UNIVERSIDADE ESTADUAL PAULISTA “JÚLIO DE MESQUITA FILHO”
FACULDADE DE ENGENHARIA
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A STOCHASTIC PROGRAMMING MODEL FOR THE OPTIMAL ALLOCATION OF DISTRIBUTED GENERATION IN ELECTRICAL DISTRIBUTION SYSTEMS CONSIDERING LOAD VARIATIONS AND GENERATION UNCERTAINTY

Ilha Solteira
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MSc thesis presented to the Faculty of Engineering, Campus of Ilha Solteira, Sao Paulo State University, as part of the requirements for obtaining the title of MSc in Electrical Engineering. Field of knowledge: Automation.

Advisor:
Prof. Dr. John Fredy Franco Baquero

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TÍTULO DA DISSERTAÇÃO: A Stochastic Programing Model for the Optimal Allocation of Distributed Generation in Electrical Distribution Systems Considering Load Variations and Generation Uncertainty

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ABSTRACT

Nowadays, the penetration of distributed generation (DG) units in power systems is increasing because of their important impacts on the main features of power systems. Place, type, and size of DG play an important role in power loss reduction, power quality improvement, security enhancement, and cost reduction. Therefore, optimal placement and sizing of DGs in electric power systems are one of the most important problems that should be evaluated carefully. DG allocation is a constrained optimization problem with different objectives such as power loss minimization, voltage profile improvement, reliability enhancement, investment, and operation cost reduction. In this dissertation, photovoltaic DG allocation problem is solved for photovoltaic units, aiming minimization of energy and investment costs considering generation uncertainty and load variation. Due to high uncertainties of solar energy resource, the problem is evaluated under different scenarios of solar radiation under a stochastic programming approach. In present work, photovoltaic DG allocation is formulated as a mixed-integer second-order conic programming problem. Tests were carried out using the 33-node and 136-node distribution systems and the obtained results demonstrate the advantage of optimal DG allocation as well as the efficiency of the adopted mathematical to find the optimal solution.

Keywords: Distributed generation. Mathematical modeling. Optimal allocation. Solar uncertainty. Stochastic programming.

RESUMO

Atualmente, a penetração de unidades de geração distribuída (DG) em sistemas de energia está aumentando devido aos seus importantes impactos nas principais características dos sistemas de energia. O local, o tipo e o tamanho da DG desempenham um papel importante na redução de perda de energia, melhoria da qualidade de energia, aprimoramento da segurança e redução de custos. Portanto, a localização e o dimensionamento ideais das DGs em sistemas de energia elétrica são um dos problemas mais importantes que devem ser avaliados cuidadosamente. A alocação de DG é um problema de otimização restrito com objetivos diferentes, como minimização de perda de energia, melhoria do perfil de tensão, aprimoramento da confiabilidade, investimento e redução de custos operacionais. Nesta dissertação, o problema de alocação de DG fotovoltaica é resolvido para unidades fotovoltaicas, visando minimizar os custos de energia e investimento, considerando a incerteza de geração e a variação de carga. Devido às altas incertezas dos recursos de energia solar, o problema é avaliado em diferentes cenários de radiação solar sob uma abordagem de programação estocástica. No presente trabalho, a alocação de DG fotovoltaica é formulada como um problema de programação cônica de segunda ordem com número inteiro misto. Os testes foram realizados usando os sistemas de distribuição de 33 e 136 nós e os resultados obtidos demonstram a vantagem da alocação ótima de DG, bem como a eficiência da matemática adotada para encontrar a solução ideal.

Palavras-chave: Geração distribuída. Modelagem matemática. Alocação ideal. Incerteza solar. Programação estocástica.

Nomenclature

Sets:

$\boldsymbol{\Omega_b}$: Set of buses.

$\boldsymbol{\Omega_d}$: Set of load levels.

$\boldsymbol{\Omega_g}$: Set of DG

$\boldsymbol{\Omega_l}$: Set of branches.

$\boldsymbol{\Omega_t}$: Set of scenarios.

Parameters:

$\alpha_{d,t}$: Number of days in one year of scenario *t* in load level *d* (h).

c_g: Annualized installation cost of DG of type *g* ($/kW).

$CDG_{d,g}$: Energy cost for DG of type *g* ($/kWh).

CS_d: Energy cost of energy supplied by the substation ($/kWh).

f_t^{DG}: Generation factor of DG in scenario *t*.

I_{ij}^{max}: Maximum current magnitude of branch *ij* (A).

N_{DG}^{max}: Maximum number of DG units.

$N_{DG,i,g}^{max}$: Maximum number of DG of type *g* on each bus.

P_g^{max}: Maximum active power provided by DG of type *g* (kW).

pf^{DG}: Power factor limit for DG.

pf_{min}^S: Minimum leading and lagging substation power factor.

Pr_t: Probability of Scenario *t*.

Q_g^{max}: Maximum reactive power provided by DG of type *g* (kVAr).

Q_g^{min}: Minimum reactive power provided by DG of type *g* (kVAr).

R_{ij}: Resistance of branch *ij* (Ω).

V_{min}: Minimum voltage magnitude (kV).

V_{max}: Maximum voltage magnitude (kV).

X_{ij}: Reactance of branch *ij* (Ω).

Z_{ij}: Impedance of branch *ij* (Ω).

Variables:

C_{DG}: Cost of generated energy by DG units ($).

C_S: Cost of generated energy by substation ($).

C_T: Total cost ($).

IC: Investment cost ($).

$I_{ij,d,t}$: Current magnitude of branch *ij* at load level *d* in scenario *t* (A).

$P_{ij,d,t}$: Active power of branch *ij* at load level *d* in scenario *t* (kW).

$P_{i,d,t}^{S}$: Active power provided by substation on bus *i* at load level *d* in scenario *t* (kW).

$P_{i,d}^{D}$: Active power demanded at node *i* in load level *d* (kW).

$P_{i,d,g,t}^{DG}$: Active power provided by DG of type *g* on bus *i* at load level *d* in scenario *t* (kW).

$Q_{ij,d,t}$: Reactive power of branch *ij* at load level *d* in scenario *t* (kVAr).

$Q_{i,d}^{D}$: Reactive power demanded at node *i* in load level *d* (kVAr).

$Q_{i,d,g,t}^{DG}$: Reactive power provided by DG of type *g* on bus *i* at load level *d* in scenario *t* (kVAr).

$Q_{i,d,t}^{S}$: Reactive power provided by substation on bus *i* at load level *d* in scenario *t* (kVAr).

$V_{i,d,t}$: Voltage magnitude on bus *i* at load level *d* in scenario *t* (kV).

$y_{i,g}$: Binary decision variable for DG of type *g* (one if DG of type g is installed, otherwise 0).

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1 INTRODUCTION

Distribution network plays an important role for delivering electrical power from transmission system to individual consumers (BADRAN et al., 2017), (PATERAKIS et al., 2016), in which providing a safe and economic electrical energy for customers is essential (DUTTAGUPTA; SINGH, 2008). Seventy percent of the total losses happens in distribution system, because of its radial topology and higher ratio of current to voltage. Therefore, minimization of distribution losses is important, because it affects the operational cost and the voltage profile (BADRAN et al., 2017), (RAHMANI-ANDEBILI; FOTUHI-FIRUZABAD, 2018). Capacitor placement, distributed generation (DG) allocation, and feeder reconfiguration are common methods for minimization of distribution losses (ACHARYA; MAHAT; MITHULANANTHAN, 2006).

In the power system, distribution network plays main role in outages, because 80% of interruptions take places in the distribution system (SULTANA et al., 2016), while in radial topologies, there are single sources of supply for each consumer.

1.1. DISTRIBUTED GENERATION

DG is not a new concept, because in 1882 when the first power plant was built electric power was generated near to load centers. Nevertheless, the role of DGs in power systems decreased in the beginning of twentieth century due to growth of large power plants. Unlike conventional power plants, which have high potential to provide energy for load centers from far distance by transmission and distribution lines, distributed generation leads to a decentralized power system in which DG units meet local demand. Recently, DG usages are increasing because of its small size, lower investment, and faster installation compared to large power plants (ANAYA; POLLITT, 2015).

DGs are less than 100 MW (according to CIGRE definition between 50-100 MW and EPRI definition from few kilowatts (kW) up to 50 MW (ACKERMANN, 2001)) in size and below 69 kV in voltage. These highly flexible and suitable energy resources include natural gas and diesel-powered generators, micro turbines, mini-hydro power plants, biomass, photovoltaic (PV), fuel cell, wind generator, and energy storage systems (ESSs) (FORSTEN; EPRI, 2015). Although, some of DGs have very *NOx* and carbon dioxide emission (CO_2) such as fuel cells and diesel or natural gas-powered generators, some others like renewable energy sources (RES) make short the time to achieve goal of zero greenhouse gas emission. They can be used to provide demand between minimum and peak load (e.g. in USA) (DUGAN; PRICE, 2003) or even back-up power.

Even though DGs are more available, smaller, and cheaper nowadays, large power plants are necessary for power generation due to increase of urbanization. In this way, a new concept rises: integration of large power plants and distributed generators in order to meet load demand (OWENS, 2014).

Moreover, high flexibility, environmental and economical attractiveness of DG are other causes for increase of distributed generation usage today. Indeed, application of distributed generators is now an efficient and economic way for power generation, because of technology innovations during recent 50 years. In addition, the following factors have important role in increase of DG usage all over world:

1) Natural gas network growth;

2) Distribution network construction restrictions;

3) Digital technologies and the internet;

4) Natural disasters.

Over the last decades, DG installations have been increasing. For example, in 2000, 21% of generation capacity in the world belonged to distributed power, while in 2017, only development of DG in European countries like Denmark and Netherland were 40 percent (IMPACT OF INCREASING CONTRIBUTION OF DISPERSED GENERATION ON THE POWER SYSTEM, CIGRE SC #37, 1998., 2019). Also, globally investment in distributed power was $150 billion, which is five times more than the investment of DG in 2000 ($30 billion). Accordingly, it is expected that DG will play greater role in 2020, because of $205 billion in DG investment and its 41%-portion in generation capacity.

Asia, Africa and the Middle East, have more chance for DG growth, because demand is growing faster in developing countries. In developed world like Great Britain (GB), the capacity of DG generation increased from 12.4 GW to 19.1 GW between 2011 and 2014, in which the largest areas of growth have been solar and wind (PACE et al., 2016).

Also, in poor electricity networks like Sudan's electrical power system, thermal and hydro power plants have essential role in electricity generation, where electricity is provided only for 30% of Sudan’s population who live in big cities like Khartoum. However, most of peoples in rural areas do not have any access to electricity. There are different types of renewable energy resources such as solar energy (in most areas), wind (in the Red Sea, Darfur and north), biomass (Jebel Mara, Sahara desert and the Red Sea) in Sudan. Sudan has good potential for growth of DG usage because of large-water resources and small sized-power plants in rural areas (CHEN et al., 2016).

1.1.1. Advantages of DG

Some advantages of DG in comparison with conventional power plants are described as follows.

1) Small Size: Small size is the most important benefit of DG leading to following advantages: modular DG units can be installed in shorter time compared to conventional power plants; price, installation and operation costs of DG units are less than conventional units; the generation level of DGs can be changed quickly according to load demand; DG units enhance local control, operation and maintenance of power system (OWENS, 2014).

2) Energy Efficiency: In developed countries, DG links energy with the human development. However, in these countries, there is less motivation for new energy supplies. Therefore, Combined Heat and Power (CHP) is the main solution for increasing energy efficiency. CHP systems transfer the heat produced from electricity generation to thermal applications, in which it provides significant economic and environmental benefits for both energy producers and consumers (society). CHP can increase efficiency of system from 36% to 90% by converting electrical power losses heat to useful heat (ELMUBARAK; ALI, 2016).

3) Operation Costs: Averagely, costs of transmission and distribution consist of 30% of the delivery cost of electricity that its main reason is its high losses. The lowest delivery cost is for industrial customers and the highest is for small ones. Thus, DGs decreases the distribution costs, because they installed near to load centers (LOPES et al., 2007).

4) Network Investment: DG can reduce costly investments for constructing new transmission and distribution lines, because transmission and distribution (T&D) budget is the biggest part of the capital investment of power system (almost two thirds) (BORGES, CARMEN; FALCÃO, 2003).

5) Environmental Effects: Greenhouse gas emissions cause numerous health and environmental problems. In United States, the portion of electricity sector in NO_x, CO_2, SO_2 emissions are $^1/_4$, $^1/_3$ and $^2/_3$, respectively. Therefore, DG development is so useful for reduction of the environmental concerns in existing power systems (PEPERMANS et al., 2005).

6) Power Quality Enhancement: DG units affect directly voltage quality and power flow. DG usage can decrease the voltage drops, i.e. improve voltage quality. Thus, increase investment in distributed generation will improve the voltage and therefore power quality especially for larger consumers (PEPERMANS et al., 2005).

1.1.2. DG Challenges

According to type of DG, some disadvantages of distributed generators are mentioned as follows.

1) Power Reliability Degradation: Wind and solar DG units cannot generate electricity continuously, because of uncertainty in wind speed and sun radiation. These power disturbances degrade network stability and security as well as network reliability (SANDHU; THAKUR, 2014).

2) Low Generation Capacity: Because of small size of DG units, their generation capacity is low (SANDHU; THAKUR, 2014).

3) Complexity Increase: The complexity of transmission and distribution systems increases by installation of DG units, therefore network monitoring will be more complicated when DG units are connected to the network.

1.1.3. Renewable DG

In non-renewable DG units such as diesel generators, micro turbines and natural gas-powered generators, the power of DG is provided by fossil fuels, while in renewable distributed generators such as mini-hydro power plants, biomass fueled generators, PV, and wind farms, no pollution and greenhouse gases like NOx and CO_2 are created. Among renewable distributed generators, PV is more economic and efficient in countries with high amount of solar radiation, because free energy that comes from sun, in which the solar radiation is transformed into electricity by photovoltaic cells. The generated electricity by PV is in direct current (DC), which can be converted into AC or stored in batteries.

More than two-thirds of worldwide newly installed electricity capacity is renewable. Growth in consumption of coal and oil could end by 2020 due to increased uptake of renewables and natural gas. In addition, electrification with renewable energy is more efficient and therefore leads to significant reductions in primary energy requirements (MATHIESEN et al., 2015).

1.2. DG ALLOCATION

In distribution systems, the goal of DG allocation is determining the place, size, and type of distributed generators, i.e. the appropriate type of DG with the optimal capacity should be installed at the suitable location of network (AMAN et al., 2013). Losses, voltage profile, and network reliability will be improved by optimal placement of DG (LETSELA; WITKOWSKI; BALKWILL, 2002). Therefore, DG allocation is very important for optimization of the operational conditions of the distribution network (HAMIDI; CHABANLOO, 2018).

The objectives of DG allocation problem can be minimization of distribution losses (active or reactive power), power flow of weak lines and generation costs (DG installation, operation, and maintenance expenses) (BISWAS; GOSWAMI; CHATTERJEE, 2012), maximization of voltage stability (MUTTAQI et al., 2014), spinning reserve, system security and reliability (BORGES FALCÃO, 2006), DG and lines' capacities (RAJ et al., 2008).

Generally, the basic objectives of DG allocation problem are the active power loss minimization and the voltage profile improvement. The majority of literature about DG allocation minimizes the active losses.

The general objective of this thesis is to develop a stochastic programming formulation for the optimal allocation of distributed generation units in electrical distribution systems considering load variation and a set of scenarios for PV generation.

2 OPTIMAL ALLOCATION OF DISTRIBUTED GENERATION

DG allocation (DGA) is an optimization problem with technical and operational constraints that can be solved using classic and metaheuristic methods. Since the DGA problem was proposed, one of the important tools to solve the problem was classical optimization methods. Later, metaheuristics were used in DGA to find high-quality solutions with less computational burden in comparison with mathematical methods.

2.1. CLASSIC OPTIMIZATION METHODS

Classical methods, based on mathematical programming, are widely used to solve the DG allocation problem. They are efficient methods for solving linear optimization problems that guarantees the optimal solution. Nevertheless, a high computational effort could be needed for solving large scaled optimization problems by these methods. Until now, many studies regarding DG allocation have utilized mathematical methods. Rueda-Medina et al. (2013) (RUEDA-MEDINA et al., 2013), proposed a mixed-integer linear programming approach to solve the optimal allocation problem of DG units. That method defines the optimal type, size, and allocation of DG units in order to minimize the total cost (installation and operation costs) considering different topologies and load levels. Linear expressions were used to represent the steady-state of the network, short-circuits, and DG capability curves. The method was implemented in AMPL and solved using CPLEX and was tested using the 33-bus distribution system. Abri et al. (AL ABRI; EL-SAADANY; ATWA, 2013) proposed mixed-integer non-linear programming (MINLP) approach to determine the optimal locations and sizes of DGs, aiming active loss minimization and voltage stability maximization considering load and generation variations. In this model, optimal placement of CHP-based distributed generators in urban distribution systems considering water and gas networks was formulated based on voltage sensitivity indices as an non-linear mathematical programing problem using AIMMS (REFERENCE, 2016); ZHANG; KARADY; ARIARATNAM, 2014).

Also, Acharya et al. (ACHARYA; MAHAT; MITHULANANTHAN, 2006) proposed an analytical technique based on sensitivity analysis to solve DGA problem with objective of distribution loss minimization. In this approach, first, the best sizes of DG on each bus are determined using the exact loss formula when the losses reduction is stopped by increasing DG power injection. Then, the optimal place of DG is identified based on the linear approximated value of losses instead of accurate load flow calculation by installing optimal sizes of DG. The proposed method is computationally efficient, because the DGA problem is solved after two power flow iterations (DG sizes are calculated in the first iteration, while the best site of DG is determined in the second one). However, application of this approach to large-scale distribution networks leads to computational inefficiency because of calculation of the bus impedance matrix.

In order to introduce a comprehensive model for DG placement, Hung et al. (HUNG; MITHULANANTHAN; BANSAL, 2010) included reactive power of DGs in the problem formulation of (ACHARYA; MAHAT; MITHULANANTHAN, 2006). Simulation results reveal that the proposed framework improves the model presented in (ACHARYA; MAHAT; MITHULANANTHAN, 2006). However, this approach cannot be employed for DG allocation in large-scale distribution networks. Mahmoud et al. (MAHMOUD; YORINO; AHMED, 2016) proposed an efficient method for loss minimization in DGA considering DG types. The proposed technique was an integration of analytical method with optimal power flow (OPF). It was concluded that the proposed approach provides faster and more accurate solution than other existing classical techniques. Mena and Martin Garcia (MENA; MARTÍN GARCÍA, 2015) presented an efficient MINLP approach to solve DGA problem considering network losses and generation cost of both conventional power plants and distributed generators. In this method, the problem is divided into two sub-problems, i.e. the optimal place of each DG is found in the first one, while the optimal generation of each location is determined in the second sub-problem. The result evaluation indicates that the proposed method can reach the optimal solution in an acceptable computation time.

In addition, Murty and Kumar (MURTY; KUMAR, 2015) defined a new index known as voltage stability in DGA, aiming active and reactive losses minimization considering future load growth with different load factors. Simulation results indicated that DG utilization with lagged power factor decreases power losses more. Gosh et al. (GHOSH; GHOSHAL; GHOSH, 2010) used a simple Newton-Raphson based search method to find the best location of distributed generators with objective of minimum losses and DG operation cost. Viral and Khatod (VIRAL; KHATOD, 2015) minimized network losses by DG allocation using an analytical technique. The numerical results show that the total number of load flow do not increase with size of system, but the proposed method cannot be applied to meshed distribution networks.

Recently, Rueda-Mendina et al. (RUEDA-MEDINA et al., 2013) and Melgar-Dominguez et al. (MELGAR DOMINGUEZ et al., 2018) formulated the DGA as a mixed-integer linear programming (MILP) problem in AMPL (FOURER; GAY; KERNIGHAN, 2003) and solved it by CPLEX. Rider et al. (RIDER et al., 2013) minimized the investment and operational costs of DG as well as active losses were by optimal placement and sizing of DGs considering load variations and short-circuit level. Also, Melgar-Dominguez et al. (MELGAR DOMINGUEZ et al., 2018) minimized the delivery cost of energy and the investment by DG allocation with consideration of ESSs and capacitor banks.

2.2. METAHEURISTIC OPTIMIZATION METHODS

Solving DGA by mathematical optimization have some limitations like high computational effort. Therefore, metaheuristics can be employed to remove these limitations. These methods find feasible solutions with low computational effort, but they cannot guarantee the global optimum. In this section, some of the most specialized papers that have solved the DGA problem by metaheuristics are presented. In (GANGULY; SAMAJPATI, 2015), an adaptive genetic algorithm (GA) was proposed to solve DGA problem, aiming power loss minimization of radial distribution systems under load and generation uncertainties. The objective function considers a weighted sum of the minimization of power losses and voltage deviation. Results using the IEEE 33-node system and a 52-node Indian distribution network show that the fuzzy-based method is efficient dealing with load growth. Ali et al. (ALI; ABD ELAZIM; ABDELAZIZ, 2017) presented ant lion optimization (ALO) algorithm to minimize network losses and maximize voltage stability by placement of PV and wind turbines distributed generators. ALO is a novel nature-inspired algorithm that adopted from the hunting mechanism of ant lions in nature. Numerical results show that this algorithm can reduce the losses and enhance voltage profile effectively.

Moreover, Sanchez Mora et al. (SANCHEZ MORA; TAMAYO; LOPEZ-LEZAMA, 2018) employed a useful iterative sampling technique known as GRASP to solve DGA problem. Nevertheless, in this method, some difficulties are raised because of the local search procedure. Kansal et al. (KANSAL; KUMAR; TYAGI, 2013) minimized network losses in DGA considering different types of distributed generators using particle swarm optimization (PSO) algorithm. Numerical results show that PSO not only reduces the power losses but minimize DG sizes in large-scale distribution networks more efficiently compered to analytical methods. Later, Karimyan et al. (KARIMYAN et al., 2014) solved the long term DGA problem in order to optimize line losses and voltage profile considering load variations and DG type using PSO. The results show the robustness and good performance of the proposed approach.

In proposal of (NARA et al., 2002), Tabu search (TS) algorithm was applied to determine the best place of DG units in radial distribution systems from viewpoint of loss reduction. Movements and memory are two important components of TS. The solution jumps to another one by movement operation, while search cycles are avoided by memory operator. The results verify TS is an appropriate method to solve the DGA problem. Poornazariyan et al. (POORNAZARYAN et al., 2016) solved DGA problem in order to loss minimization and voltage stability enhancement considering load variations using Imperialistic Competitive Algorithm (ICA). ICA is a new evolutionary algorithm that includes the initial population (countries). Imperialists are countries with low-cost functions and other countries (colonies) are divided among them. Each colony moves toward its best cost function. It was shown that the performance of ICA is better than Cuckoo search algorithm for solving DGA problem.

In following, the proposed DGA is formulated as a constrained stochastic optimization problem, in which objective function components and restrictions are described.

3 PROBLEM FORMULATION

In this research, the problem is the determination of size, location, and type of photovoltaic DG units (decision variables of the problem) in order to minimize investment cost of DGs, cost of energy supplied by substation and DG units under load variation and generation uncertainties (related to solar irradiation). The DGA problem is formulated as a stochastic programming problem because of considering load and generation uncertainties and their occurrence probabilities.

The investment decisions are the optimal number of DG units in the suitable buses of network according to minimum installation cost of distributed generators. The decision variables are integer variables $y_{i,g}$ (number, type and allocation of DG units) as well as real variables active and reactive powers of branches ($P_{ij,d,t}$ and $Q_{ij,d,t}$), branch currents ($I_{ij,d,t}$), active and reactive power generated by DG units ($P^{DG}_{i,d,g,t}$ and $Q^{DG}_{i,d,g,t}$) and substation ($P^{S}_{i,d,t}$ and $Q^{S}_{i,d,t}$), and nodal voltages ($V_{i,d,t}$). The network is represented as receiving buses and end buses that are connected by distribution lines. It should be noted that DG units can be installed on each load buses. The objective function consists of three parts including installation cost, cost of energy supplied by DG units and cost of energy supplied by substation. According to Figure 1, the DGA problem based on minimization of total cost for different load levels is modeled as follows.

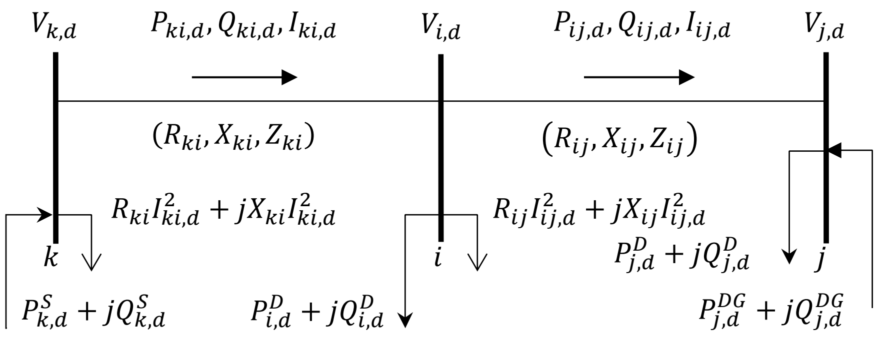


Figure 1: Example network (RUEDA-MEDINA et al., 2013)

$$\min C_T = IC + C_{DG} + C_S \tag{1}$$

where:

$$IC = \sum_{i \in \Omega_b} \sum_{g \in \Omega_g} c_g y_{i,g} \, P_g^{max} \tag{2}$$

$$CE_{DG} = \sum_{t\in\Omega_t}\sum_{g\in\Omega_g}\sum_{d\in\Omega_d}\sum_{i\in\Omega_b}\alpha_{d,t}CDG_{d,g}P^{DG}_{i,d,g,t}Pr_t \tag{3}$$

$$CE_S = \sum_{t\in\Omega_t}\sum_{d\in\Omega_d}\sum_{i\in\Omega_b}\alpha_{d,t}CS_d P^S_{i,d,t}Pr_t \tag{4}$$

s. to:

$$\sum_{ki\in\Omega_l}P_{ki,d,t} - \sum_{ij\in\Omega_l}\left(P_{ij,d,t} + R_{ij}I^2_{ij,d,t}\right) + P^S_{i,d,t} + \sum_{g\in\Omega_g}P^{DG}_{i,d,g,t} = P^D_{i,d} \quad \forall i\in\Omega_b, \forall d\in\Omega_d, \forall t\in\Omega_t \tag{5}$$

$$\sum_{ki\in\Omega_l}Q_{ki,d,t} - \sum_{ij\in\Omega_l}\left(Q_{ij,d,t} + X_{ij}I^2_{ij,d,t}\right) + Q^S_{i,d,t} + \sum_{g\in\Omega_g}Q^{DG}_{i,d,g,t} = Q^D_{i,d} \quad \forall i\in\Omega_b, \forall d\in\Omega_d, \forall t\in\Omega_t \tag{6}$$

$$V^2_{i,d,t} - 2\left(R_{ij}P_{ij,d,t} + X_{ij}Q_{ij,d,t}\right) - \left(R^2_{ij} + X^2_{ij}\right)I^2_{ij,d,t} - V^2_{j,d,t} = 0 \quad \forall ij\in\Omega_l, \forall d\in\Omega_d, \forall t\in\Omega_t \tag{7}$$

$$V^2_{j,d,t}I^2_{ij,d,t} = P^2_{ij,d,t} + Q^2_{ij,d,t} \qquad \forall ij\in\Omega_l, \forall d\in\Omega_d, \forall t\in\Omega_t \tag{8}$$

$$V^2_{min} \le V^2_{i,d,t} \le V^2_{max} \qquad \forall i\in\Omega_b, \forall d\in\Omega_d, \forall t\in\Omega_t \tag{9}$$

$$0 \le I^2_{ij,d,t} \le \left(I^{max}_{ij}\right)^2 \qquad \forall ij\in\Omega_l, \forall d\in\Omega_d, \forall t\in\Omega_t \tag{10}$$

$$0 \le P^{DG}_{i,d,g,t} \le f^{DG}_t P^{max}_g y_{i,g} \qquad \forall i\in\Omega_b, \forall d\in\Omega_d, \forall g\in\Omega_g, \forall t\in\Omega_t \tag{11}$$

$$Q^{min}_g y_{i,g} \le Q^{DG}_{i,d,g,t} \le Q^{max}_g y_{i,g} \qquad \forall i\in\Omega_b, \forall d\in\Omega_d, \forall g\in\Omega_g, \forall t\in\Omega_t \tag{12}$$

$$-P^{DG}_{i,d,g,t}\tan(\arccos(pf^{DG})) \le Q^{DG}_{i,d,g,t} \le P^{DG}_{i,d,g,t}\tan(\arccos(pf^{DG})) \quad \forall i\in\Omega_b, \forall d\in\Omega_d, \forall g\in\Omega_g, \forall t\in\Omega_t \tag{13}$$

$$-P^S_{i,d,t}\tan\left(\arccos\left(pf^S_{min}\right)\right) \le Q^S_{i,d,t} \le P^S_{i,d,t}\tan(\arccos(pf^S_{min})) \quad \forall i\in\Omega_b, \forall d\in\Omega_d, \forall g\in\Omega_g, \forall t\in\Omega_t \tag{14}$$

$$0 \le \sum_{i\in\Omega_b}\sum_{g\in\Omega_g}y_{i,g} \le N^{max}_{DG} \tag{15}$$

$$0 \le y_{i,g} \le N^{max}_{DG,i,g} \qquad \forall i\in\Omega_b \tag{15}$$

$$\sum_{i\in\Omega_b}\sum_{g\in\Omega_g}y_{i,g}P^{max}_g \le \beta max\left\{\sum_{i\in\Omega_b}P^D_{i,d}\right\} \tag{17}$$

First part of objective function (1) describes annual investment of distributed generators, in which number of DG units on each bus is multiplied by generation cost of each DG unit on the same bus. In (2), Ω_b and Ω_g are the set of buses and DG units, respectively. Also, c_g is annualized installation cost of DG of type g ($) and $y_{i,g}$ is binary decision variable for allocation of DG type g. The second part calculates the expected cost of energy generated by DGs, i.e. total generation cost of DG units is equal to cost of energy generated by each DG in each load level and duration at each scenario multiplied by the occurrence probability of that scenario. In (3), Ω_t and Ω_d are set of scenarios and load levels, respectively. Moreover, $\alpha_{d,t}$ is number of days in one year of scenario *t* in load level d (h) and $CDG_{d,g}$ is energy cost for DG of type g (\$/kWh). $P_{i,d,g,t}^{DG}$ is active power provided by DG of type g on bus *i* at load level d in scenario *t* (kW) and Pr_t is probability of Scenario *t*. The third one indicates energy generated by substation. In simple terms, total generation cost of substations is equal to cost of energy generated by each substation in each load level and duration at each scenario multiplied by the occurrence probability of that scenario. In (4), CS_d and $P_{i,d,t}^{S}$ are the energy cost for substation (\$/kWh) and active power provided by substation on bus *i* at load level *d* in scenario *t* (kW), respectively.

Equation (5) is the nodal active power balance in presence of DG showing that total active power injected to each bus by lines, substation and DG units are equal to total active power consumed by lines and loads and active losses of lines connected to that bus. In this equation, Ω_l and R_{ij} are set of all branches and resistance of branch *ij* (Ω), respectively. $P_{ij,d,t}$ and $P_{ki,d,t}$ are active power of branch *ij* and *ki* (kW), respectively, and $I_{ij,d,t}$ is current magnitude of branch *ij* (A) at load level *d* in scenario *t*. Also, $P_{i,d}^{D}$ indicates active power demanded at node *i* in load level *d* (kW). Equation (6) represents the nodal reactive power balance in presence of DG. It means that total reactive power injected to each bus by lines, substation and DG units are equal to total reactive power consumed by lines and loads and active losses of lines connected to that bus. In (6), $Q_{ij,d,t}$ and $Q_{ki,d,t}$ are reactive power of branch *ij* and *ki* at load level *d* in scenario *t* (kVAr), respectively. In addition, $Q_{i,d,g,t}^{DG}$ is reactive power provided by DG of type g and $Q_{i,d,t}^{S}$ is reactive power provided by substation on bus *i* at load level *d* in scenario *t* (kVAr), respectively. $Q_{i,d}^{D}$ represents reactive power demanded at node *i* in load level *d* (kVAr).

Equation (7) indicates the Kirchhoff voltage law (KVL) (net summation of voltage magnitudes in each loop is zero), in which $V_{i,d,t}$ and $V_{j,d,t}$ are voltage magnitudes on buses *i* and *j* at load level d in scenario t (kV), respectively, and X_{ij} is reactance of branch ij (Ω^{-1}). Equation (8) explains relationship between apparent power of each branch and its active and reactive components. Also, (9) shows that voltage magnitude of each bus is limited by its minimum and maximum values, in which V_{min} and V_{max} are minimum and maximum voltage magnitudes (kV). Expression (10) indicates that current of each branch is between zero and its maximum amount, where I_{ij}^{max} is maximum current magnitude of branch ij (A).

Furthermore, (11) illustrates that active power generated by each DG unit must be less or equal to its maximum active power capacity multiplied by its generation factor. In this equation, P_{g}^{max} is maximum active power provided by DG of type g (kW) and f_{t}^{DG} is DG generation factor in scenario t. Constraint (12) describes that reactive power generated by each distributed generator is between its minimum and maximum reactive generation capability. In (12), Q_{g}^{min} and Q_{g}^{max} are minimum and maximum reactive power provided by DG of type g (kVAr), respectively, and pf^{DG} is DG power factor limit. Expressions (13) and (14) explain reactive power generated by a DG and a substation are limited by their active generation and power factors, in which $pf_{min}^{S}$is minimum leading and lagging substation power factor.

Moreover, (15) and (16) show maximum numbers of DG that can be installed in network (N_{DG}^{max}) and maximum numbers of each DG type that can be installed on each bus ($N_{DG,i,g}^{max}$) because of investment and technical limitations, respectively. Expression (17) show that maximum active power provided by DG units are equal to a fraction ($0 < \beta \leq 1$) of total active peak load of system. This equation shows that maximum number of DG units that are installed in network are limited because N_{DG}^{max} should be chosen so that total generation of DG units does not exceed a given percentage, defined percent, of the total peak load.

3.1. MIXED-INTEGER CONIC FORMULATION OF THE DGA

The proposed DGA problem including binary variables ($y_{i,g}$) and real variables ($I_{ij,d,t}$, $V_{i,d,t}$, $P_{ij,d,t}$, $Q_{ij,d,t}$, $P_{i,d,t}^{S}$, $Q_{i,d,t}^{S}$, $P_{i,d,g,t}^{DG}$, and $Q_{i,d,g,t}^{DG}$) cannot be solved by convex commercial tools because of non-linear terms $I_{ij,d,t}^{2}$ and $V_{j,d,t}^{2}$. Thus, the model has to be linearized using various linear programing methods or converted to a mixed-integer conic formulation using the variable change technique used in proposal of (RUEDA-MEDINA et al., 2013). The variable change method is easier and more accurate than the linearization, because many assumptions and approximations need to be considered in linear optimization methods that can decrease the quality of solutions for large-scale distribution systems. Here, the variable change method was used to reformulate the problem by replacing square variables $I_{ij,d,t}^2$, $V_{j,d,t}^2$ and $V_{i,d,t}^2$ with $I_{ij,d,t}^{sqr}$, $V_{j,d,t}^{sqr}$ and $V_{i,d,t}^{sqr}$, respectively, as follows.

$$\min C_T = \sum_{i\in\Omega_b}\sum_{g\in\Omega_g} c_g y_{i,g} P_g^{max} + \sum_{t\in\Omega_t}\sum_{g\in\Omega_g}\sum_{d\in\Omega_d}\sum_{i\in\Omega_b} \alpha_{d,t} CDG_{d,g} P_{i,d,g,t}^{DG} Pr_t + \sum_{t\in\Omega_t}\sum_{d\in\Omega_d}\sum_{i\in\Omega_b} \alpha_{d,t} CS_d P_{i,d,t}^{S} Pr_t \tag{16}$$

Subjected to (11)-(17) and:

$$\sum_{ki\in\Omega_l} P_{ki,d,t} - \sum_{ij\in\Omega_l}\left(P_{ij,d,t} + R_{ij} I_{ij,d,t}^{sqr}\right) + P_{i,d,t}^{S} + \sum_{g\in\Omega_g} P_{i,d,g,t}^{DG} = P_{i,d}^{D} \quad \forall i \in \Omega_b, \forall d \in \Omega_d, \forall t \in \Omega_t \tag{19}$$

$$\sum_{ki\in\Omega_l} Q_{ki,d,t} - \sum_{ij\in\Omega_l}\left(Q_{ij,d,t} + X_{ij} I_{ij,d,t}^{sqr}\right) + Q_{i,d,t}^{S} + \sum_{g\in\Omega_g} Q_{i,d,g,t}^{DG} = Q_{i,d}^{D} \quad \forall i \in \Omega_b, \forall d \in \Omega_d, \forall t \in \Omega_t \tag{20}$$

$$V_{i,d,t}^{sqr} - 2\left(R_{ij}P_{ij,d,t} + X_{ij}Q_{ij,d,t}\right) - \left(R_{ij}^2 + X_{ij}^2\right) I_{ij,d,t}^{sqr} - V_{j,d,t}^{sqr} = 0 \quad \forall ij \in \Omega_l, \forall d \in \Omega_d, \forall t \in \Omega_t \tag{21}$$

$$P_{ij,d,t}^2 + Q_{ij,d,t}^2 \le V_{j,d,t}^{sqr} I_{ij,d,t}^{sqr} \quad \forall ij \in \Omega_l, \forall d \in \Omega_d, \forall t \in \Omega_t \tag{22}$$

$$V_{min}^2 \le V_{i,d,t}^{sqr} \le V_{max}^2 \quad \forall i \in \Omega_b, \forall d \in \Omega_d, \forall t \in \Omega_t \tag{23}$$

$$0 \le I_{ij,d,t}^{sqr} \le \left(I_{ij}^{max}\right)^2 \quad \forall ij \in \Omega_l, \forall d \in \Omega_d, \forall t \in \Omega_t \tag{24}$$

Where, (22) converts the non-convex equation (8) to a convex equation in order to have a convex optimization problem. Therefore, the above mixed integer non-linear programming (MINLP) problem can be recast as a second-order conic programming problem. This convex formulation ensures that optimal solutions can be obtained and can be solved by commercial solvers such as CPLEX.

4 CASE STUDY

In order to verify the efficiency of the proposed model, the formulation was used to study the DGA on the 33-bus and 136-bus distribution systems using a computer with a 64-bit processor and an Intel i7 3.6GHz processor. Solution times for 33-bus and 136-bus test systems are 595.81 seconds (s) and 207252 s, respectively. It should be mentioned that β, V_{min}, and V_{max} were considered to be 0.5, 0.9 per unit (p.u.), and 1 p.u., respectively, the voltage magnitude of substation nodes has been fixed on 1 p.u and power factor limit was adopted 0.85.

In the stochastic programming model, a set of generation scenarios are considered for the stochastic DG allocation. However, there are many different scenarios for photovoltaic DG generation profile that considering all of them increases the computational time of DGA problem considerably. On the other hand, there are many similar scenarios that one of them can be selected as a sample to analyze the model. In fact, the network performance can be properly estimated by selecting a few appropriate scenarios (sample scenarios). A way to reduce the computational burden is using scenario reduction method. In this method, scenarios with low or similar probabilities are eliminated from set of scenarios. Franco et al. (FRANCO; OCHOA; ROMERO, 2018) considered four important generation scenarios for photovoltaic DG units as shown in Figure 2. Therefore, in present dissertation, these four generation profiles ($\Omega_t = \{1, 2, 3, 4\}$) are considered to represent uncertainty in PV generation that each scenario indicates situation of solar irradiation. For example, t=1 indicates that generation of PV is zero in night because of lack of solar irradiation, or t=2 shows that PV works with full capacity in sunny hours.

Also, different load levels (daily load profile), as shown in Figure 3, are considered for the DGA problem in order to show load variations. Table 1 lists capacity, energy cost and investment of different DG types for both case studies. According to data given in proposal of (MARGOLIS; FELDMAN; BOFF, 2018), investment costs (*P*) of DG types 1, 2 and 3 are 3510 $/kW, 2650 $/kW, and 2040 $/kW, respectively, for average lifetime (*n*) 20 years. Therefore, annual installation costs (*A*) of PV units are calculated considering annual interest rate (*i*) of 8% according to equation (25) (LEBLOND et al., [s.d.])) (please see column 5 of Table 1).

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} \tag{25}$$

Also, energy costs (operation and maintenance costs) of DG types 1, 2, and 3 are 19 $/kW-year, 19 $/kW-year, and 16 $/kW-year, respectively (THEO et al., 2017). Furthermore, energy cost for substation is considered to be 0.3 $/kWh obtained from (RUEDA-MEDINA et al., 2013).

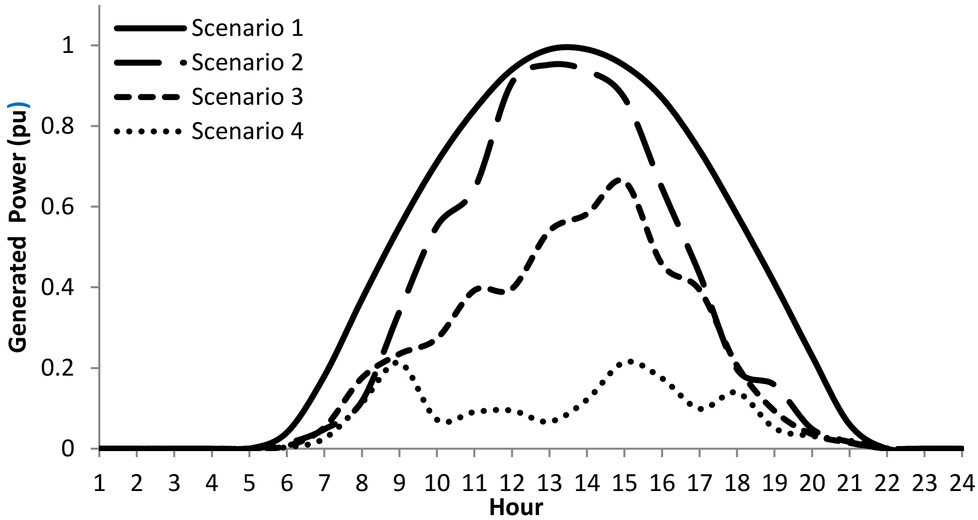


Figure 2: PV generation profile for each scenario (FRANCO; OCHOA; ROMERO, 2018)

Table 1: Capacity, and installation and energy costs for different DG types (THEO et al., 2017)

Type	Maximum Active Power (kW)	Maximum reactive power (kVAr)	Minimum reactive power (kVAr)	Installation cost ($/kW-yr)	Energy cost ($/kWh)
1	100	40	-40	357.50	0.0022
2	500	200	-200	269.90	0.0022
3	1000	400	-400	207.78	0.0018

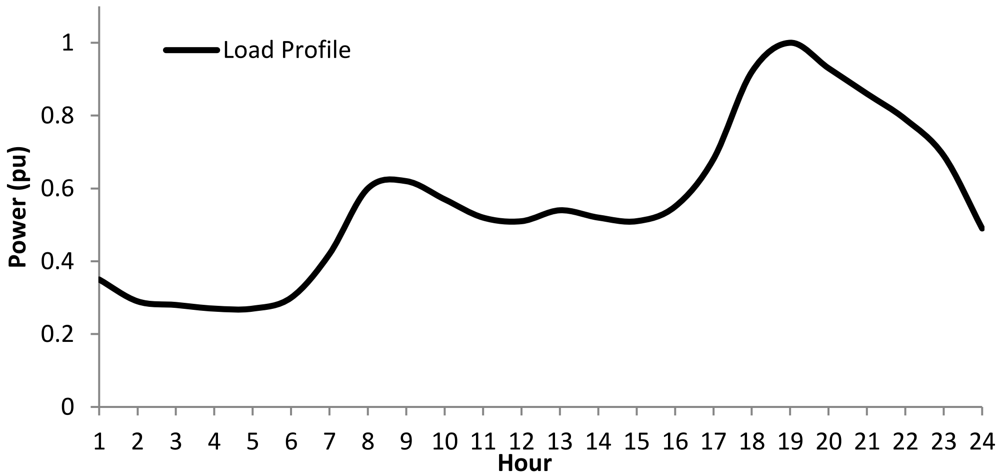


Figure 3: Normalized load profile (FRANCO; OCHOA; ROMERO, 2018)

4.1. 33-BUS TEST SYSTEM

The proposed DGA problem (set of equations (11)–(24)) was applied to 33-bus distribution system, as shown in Figure 4, (BARAN; WU, 1989) using CPLEX in AMPL under four scenarios and daily load level ($\Omega_d = \{1, 2, 3, \dots, 24\}$) considering allocation of photovoltaic DG units. Probability of each scenario is considered to be equal, i.e. 1 divided by number of scenarios (25%). The data of 33-bus distribution network is given in Appendix A. The distribution system was simulated with and without DG units, in which the voltage profile for peak load in different scenarios is shown in Figure 5.

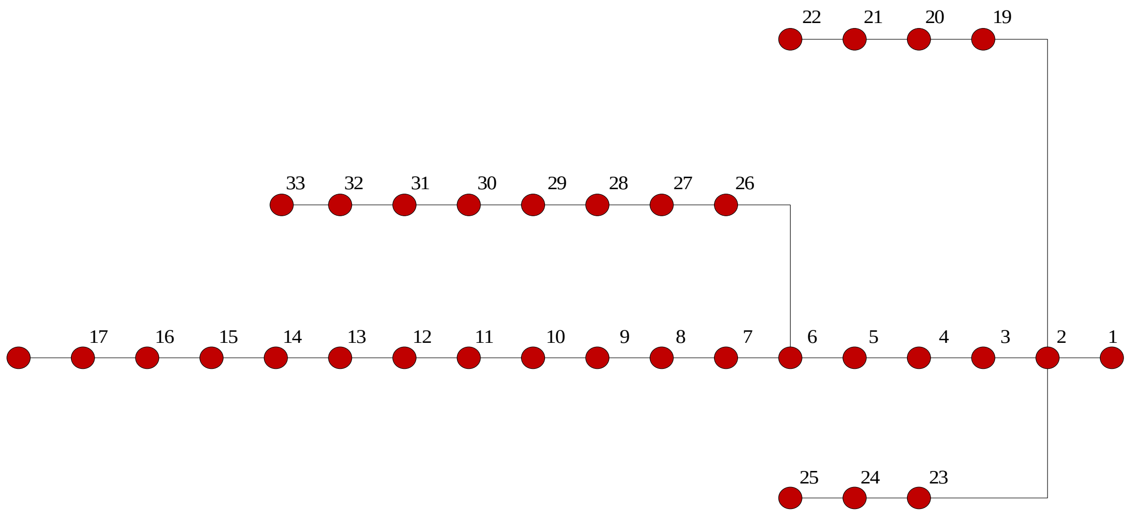
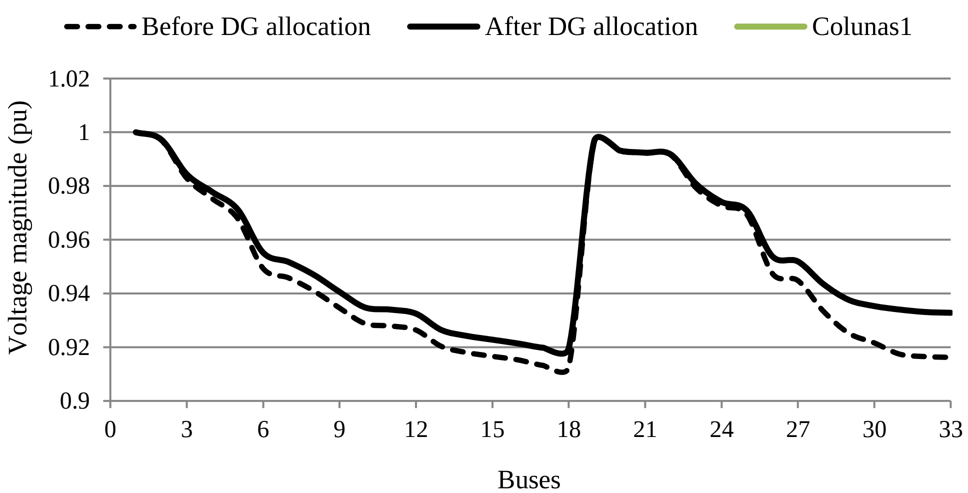
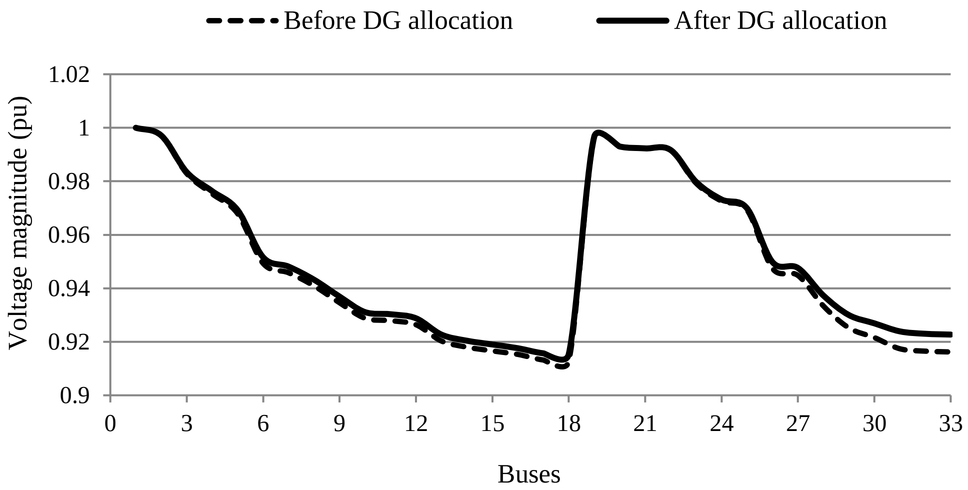


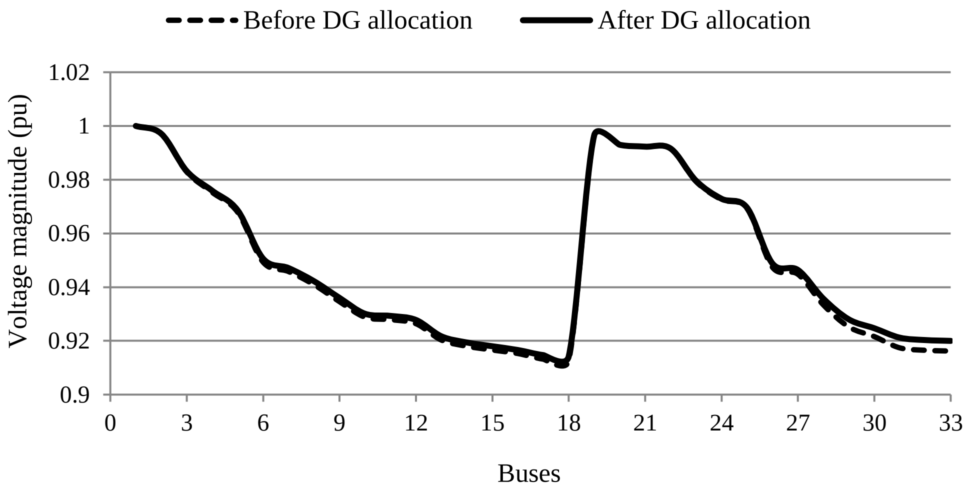
Figure 4: IEEE 33-bus Test System



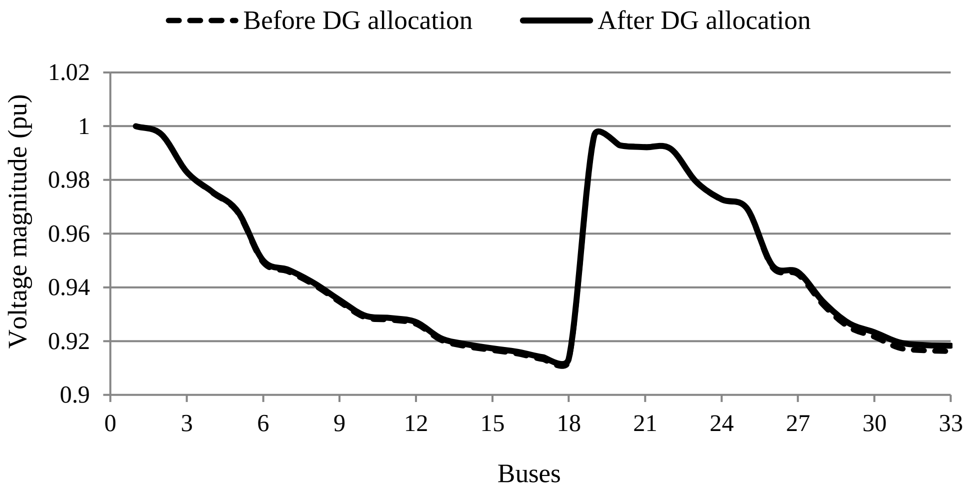
a) Scenario 1



b) Scenario 2



c) Scenario 3



d) Scenario 4

Figure 5: Voltage profile before and after DG allocation in different scenarios for IEEE 33-bus Test System

Table 1 shows the results about selected buses for installation of each type of DGs, number of installed DG units and their generation. Also Table 2 describes investment and cost of energy generated by DGs and substation. In addition, generation at substation bus and network losses are compared in Figures 6 and 7 before and after DG allocation, respectively.

Table 1: Optimal place, type, number and generation of selected DGs for 33-bus network

Bus	Number			Generation in Every Scenario on Each Bus (kWh)			
	Type 1	Type 2	Type 3	t=1	t=2	t=3	t=4
18	2	0	0	0.1394	0.1016	0.0667	0.0227
31	0	0	1	0.6962	0.5070	0.3332	0.1129
32	0	1	0	0.3484	0.2538	0.1666	0.0565
33	1	0	0	0.0700	0.0510	0.0334	0.0114

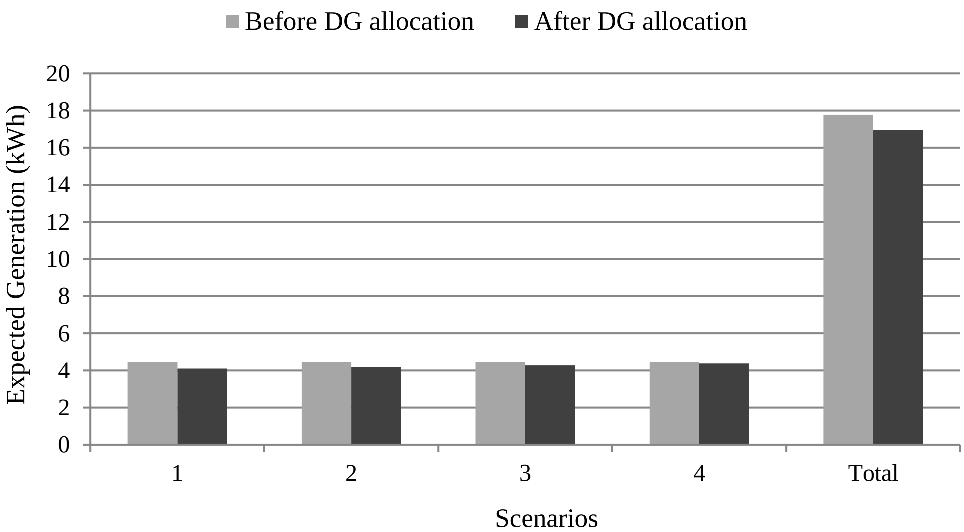


Figure 6: Generation at substation bus for 33-bus network

Table 2: The annual investment and operation costs for 33-bus network ($)

	Before DG allocation	After DG allocation
IC [1]	0	449980.3
C_{DG} [2]	0	4696.73
C_S [3]	17044786.84	16268149.95
C_T [4]	17044786.84	16722826.98

[1]Investment cost, [2]Cost of energy generated by DGs, [3]Cost of energy generated by substation, [4]Total cost

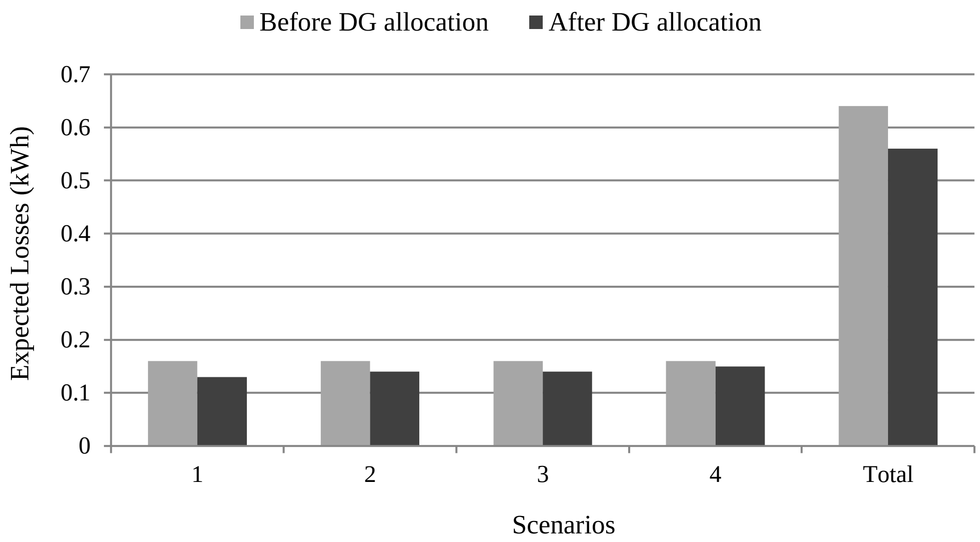


Figure 7: Active power losses for 33-bus network

From Table 1, it can be seen that one DG unit and two DG units with capacity of 100 kW on buses 18 and 33, one 500-kW DG unit on bus 32, and one DG unit with capacity of 1000 kW on bus 31 are needed to be installed. This fact causes total expected network losses (please see Figure 7) is reduced by 0.08 kWh (12. 5%).

According to Figure 6, total expected power generated at substation bus is reduced by 0.81 kWh (5.1%) after DG allocation. It means that rest of generation is compensated by DG units.

The results reported in Table 2 indicate that installation of DG units causes 449980.3 $ and 4696.73 $ more for investment and operation costs of DG units. However, it can decrease the cost of energy generation of substation by 776636.89 $ (4.55%) and therefore 321959.86 $ savings in total cost.

From Figure 5, it can be seen that the voltage profile is improved after DG installation, in which the best improvement happens in scenario 1.

4.2. 136-BUS TEST SYSTEM

The proposed formulation was tested on 136-bus distribution network under four scenarios and daily load level considering and disregarding PV units. It is a part of Treês Lagoas distribution network in Brazil that its schematic diagram is shown in Figure 8 and its data is given in Appendix B. Also, probability of each scenario is 25%. Figure 9 shows the voltage profile for peak load in different scenarios.

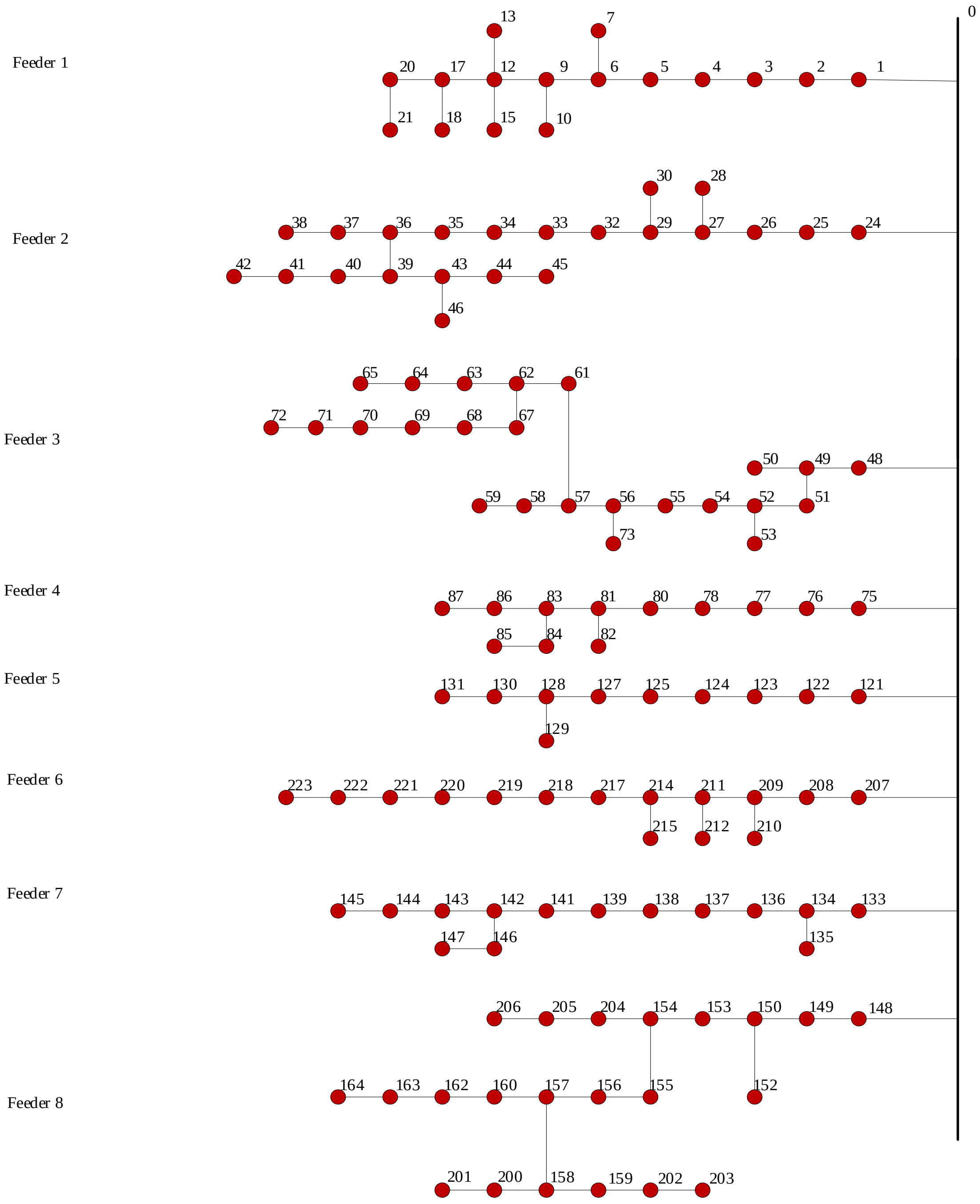
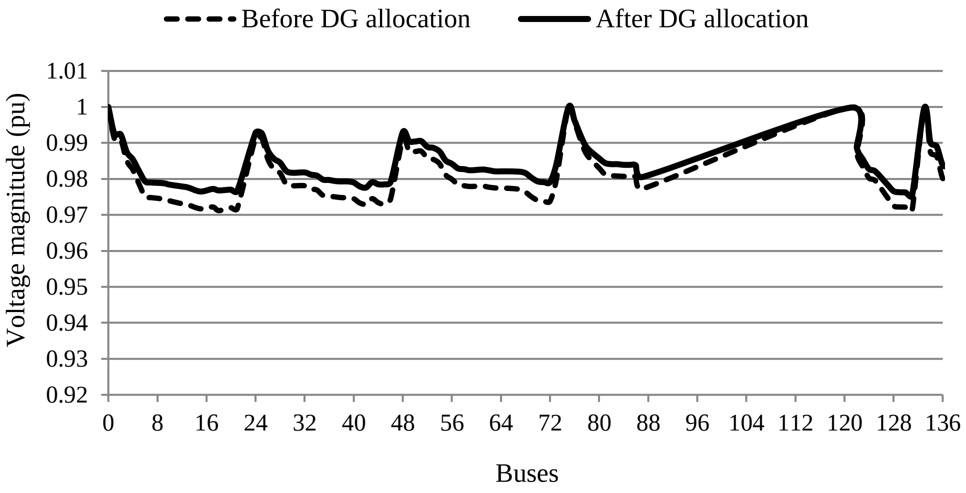
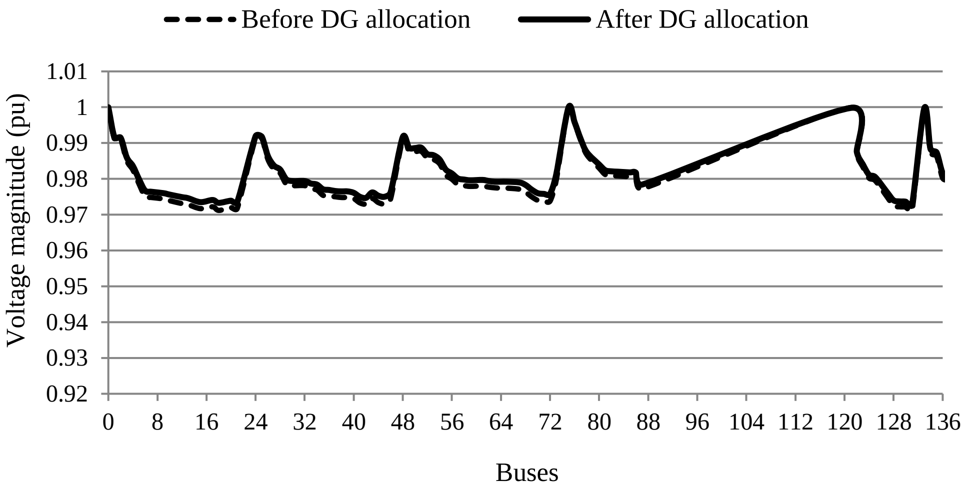


Figure 8: Diagram of 136-bus distribution system (MANTOVANI; CASARI; ROMERO, 2000)

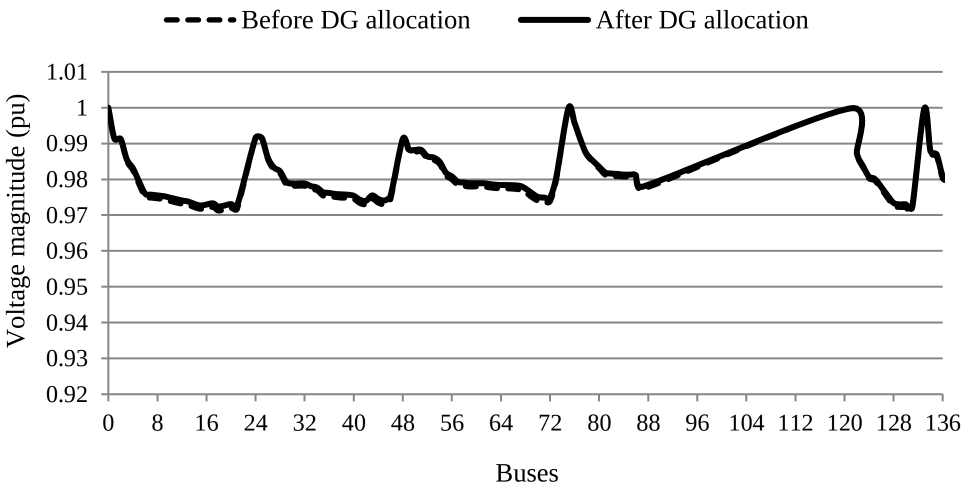
Also, Table 3 lists optimal buses for installation of each type of DG, number of required DG units and their generation. Furthermore, Table 4 describes investment and generation cost of PV units and substation. Moreover, Figures 10 and 11 illustrate substation bus generation and active losses before and after DG installation, respectively.



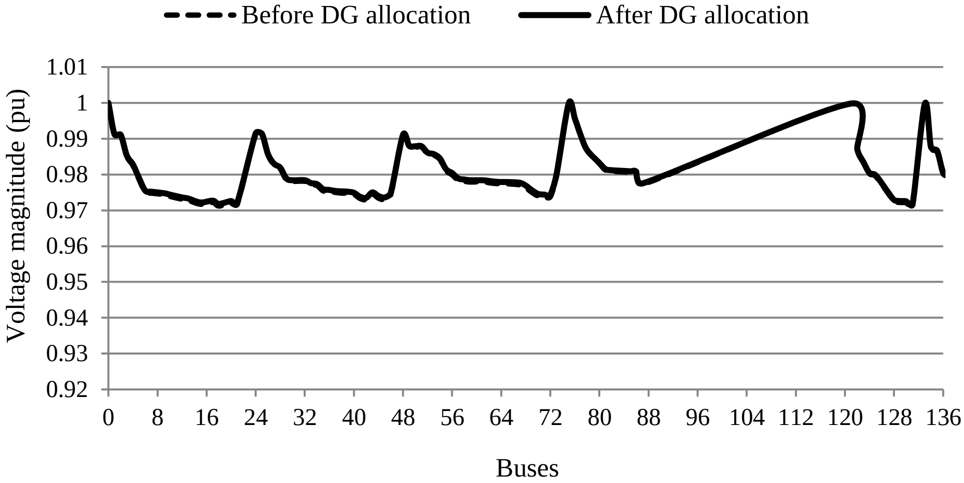
a) Scenario 1



b) Scenario 2



c) Scenario 3



d) Scenario 4

Figure 9: Voltage profile before and after DG allocation in different scenarios for IEEE 136-bus Test System

Table 3: Optimal place, number and generation of selected DGs for 136-bus network

Bus	Number			Generation in Every Scenario on Each Bus (kWh)			
	Type 1	Type 2	Type 3	*t*=1	*t*=2	*t*=3	*t*=4
12	0	1	0	4723.9436	3439.8578	2259.5371	766.1263
18	0	1	0	4723.8031	3439.7464	2259.5372	766.1265
35	0	1	0	4722.9389	3439.0064	2259.5366	766.1259
45	0	1	0	4722.1478	3438.4688	2259.5367	766.1261
49	0	1	0	4145.2614	3237.8949	2259.5357	766.1249
56	0	1	0	4709.1486	3434.8793	2259.5362	766.1255
68	0	1	0	4675.0970	3432.8923	2259.5363	766.1258
83	0	1	0	4723.5758	3439.5030	2259.5363	766.1255
121	0	1	0	4564.7777	3407.6238	2259.5349	766.1238
128	0	1	0	4724.0434	3440.1080	2259.5369	766.1262
134	0	2	0	9425.4331	6869.7216	4518.4805	1531.6556
141	0	1	0	4721.4570	3438.2535	2259.5362	766.1256
155	0	2	0	9444.4688	6878.8328	4518.4864	1531.6613
158	0	1	0	4697.0439	3438.0799	2259.5413	766.1309
203	1	0	0	945.5404	688.6946	452.3912	153.7093
217	0	1	0	4715.1912	3434.6810	2259.5361	766.1254
221	0	1	0	4706.6682	3433.5158	2259.5363	766.1257

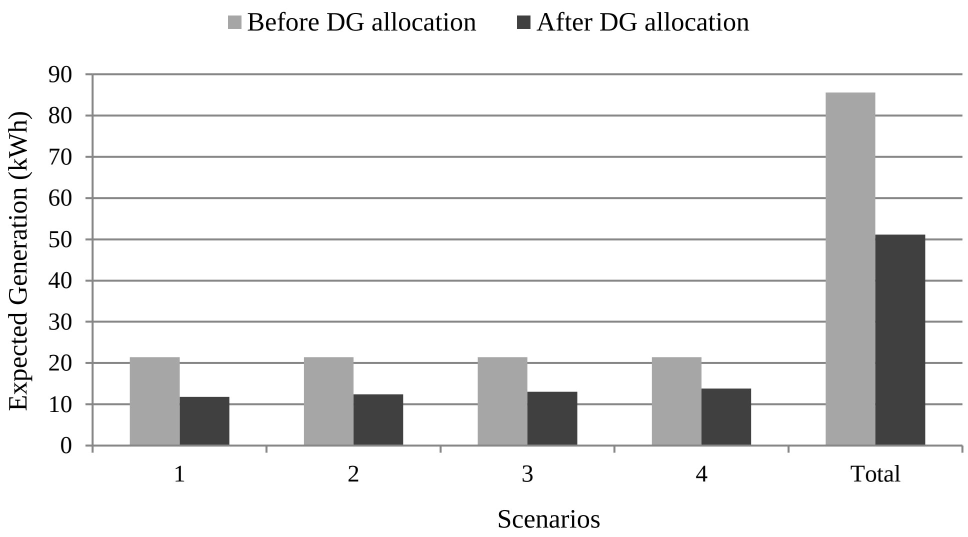


Figure 10: Generation at substation bus for 136-bus network

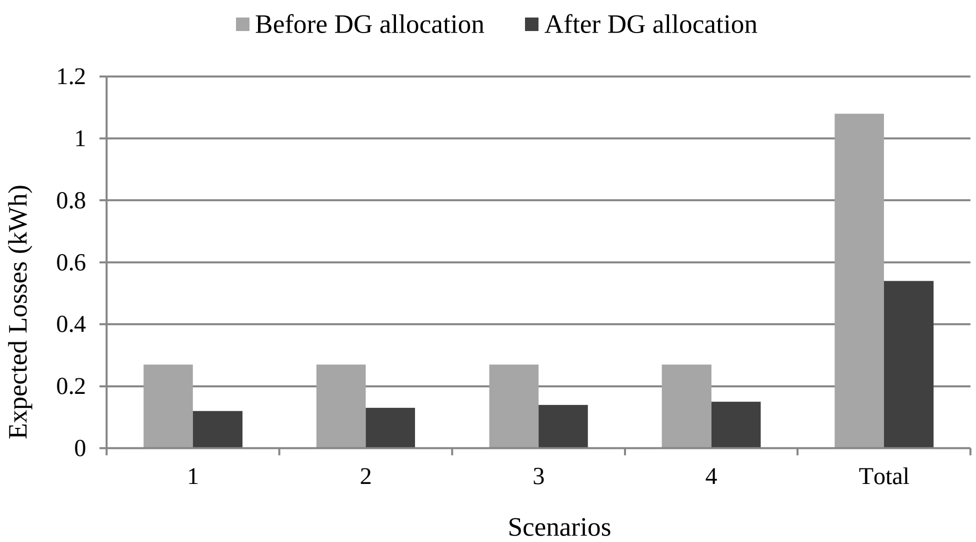


Figure 11: Active power losses for 136-bus network

Table 4: The annual investment and operation costs for 136-bus network ($)

	Before DG allocation	After DG allocation
IC	0	2464850.1
C_{DG}	0	40649.46
C_S	82115395	49041739
C_T	82115395	51547238.57

According to Figure 11, the total network losses is reduced by 0.54 kWh (50%) because eighteen 500-kW and one 100-kW PV units are installed in network (please see Table 3).

According to Figure 10, power generated by substation bus is reduced by

34.4693 kW (40.26%) after DG allocation. The results reported in Table 4 indicate that installation of DG units causes 2464850.1 $ and 40649.46 $ more for investment and operation costs of DG units. However, it can decrease the cost of energy generation of substation by 33073656 $ (40.27%) and therefore 30568156.43 $ savings in total cost.

From Figure 9, it can be seen that the voltage profile is improved after DG installation, especially in scenario 1.

5 CONCLUSIONS

Distribution network has important role for delivering electrical power generated by power stations to consumers, in which its power losses is more than generation and transmission systems. A way for reduction of this losses is optimal allocation of distributed generation (DG) units in distribution network. Optimal allocation of DG units means finding the suitable place, appropriate type and optimal size of distributed generators in distribution network in order to reduce total costs. Also, existing uncertainties in network such as load changes and generation variations should be included in the problem.

Power generation of PV units is variable and depends on sun radiation and is a probable event. Therefore, in this research, a stochastic formulation is presented for the optimal allocation of DG units in distribution networks under load and generation uncertainties. The daily load variations and different scenarios for solar generation are considered in the proposed model. The objective is to minimize cost of energy generated by substation as well as investment and operation costs of distributed generators. The proposed DGA is a constrained mixed-integer conic optimization problem that is solved by CPLEX in AMPL. CPLEX is an optimization solver to solve linear optimization problems using mathematical classic methods that has good performance in finding high-quality solutions. The proposed model is tested on IEEE 33-bus and 136-bus distribution networks considering and without considering distributed generators under daily load variations and generation uncertainty of PV units.

The simulation results show that although employing DG units increases the investment and operation cost of distributed generators, the cost of energy generated by substation is reduced, decreasing total cost of network. In other words, although DG installation imposes investment and operation costs of distributed generators to network, it leads to cost savings. DG units can reduce network losses by generating the power at the load points and therefore they reduce the network costs.

In future research, DG allocation considering demand response (DR) will be formulated as a mixed-integer conic stochastic programming problem under different types of DG such as wind and solar generating units. DR is defined as changes in electric usage by end use customers from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices. Consequently, the DR can be seen as a negative load or even a virtual DG and so, it can be considered as a new solution along with DG allocation problem.

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APPENDIX

Data of both case study systems are described in this section.

APPENDIX A – 33-BUS SYSTEM DATA

Data of 33-bus test system is listed in Tables A1–A7.

Table A1: Load variations

Hour	Load factor	Hour	Load factor	Hour	Load factor	Hour	Load factor
1	0.35	7	0.42	13	0.54	19	1
2	0.29	8	0.60	14	0.52	20	0.93
3	0.28	9	0.62	15	0.51	21	0.86
4	0.27	10	0.57	16	0.55	22	0.79
5	0.27	11	0.52	17	0.68	23	0.69
6	0.30	12	0.51	18	0.92	24	0.49

Table A2: Branch characteristics

Branch	Resistance (Ω)	Reactance (Ω)	Maximum current (A)
1-2	0.0922	0.0477	300
2-3	0.4930	0.2511	300
3-4	0.3660	0.1864	300
4-5	0.3811	0.1941	300
5-6	0.8190	0.7070	300
6-7	0.1872	0.6188	300
7-8	0.7114	0.2351	300
8-9	1.0300	0.7400	300
9-10	1.0440	0.7400	300
10-11	0.1966	0.0650	300
11-12	0.3744	0.1238	300
12-13	1.4680	1.1550	300
13-14	0.5416	0.7129	300
14-15	0.5910	0.5260	300
15-16	0.7463	0.5450	300
16-17	1.2890	1.7210	300
17-18	0.7320	0.5740	300
2-19	0.1640	0.1565	300
19-20	1.5042	1.3554	300
20-21	0.4095	0.4784	300
21-22	0.7089	0.9373	300
3-23	0.4512	0.3083	300
23-24	0.8980	0.7091	300
24-25	0.8960	0.7011	300
6-26	0.2030	0.1034	300
26-27	0.2842	0.1447	300
27-28	1.0590	0.9337	300
28-29	0.8042	0.7006	300
29-30	0.5075	0.2585	300
30-31	0.9744	0.9630	300
31-32	0.3105	0.3619	300
32-33	0.3410	0.5302	300

Table A3: Load data

bus	Active power ($P_{i,d}^D$) (kW)	Reactive power ($Q_{i,d}^D$) (kVAr)	bus	$P_{i,d}^D$ (kW)	$Q_{i,d}^D$ (kVAr)	bus	$P_{i,d}^D$ (kW)	$Q_{i,d}^D$ (kVAr)
1	0	0	12	60	35	23	90	50
2	100	60	13	60	35	24	420	200
3	90	40	14	120	80	25	420	200
4	120	80	15	60	10	26	60	25
5	60	30	16	60	20	27	60	25
6	60	20	17	60	20	28	60	20
7	200	100	18	90	40	29	120	70
8	200	100	19	90	40	30	200	600
9	60	20	20	90	40	31	150	70
10	60	20	21	90	40	32	210	100
11	45	30	22	90	40	33	60	40

Table A4: PV generation profile in Scenario 1

Hour	Power factor	Hour	Power factor	Hour	Power factor	Hour	Power factor
1	0	7	0.18	13	0.99	19	0.41
2	0	8	0.37	14	0.99	20	0.23
3	0	9	0.55	15	0.95	21	0.06
4	0	10	0.71	16	0.87	22	0
5	0	11	0.84	17	0.74	23	0
6	0.04	12	0.94	18	0.58	24	0

Table A5: PV generation profile in Scenario 2

Hour	Power factor	Hour	Power factor	Hour	Power factor	Hour	Power factor
1	0	7	0.0481	13	0.9520	19	0.1579
2	0	8	0.1175	14	0.9370	20	0.0490
3	0	9	0.3394	15	0.8695	21	0.0167
4	0	10	0.5513	16	0.6466	22	0.0006
5	0	11	0.6443	17	0.4335	23	0
6	0.0134	12	0.9073	18	0.1970	24	0

Table A6: PV generation profile in Scenario 3

Hour	Power factor	Hour	Power factor	Hour	Power factor	Hour	Power factor
1	0	7	0.0529	13	0.54	19	0.0935
2	0	8	0.1754	14	0.582	20	0.0357
3	0	9	0.2348	15	0.6627	21	0.0144
4	0	10	0.273	16	0.4575	22	0.0003
5	0	11	0.3926	17	0.3937	23	0
6	0.0067	12	0.3966	18	0.2062	24	0

Table A7: PV generation profile in Scenario 4

Hour	Power factor	Hour	Power factor	Hour	Power factor	Hour	Power factor
1	0	7	0.0251	13	0.0669	19	0.0505
2	0	8	0.1134	14	0.1214	20	0.0316
3	0	9	0.2133	15	0.2144	21	0.0199
4	0	10	0.0719	16	0.1745	22	0.0008
5	0	11	0.0909	17	0.1745	23	0
6	0.0037	12	0.0946	18	0.1393	24	0

APPENDIX B – 136-BUS SYSTEM DATA

Data of 136-bus real test system is listed in Tables B1 and B2. It should be mentioned that load variations, PV generation profiles in four scenarios, and data of DG costs are similar to 33-bus test system.

Table B1: Branch characteristics

Branch	Resistance (R_{ij}) (Ω)	Reactance (X_{ij}) (Ω)	Branch	R_{ij} (Ω)	X_{ij} (Ω)	Branch	R_{ij} (Ω)	X_{ij} (Ω)
0-1	0.33205	0.76653	54-55	0.13132	0.30315	138-139	0.33205	0.76653
1-2	0.00188	0.00433	55-56	0.06191	0.14291	139-141	0.08442	0.19488
2-3	0.22324	0.51535	56-57	0.11444	0.26417	141-142	0.13320	0.30748
3-4	0.09943	0.22953	57-58	0.28374	0.28331	142-143	0.29320	0.29276
4-5	0.15571	0.35945	58-59	0.28374	0.28331	143-144	0.21753	0.21721
5-6	0.16321	0.37677	57-61	0.04502	0.10394	144-145	0.26482	0.26443
6-7	0.11444	0.26417	61-62	0.02626	0.06063	142-146	0.10318	0.23819
6-9	0.05675	0.05666	62-63	0.06003	0.13858	146-147	0.13507	0.31181
9-10	0.52124	0.27418	63-64	0.03002	0.06929	0-148	0.00938	0.02165
9-12	0.10877	0.10860	64-65	0.02064	0.04764	148-149	0.16884	0.38976
12-13	0.39803	0.20937	62-67	0.10881	0.25118	149-150	0.11819	0.27283
12-15	0.91744	0.31469	67-68	0.25588	0.13460	150-152	2.28608	0.78414
12-17	0.11823	0.11805	68-69	0.41699	0.21934	150-153	0.45587	1.05236
17-18	0.50228	0.26421	69-70	0.50228	0.26421	153-154	0.69600	1.60669
17-20	0.05675	0.05666	70-71	0.33170	0.17448	154-155	0.45774	1.05669
20-21	0.29379	0.15454	71-72	0.20849	0.10967	155-156	0.20298	0.26373
0-24	0.33205	0.76653	56-73	0.13882	0.32047	156-157	0.21348	0.27737
24-25	0.00188	0.00433	0-75	0.00750	0.01732	157-158	0.54967	0.28914
25-26	0.22324	0.51535	75-76	0.27014	0.62362	158-159	0.54019	0.28415
26-27	0.10881	0.25118	76-77	0.38270	0.88346	157-160	0.04550	0.05911
27-28	0.71078	0.37388	77-78	0.33018	0.76220	160-162	0.47385	0.24926
27-29	0.18197	0.42008	78-80	0.32830	0.75787	162-163	0.86241	0.45364
29-30	0.30326	0.15952	80-81	0.17072	0.39409	163-164	0.56862	0.29911
29-32	0.02439	0.05630	81-82	0.55914	0.29412	158-200	0.77711	0.40878
32-33	0.04502	0.10394	81-83	0.05816	0.13425	200-201	1.08038	0.56830
33-34	0.01876	0.04331	83-84	0.70130	0.36890	159-202	1.09933	0.57827
34-35	0.11823	0.11805	84-85	1.02352	0.53839	202-203	0.47385	0.24926
35-36	0.02365	0.02361	83-86	0.06754	0.15591	154-204	0.32267	0.74488
36-37	0.18954	0.09970	86-87	1.32352	0.45397	204-205	0.14633	0.33779
37-38	0.39803	0.20937	0-121	0.01126	0.02598	205-206	0.12382	0.28583
36-39	0.05675	0.05666	121-122	0.72976	1.68464	0-207	0.01126	0.02598
39-40	0.09477	0.04985	122-123	0.22512	0.51968	207-208	0.64910	1.49842
40-41	0.41699	0.21934	123-124	0.20824	0.48071	208-209	0.04502	0.10394
41-42	0.11372	0.05982	124-125	0.04690	0.10827	209-210	0.52640	0.18056
39-43	0.07566	0.07555	125-127	0.61950	0.61857	209-211	0.02064	0.04764
43-44	0.36960	0.19442	127-128	0.34049	0.33998	211-212	0.53071	0.27917
44-45	0.26536	0.13958	128-129	0.56862	0.29911	211-214	0.09755	0.22520
43-46	0.05675	0.05666	128-130	0.10877	0.10860	214-215	0.11819	0.27283
0-48	0.33205	0.76653	130-131	0.56862	0.29911	214-217	0.13882	0.32047
48-49	0.11819	0.27283	0-133	0.01126	0.02598	217-218	0.04315	0.09961
49-50	2.96288	1.01628	133-134	0.41835	0.96575	218-219	0.09192	0.21220
49-51	0.00188	0.00433	134-135	0.10499	0.13641	219-220	0.16134	0.37244
51-52	0.06941	0.16024	134-136	0.43898	1.01338	220-221	0.37832	0.37775
52-53	0.81502	0.42872	136-137	0.07520	0.02579	221-222	0.39724	0.39664
52-54	0.06378	0.14724	137-138	0.07692	0.17756	222-223	0.29320	0.29276

Table B2: Load data

bus	Active power ($P_{i,d}^{D}$) (kW)	Reactive power ($Q_{i,d}^{D}$) (kVAr)	bus	$P_{i,d}^{D}$ (kW)	$Q_{i,d}^{D}$ (kVAr)	bus	$P_{i,d}^{D}$ (kW)	$Q_{i,d}^{D}$ (kVAr)	bus	$P_{i,d}^{D}$ (kW)	$Q_{i,d}^{D}$ (kVAr)
0	0	0	42	396.735	193.96	81	176.408	70.184	152	9.065	3.843
1	0	0	43	0	0	82	83.015	33.028	153	2.092	0.887
2	47.78	19.009	44	181.152	88.563	83	217.917	86.698	154	16.735	7.094
3	42.551	16.929	45	242.172	118.395	84	23.294	9.267	155	1506.522	638.634
4	87.022	34.622	46	75.316	36.821	85	5.075	2.019	156	313.023	132.694
5	311.31	123.855	48	0	0	86	72.638	28.899	157	79.831	33.842
6	148.869	59.228	49	1.254	0.531	87	405.99	161.523	158	51.322	21.756
7	238.672	94.956	50	6.274	2.66	121	0	0	159	0	0
9	62.299	24.786	51	0	0	122	100.182	42.468	160	202.435	85.815
10	124.598	49.571	52	117.88	49.971	123	142.523	60.417	162	60.823	25.784
12	140.175	55.768	53	62.668	26.566	124	96.042	40.713	163	45.618	19.338
13	116.813	46.474	54	172.285	73.034	125	300.454	127.366	164	0	0
15	249.203	99.145	55	458.556	194.388	127	141.238	59.873	200	157.07	66.584
17	291.447	115.952	56	262.962	111.473	128	279.847	118.631	201	0	0
18	303.72	120.835	57	235.761	99.942	129	87.312	37.013	202	250.148	106.041
20	215.396	85.695	58	0	0	130	243.849	103.371	203	0	0
21	198.586	79.007	59	109.215	46.298	131	247.75	105.025	204	69.809	29.593
24	0	0	61	0	0	133	0	0	205	32.072	13.596
25	0	0	62	72.809	30.865	134	89.878	38.101	206	61.084	25.894
26	0	0	63	258.473	109.57	135	1137.28	482.108	207	0	0
27	30.127	14.729	64	69.169	29.322	136	458.339	194.296	208	94.622	46.26
28	230.972	112.92	65	21.843	9.26	137	385.197	163.29	209	49.858	24.375
29	60.256	29.458	67	0	0	138	0	0	210	123.164	60.214
30	230.972	112.92	68	20.527	8.702	139	79.608	33.747	211	78.35	38.304
32	120.507	58.915	69	150.548	63.819	141	87.312	37.013	212	145.475	71.121
33	0	0	70	220.687	93.552	142	0	0	214	21.369	10.447
34	56.981	27.857	71	92.384	39.163	143	74.001	31.37	215	74.789	36.564
35	364.665	178.281	72	0	0	144	232.05	98.369	217	227.926	111.431
36	0	0	73	226.693	96.098	145	141.819	60.119	218	35.614	17.411
37	124.647	60.939	75	0	0	146	0	0	219	249.295	121.877
38	56.981	27.857	76	294.016	116.974	147	76.449	32.408	220	316.722	154.842
39	0	0	77	83.015	33.028	148	0	0	221	333.817	163.199
40	85.473	41.787	78	83.015	33.028	149	51.322	21.756	222	249.295	121.877
41	0	0	80	103.77	41.285	150	59.874	25.381	223	0	0