0$) endowed to an orbifold topology, for instance. Besides, in this four-dimensional brane context, we can implement the physical constraint ($\bar{R} = 0$) in trying to describe our universe in large scales. Therefore, after these particularizations, and using eqs. (3) and (4) for the sources, we have the following set of conditions:

$$
\begin{aligned}
0 = 8\pi G_5 \oint \frac{W^{\alpha+1}}{F(\mathcal{R})} \{(3-\alpha)\Phi' \cdot \Phi' + 2(\alpha+1)V(\Phi)\} \\
+ (\alpha+1)\Bigg\{ 4 \oint \frac{W^{\alpha-1}\nabla_\mu \nabla^\mu F(\mathcal{R})^{2/3}}{F(\mathcal{R})^{2/3}} + \oint W^{\alpha+1} \left[\left(\mathcal{R} - \frac{f(\mathcal{R})}{F(\mathcal{R})} \right) + \frac{(\nabla_a \nabla^a F(\mathcal{R})^{2/3})}{F(\mathcal{R})^{2/3}} \right] \\
- \frac{1}{6} \oint \frac{W^{\alpha-1}}{F(\mathcal{R})^{4/3}} \nabla_\mu F(\mathcal{R})^{2/3} \nabla^\mu F(\mathcal{R})^{2/3} + \frac{1}{6} \oint \frac{W^{\alpha-1}}{F(\mathcal{R})^{4/3}} \nabla_a F(\mathcal{R})^{2/3} \nabla^a F(\mathcal{R})^{2/3} \Bigg\} \\
+ \frac{4}{3}\alpha \oint \frac{W^{\alpha-1}}{F(\mathcal{R})^{4/3}} \nabla_\mu F(\mathcal{R})^{2/3} \nabla^\mu F(\mathcal{R})^{2/3} + \frac{4}{3}(3-\alpha) \oint \frac{W^{\alpha+1}}{F(\mathcal{R})^{4/3}} \nabla_a F(\mathcal{R})^{2/3} \nabla^a F(\mathcal{R})^{2/3}.
\end{aligned}
\tag{24}
$$

There are many irrelevant consistency conditions among all encoded in (24), each related to a given α. As a matter of fact, in order to explore the smooth branes possibility the choice $\alpha = -1$ is particularly elucidative, since it eliminates the overall warp factor. In fact, this choice simply provides

$$
\oint \frac{\Phi' \cdot \Phi'}{F(\mathcal{R})} + \frac{1}{6\pi G_5} \oint \frac{\nabla_a F(\mathcal{R})^{2/3} \nabla^a F(\mathcal{R})^{2/3}}{F(\mathcal{R})^{4/3}} = 0, \tag{25}
$$

in which $\nabla_\mu F(R)^{2/3} = 0$ was already taken into account. Interestingly enough, eq. (25) may be rewritten as

$$
\oint \frac{\Phi' \cdot \Phi'}{F(\mathcal{R})} + \frac{1}{27\pi G_5} \oint (\ln|F(\mathcal{R})|)' \cdot (\ln|F(\mathcal{R})|)' = 0. \tag{26}
$$

Now it turns out that whether $F(\mathcal{R})$ is positive, then it is impossible to achieve a smooth generalization of usual braneworld models, since the resulting constraint

$$
\oint \left(\frac{1}{F(\mathcal{R})^{1/2}} \frac{\mathrm{d}\Phi}{\mathrm{d}r} \right) \cdot \left(\frac{1}{F(\mathcal{R})^{1/2}} \frac{\mathrm{d}\Phi}{\mathrm{d}r} \right) + \frac{1}{27\pi G_5} \oint (\ln|F(\mathcal{R})|)' \cdot (\ln|F(\mathcal{R})|)' = 0 \tag{27}
$$

can never be satisfied. The situation is utterly different in the case of a negative $F(\mathcal{R})$. Obviously, in this last case the balance relation

$$
\oint \left(\frac{1}{|F(\mathcal{R})|^{1/2}} \frac{\mathrm{d}\Phi}{\mathrm{d}r} \right) \cdot \left(\frac{1}{|F(\mathcal{R})|^{1/2}} \frac{\mathrm{d}\Phi}{\mathrm{d}r} \right) = \frac{1}{27\pi G_5} \oint (\ln|F(\mathcal{R})|)' \cdot (\ln|F(\mathcal{R})|)' \tag{28}
$$

may be satisfied. Equation (28) performs, then, a clear criteria – $F(\mathcal{R}) < 0$ – for the possibility of smooth 3-branes in a five-dimensional bulk. Compared with the metric approach, where the negative quantity was proportional to $\oint F(\mathcal{R})^{-1} \nabla^2 F(\mathcal{R})$, the result obtained in the Palatini context is indeed exhaustive. As a final remark, notice that in the limit $F(\mathcal{R}) \to 1$, *i.e.* $f(\mathcal{R}) \to \mathcal{R}$, eq. (28) reduces to $\oint \Phi' \cdot \Phi' = 0$, just as in the usual General Relativity (as expected), a constraint which can never be reached.

5 Concluding remarks

It is shown that smooth generalizations of the usual Randall-Sundrum braneworld model can be achieved in $f(\mathcal{R})$ gravity. This is already known from previous work [9], but here we have worked on the Palatini formalism to $f(\mathcal{R})$. Apart from the fact that the metric and the Palatini formalisms to $f(\mathcal{R})$ are inequivalent, the analysis performed here has culminated into a more clear and assertive constraint to be fulfilled.

The modeling of warped smooth branes has given rise to a somewhat more formal branch of research in the context of braneworld gravity. In turn, this line of investigation has led to the solidification of braneworld models in several different perspectives, such as non-compact extra dimension [16], different bulk cosmological constants [17], and ingenious single thick branes approach [18], just to enumerate some. As for this paper, the general idea is not to set a specific model, but to provide instead a comprehensive scope from which consistent models can be constructed. Even though here the proposition of models is not our purpose, we shall emphasize that among all the possible generalizations leading to smooth 3-branes in five dimensions within non-separable geometry (and a compact extra dimension), the use of the $f(\mathcal{R})$-Eintein-Palatini formalism seems to be quite a promising approach. This is because, once again, in this context it is possible to extract a simple and sharp necessary criterion.

It is important to say some words about the stabilization mechanism as far as the use of our results is concerned. Soon after the publication of the Randall-Sundrum two branes model, the distance between the branes was addressed in ref. [19]. The basic idea was to provide a stabilization scheme along the extra dimension. Although the stabilization possibility had been demonstrated in ref. [19] by means of the addiction of a bulk scalar field endowed to a well-behaved potential, the proposed mechanism did not take into account the back reaction of the scalar field on the metric. A more complete realization of the idea explored in ref. [19] was performed in ref. [20], the so-called DeWolfe-Freedman-Gubser-Karch stabilization mechanism. In ref. [20] a simple and effective toy model is presented in which the distance between two branes is saturated by a given scalar field potential. Here we shall see that the additional freedom shown by the sum rules in the context just explored in this paper points to a stabilization mechanism possibility with the extra bonus of only positive tension branes.

To fix ideas let us implement two singular branes along the orbifolded extra dimension, being one of the branes placed at $r = 0$ and the other at $r = \varDelta$. The orbifold is implemented requiring $2r = 0$, leading to a compact internal space. In the presence of singular sources, the condition (26) is replaced by

$$\oint \frac{\varPhi' \cdot \varPhi'}{F(\mathcal{R})} + \oint \sum_{i=1}^{2} \frac{\lambda_i(\varPhi)\delta(r - r_i)}{F(\mathcal{R})} + \frac{1}{27\pi G_5} \oint (\ln|F(\mathcal{R})|)' \cdot (\ln|F(\mathcal{R})|)' = 0.$$

It is fairly simple to see that the integration along the orbifold is twice the integration in the range between the branes, in such a way that

$$2\int_0^{\varDelta} \mathrm{d}r \frac{\varPhi' \cdot \varPhi'}{F(\mathcal{R})} + \frac{2}{27\pi G_5} \int_0^{\varDelta} (\ln|F(\mathcal{R})|)' \cdot (\ln|F(\mathcal{R})|)' + \frac{\lambda_1(\varPhi_1)}{F(\mathcal{R}(0))} + \frac{\lambda_2(\varPhi_2)}{F(\mathcal{R}(\varDelta))} = 0.$$

In ref. [20] a suitable potential, chosen in terms of a special superpotential, was achieved as analytical solution to the scalar field and the warp factor. Here, as the same model cannot be directly implemented, we finalize the concluding remarks stressing that, for a relatively simple model for which[1] $\varDelta = f(\varPhi_1, \varPhi_2)$, taking into account the necessity of a negative $F(\mathcal{R})$, one arrives at

$$\frac{2}{27\pi G_5} \int_0^{\varDelta} (\ln|F(\mathcal{R})|)' \cdot (\ln|F(\mathcal{R})|)' = 2\int_0^{\varDelta} \mathrm{d}r \frac{\varPhi' \cdot \varPhi'}{|F(\mathcal{R})|} + \frac{\lambda_1(\varPhi_1)}{|F(\mathcal{R}(0))|} + \frac{\lambda_2(\varPhi_2)}{|F(\mathcal{R}(\varDelta))|}$$

and, therefore, the framework does not necessitate any negative brane tension. In other words, the stabilization takes part with singular branes for which the four-dimensional gravitational constant has a positive contribution coming from the brane tension. This is, indeed, an extra bonus of the considered gravitational theory.

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[1] For instance, one could insist in taking advantage of the model presented in ref. [20] and impose $\varPhi = \varPhi_1 e^{-Lr}$, where L is a constant, and search for an adequate potential.

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