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RANULFO ACIR DE OLIVEIRA RESENDE

**Nonconvexity in Optimal Control Problems:
An Approach by Automated Numerical Simulations**

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RANULFO ACIR DE OLIVEIRA RESENDE

**Nonconvexity in Optimal Control Problems:
An Approach by Automated Numerical Simulations**

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Prof. Tit. José Manoel Balthazar
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Prof. Dr. JOSE MANOEL BALTHAZAR - Orientador(a) do(a) Departamento de Engenharia Mecânica - docente voluntário / Faculdade de Engenharia de Bauru - UNESP, Prof. Dr. AIRTON NABARRETE

do(a) Departamento de Engenharia Aeroespacial e Aeronáutica / Instituto Tecnológico de Aeronáutica,

Prof. Dr. ANTONIO ROBERTO BALBO do(a) Departamento de Matemática / Faculdade de Ciências

de Bauru - UNESP, Prof. Dr. DIEGO COLON do(a) Departamento de Engenharia de

Telecomunicações e Controle / Universidade de São Paulo/Poli, Prof. Dr. GERALDO NUNES SILVA

do(a) Departamento de Matemática Aplicada / Instituto de Biociências, Letras e Ciências Exatas, sob

a presidência do primeiro, a fim de proceder a arguição pública da TESE DE DOUTORADO de

RANULFO ACIR DE OLIVEIRA RESENDE, intitulada **Nonconvexity in Optimal Control Problems:**

An Approach by Automated Numerical Simulations. Após a exposição, o discente foi arguido

oralmente pelos membros da Comissão Examinadora, tendo recebido o conceito final: aprovado

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membros da Comissão Examinadora.

Prof. Dr. JOSE MANOEL BALTHAZAR

Prof. Dr. AIRTON NABARRETE

Prof. Dr. ANTONIO ROBERTO BALBO

Prof. Dr. DIEGO COLON

Prof. Dr. GERALDO NUNES SILVA

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“O mundo é um moinho.”

Agenor de Oliveira - Cartola (1908-1980)

*“No mundo passais por aflições; mas tende bom ânimo,
eu venci o mundo.”*

Bíblia (João 16:33)

À minha família,

de quem aprendemos e pra quem ensinamos o amor, o perdão, a esperança e a fé,

com quem aprendemos e ensinamos o amor, o perdão, a esperança e a fé.

Resumo

Esta Tese de Doutorado é composta pelo estudo da aplicação de simulações numéricas automatizadas para solucionar problemas de controle ótimo (PCO) não convexos. Um algoritmo para tratar PCOs não convexos é apresentado como a principal contribuição da Tese. Outra contribuição, a estrutura *ASAF - Automated Simulation and Analysis Framework*, codificada em linguagem Python e baseada em modernas ferramentas de modelagem, simulação, *JModelica.org*, e otimização, *Interior-Point Optimizer* (IPOPT), é utilizada como auxílio para aplicação do referido algoritmo. O algoritmo proposto foi testado de forma metodológica com escala métrica e quatro PCOs não convexos com soluções analíticas exatas conhecidas. A viabilidade de aplicação do algoritmo foi verificada por clássicos métodos estatísticos aplicados aos dados das simulações, alcançando-se os resultados esperados e fundamentando-se a possibilidade de se tratar PCOs não convexos de soluções desconhecidas. Assim, esta Tese apresenta uma contribuição relevante para o tratamento de PCOs não convexos, ao propor um algoritmo para, com uso eficaz dos crescentes recursos computacionais de metaprogramação e automatização, obter solução subótima recomendada aceitável. A mediana da efetividade do algoritmo é estimada, a partir de testes estatísticos, em 93%, com significância de 0,05 (95% de confiança).

Palavras-chave: otimização. controle ótimo. modelagem matemática. simulação numérica.

Abstract

This doctoral Thesis is comprised by the study of the application of automated numerical simulations to solve nonconvex optimal control problems (OCP). An algorithm to treat nonconvex OCPs is presented as the main contribution of the Thesis. Another contribution of the Thesis, the *ASAF-Automated Simulation and Analysis Framework*, coded in Python language and based on modern tools of modeling, simulation, *JModelica.org*, and optimization, *Interior-Point Optimizer* (IPOPT), is used as an aid in the application of this algorithm. The proposed algorithm was tested by a methodological procedure with a metric scale and four nonconvex OCPs with known exact analytical solutions. The feasibility of using the algorithm was verified by classic statistical methods applied to the simulations data, achieving the expected outcome and grounding the possibility to treat nonconvex OCPs of unknown solutions. Thus, this Thesis presents a relevant contribution to the treatment of nonconvex OCPs, by proposing an algorithm to, upon the effective use of the increasing computer resources of metaprogramming and automation, obtain an acceptable sub-optimal recommended solution. The median of the algorithm effectiveness is estimated, from statistical tests, in 93%, with a significance of 0.05 (95% confidence).

Keywords: Optimization. Optimal control. Mathematical modeling. Numerical simulation.

List of Figures

Figure 1 – Graphic Description of the Contribution of the Thesis for the Treatment of Nonconvex OCPs. Source: The author.	27
Figure 2 – Sequence for the Application of the Algorithm. Source: The author.	44
Figure 3 – Description of the Process of Algorithm Application. Source: The author.	48
Figure 4 – Automated Structure to Solve Nonlinear and Nonconvex OCPs. Source: The author.	50
Figure 5 – Development of the Graphical User Interface. Source: The author	56
Figure 6 – First Analytical Solution of OCP#1. Source: The author.	60
Figure 7 – Second Analytical Solution of OCP#1. Source: The author.	61
Figure 8 – First Analytical Solution for OCP#2. Source: The author.	63
Figure 9 – Second Analytical Solution for OCP#2. Source: The author.	63
Figure 10 – Candidate Solution for OCP#1 - Context A: Control. Source: The author.	68
Figure 11 – Candidate Solution for OCP#1 - Context A: State. Source: The author.	69
Figure 12 – Rejected Solution for OCP#1 - Context A: Control. Source: The author.	70
Figure 13 – Rejected Solution for OCP#1 - Context A: State. Source: The author.	70
Figure 14 – Solution Improperly Classified as Candidate for OCP#1 - Context A: Control. Source: The author.	71
Figure 15 – Solution Improperly Classified as Candidate for OCP#1- Context A: State. Source: The author.	72
Figure 16 – Candidate Solution for OCP#1 - Context B: Control. Source: The author.	73
Figure 17 – Candidate Solution for OCP#1 - Context B: State. Source: The author.	74

Figure 18 – Rejected Solution for OCP#1 - Context B: Control. Source: The author.	75
Figure 19 – Rejected Solution for OCP#1 - Context B: State. Source: The author.	75
Figure 20 – Candidate Solution for OCP#1 - Context C: Control. Source: The author.	76
Figure 21 – Candidate Solution for OCP#1 - Context C: State. Source: The author.	76
Figure 22 – Candidate Solution for OCP#1 - Context S: Control. Source: The author.	77
Figure 23 – Candidate Solution for OCP#1 - Context S: State. Source: The author.	78
Figure 24 – Rejected Solution for OCP#1 - Context S: Control. Source: The author.	79
Figure 25 – Rejected Solution for OCP#1 - Context S: Control. Source: The author.	79
Figure 26 – Candidate Solution for OCP#1 - Context T: Control. Source: The author.	80
Figure 27 – Candidate Solution for OCP#1 - Context T: State. Source: The author.	80
Figure 28 – Rejected Solution for OCP#1 - Context T: Control. Source: The author.	81
Figure 29 – Rejected Solution for OCP#1 - Context T: State. Source: The author.	81
Figure 30 – Candidate Solution for OCP#1 - Context U: Control. Source: The author.	82
Figure 31 – Candidate Solution for OCP#1 - Context U: State. Source: The author.	82
Figure 32 – Rejected Solution for OCP#1 - Context U: Control. Source: The author.	83
Figure 33 – Rejected Solution for OCP#1 - Context U: State. Source: The author.	83

List of Tables

Table 1 – Results of Searches in Databases	33
Table 2 – Searches in Databases: Most Cited Articles on the Topic	34
Table 3 – Parameters for the Solution Classification as Candidate	43
Table 4 – Tools Used in this Thesis.	52
Table 5 – Summary of the Canonical Solutions of the Used OCPs	65
Table 6 – Optimization Parameters Used in All the Optimizations	65
Table 7 – Parameters Used in the Definitions of the Contexts	66
Table 8 – Parameters Used in the Definitions of the Simulations	67
Table 9 – Scope of Simulations	67
Table 10 – Optimizations Results for OCP#1	67
Table 11 – Estimation of the Effectiveness of the Algorithm: Summary of the Results	84

List of Abbreviations and Acronyms

HJB	Hamilton-Jacobi-Bellman (Equation of)
IPOPT	Interior-Point Optimizer (Optimization Tool)
MPC	Model Predictive Control
NLP	Nonlinear Program
NMPC	Nonlinear Model Predictive Control
NRMSE	Normalized Root-Mean-Squared Error
OCP	Optimal Control Problem
OP	Optimization Problem
PMP	Pontryagin Maximum Principle

List of Symbols

α	Significance level ¹
α	Maximum non-compliance index ^{2,3}
ρ	Maximum relative rate for growth of J ²
ε	Minimum acceptable slack to the bounds of the variables ²
$\mathbf{u}_0$	Initial condition for control variable
$\mathbf{u}^*$	Optimal control solution
$\mathbf{u}$	Control variable - control vector
$\mathbf{x}_0$	Initial condition for state variable
$\mathbf{x}^*$	Optimal state solution
$\mathbf{x}$	State variable - state vector
t_0	Initial condition for time variable
t_f	Final condition for time variable
$J[\mathbf{x}, \mathbf{u}, t]$	Performance Index, Cost Function or Objective
n_e	Number of elements for the discretization of the OCP ⁴
n_c	Number of collocation points in each element ⁴
$\tau_{i,k}$	Collocation point 'k' in element 'i' ⁴
$\tilde{\ell}_k$	Lagrange basis polynomials with interpolation point $\tau_{i,0}$ ⁴
ℓ_k	Lagrange basis polynomials without interpolation point $\tau_{i,0}$ ⁴
g_i	Inequality constraints ⁴
g_e	Equality constraints ⁴
G_i	Point inequality constraints ⁴
G_e	Point equality constraints ⁴

¹Statistical Parameter.

²Parameter for the Analysis of the Numerical Solution.

³Indicates the nonconvexity of solution $\mathbf{u}^*$ or $\mathbf{x}^*$, non-monotonicity or non-slow growth of J .

⁴Applicable in the technique of dynamic optimization by collocation points.

Contents

1	INTRODUCTION	25
2	CONSIDERATIONS ABOUT THE METHODOLOGY	31
2.1	BIBLIOGRAPHICAL REVIEW	31
2.2	THE SCIENTIFIC PROBLEM	35
2.3	THE HYPOTHESIS	35
2.4	THE REFUTATION METHOD	36
2.5	STATISTICAL ANALYSIS AND TOOLS TO VERIFY THE HYPOTHESIS	38
3	CONTRIBUTIONS	41
3.1	ALGORITHM TO TREAT NONCONVEX OPTIMAL CONTROL PROBLEMS	41
3.2	THE INTEGRATED AND AUTOMATED ENVIRONMENT FOR MODELING, SIMULATION AND OPTIMIZATION	48
3.2.1	<i>JModelica.org</i>	49
3.2.2	<i>IPOPT</i>	50
3.2.3	<i>HSL Routines</i>	51
3.2.4	<i>Python and other tools</i>	51
3.3	ASAF - AUTOMATED SIMULATION AND ANALYSIS FRAMEWORK	52
3.3.1	<i>Main Features of the ASAF Structure</i>	54
4	THE HYPOTHESIS VERIFICATION	59
4.1	FIRST OCP USED IN THE HYPOTHESIS VERIFICATION	59
4.2	SECOND OCP USED IN THE HYPOTHESIS VERIFICATION	62
4.3	THIRD OCP USED IN THE HYPOTHESIS VERIFICATION	64
4.4	FOURTH OCP USED IN THE HYPOTHESIS VERIFICATION	64
4.5	RESULTS	65
4.5.1	<i>Optimizations Results for OCP#1</i>	67
4.5.1.1	Analysis of the Optimizations Results for OCP#1	68
4.6	THE HYPOTHESIS VERIFICATION BY STATISTICAL TESTS	84

4.6.1	<i>The MATLAB® Code for the Hypothesis Verification</i>	84
5	DISCUSSION OF THE RESULTS	87
6	CONCLUSION	91
	REFERENCES	93
APPENDIX A	SUMMARY OF THE <i>DYNAMIC OPTIMIZATION BY DIRECT COLLOCATION</i>	99
A.1	COLLOCATION POLYNOMIALS	102
A.2	THE TRANSFORMATION OF THE OCP	103
APPENDIX B	STATISTICAL ANALYSIS OF THE SIMULATIONS	107
APPENDIX C	THE OUTPUT OF THE KRUSKAL-WALLIS TEST	135
	INDEX	141

— 1 —

INTRODUCTION

The theory of optimal control aims to minimize a performance index which, most of the time, is related to a function of economic cost. This theory is the core of this research and there are many excellent bibliographical sources, such as (KIRK, 1970; BRYSON, 1975; ATHANS, 2007), with no intention of discrediting others, that present various methods and techniques to solve specific categories of Optimal Control Problems (OCPs).

Classical techniques and methods to solve OCPs rely on the Theorems of Existence (CESARI, 1983, p. 367-472) to ensure optimal solution. However, there is a variety of OCPs that do not comply with the classical conditions of linearity and convexity.

Currently, the theory of optimal control shows relevant evolution in the theorems of existence, for which the non-compliance with the classical conditions (CESARI, 1974), (ANCONA, 2008), (CELLINA; COLOMBO, 1990), (MARICONDA, 1992), (EKELAND; TEMAM, 1976), (ZASLAVSKI, 2007), (CLARKE, 1983), (CLARKE, 2011), such as convexity and lower semi-continuity (CLARKE, 1983, p.103), under certain conditions, does not compromise the existence of optimal solution, even if it is very hard to find it.

With the emergence of recent concepts and approaches defined by new conditions of existence (ZASLAVSKI, 2013, p. 1) based on the performance index growth, the theoretical evolution was not followed by the practical development to obtain optimal solutions.

This Thesis seeks to fill this gap, looking to use the potential of metaprogramming,

the versatility and flexibility of the models in Modelica language and computer automation in numerical simulations, to advance the study of nonconvex OCPs and exploit these new concepts.

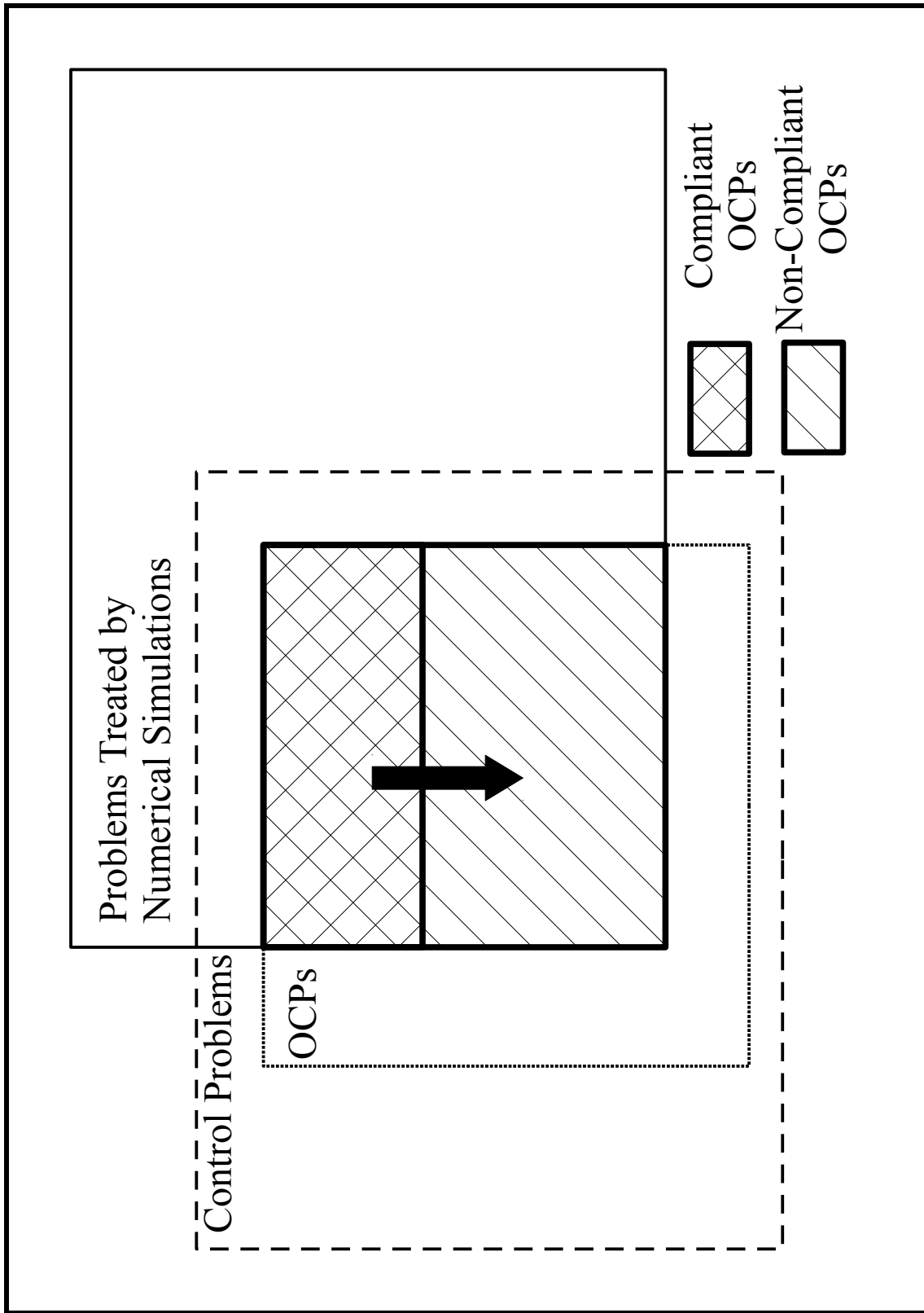


Figure 1 – Graphic Description of the Contribution of the Thesis for the Treatment of Nonconvex OCPs. Source: The author.

Figure 1 graphically describes the contribution of this Thesis to the expansion of the nonconvex set of OCPs with acceptable solutions obtained by numerical simulations.

Therefore, the overall goal of this research was to find a method to treat non-compliant OCPs and obtain acceptable solutions through automated numerical simulations.

This research presents as main result, in Section 3.1, the definition of an algorithm to achieve acceptable solutions for non-compliant OCPs (nonconvex, nonlinear) using the assistance of automated numerical simulations. In Section 3.3, the ASAF - Automated Simulation and Analysis Framework, a set of *Python scripts* used to verify the suggested algorithm is presented as secondary result.

The median of the algorithm effectiveness was estimated, from statistical tests, in 93%, with a significance of 0.05 (95% confidence). The process to verify the effectiveness of the algorithm carried out 972 simulations of four OCPs. These OCPs were specifically chosen from the literature because they present nonconvexity and known solutions for state vector.

Therefore, a relevant contribution to the treatment of nonconvex OCPs is provided by this Thesis.

The Thesis was divided into the following chapters.

Chapter 2, CONSIDERATIONS ABOUT THE METHODOLOGY, presents the relevant topics that were followed to achieve the Thesis results: Initially, the details of the bibliographical and theoretical review are described, and, after that, the Scientific Problem, the Hypothesis and The Refutation Method are defined.

Then, in Chapter 3, CONTRIBUTIONS, the main and the secondary results of the Thesis are presented, along with a short description of the software tools used.

Chapter 4, THE HYPOTHESIS VERIFICATION, contains the details about the procedures performed to verify the hypothesis of this Thesis.

Chapter 5, DISCUSSION OF THE RESULTS, discusses the implications of the achieved

results in relation to the current knowledge about the treatment of non-compliant OCPs, as well as to the possibilities of evolution of computer resources and of automated numerical simulations.

Chapter 6, *CONCLUSION*, presents a brief description of the importance of the achieved results.

Next, in *REFERENCES*, all bibliographical publications used to support this doctoral Thesis are provided.

The Appendix A presents a *SUMMARY OF THE DYNAMIC OPTIMIZATION BY DIRECT COLLOCATION*, used in the automated simulations, based on the original source (MAGNUSSON; ÅKESSON, 2012).

A report of the numerical results and the solutions classifications is presented in Appendix B.

Appendix C presents the statistical analysis of the Kruskal-Wallis Test (DANIEL, 1990), which estimated the median of the algorithm effectiveness.

The next chapter presents the details about the methodology used in this Ph.D. Thesis.

— 2 —

CONSIDERATIONS ABOUT THE METHODOLOGY

This chapter presents the methodological details, (MARCONI; LAKATOS, 2010; KOCHÉ, 2009; ECO, 2007), that have guided the development of the Thesis and explains how the results were achieved.

Initially, the procedures performed within the bibliographical review are described. Next, the Scientific Problem, the Hypothesis and the Refutation Method used in the Thesis are presented.

2.1 BIBLIOGRAPHICAL REVIEW

A bibliographical review was carried out, which started with classical literature on Control Engineering, (OGATA, 2010; OGATA, 1995; FRANKLIN; POWELL, 1980; FRANKLIN, 1991), in which some fundamental concepts related essentially to the Modern Control Theory are evidenced, such as state variables and state equations.

Next, the optimal control theory was studied, (LEWIS, 1986), (BRYSON, 1975), (KIRK, 1970), (ATHANS, 2007), (BERTSEKAS, 2005a; BERTSEKAS, 2005b), where the performance index, the Quadratic Continuous Linear Regulator, the Linear Quadratic Continuous Tracker, the Pontryagin Maximum Principle, the Bellman Optimality Principle, the Hamilton-Jacobi-Bellman and Riccati equations were defined.

Similarly, a bibliographical review and theoretical foundation related to Optimization were performed, as a science that seeks to solve a static optimization problem (OP), may it be

linear or nonlinear.

Classic literature in this area can be found on (LUENBERGER; YE, 2008), (BAZARAA, 2009), where fundamental concepts are presented, such as the Constrained Optimization Problem, Necessary Conditions of First and Second Order, Karush, Kuhn and Tucker (KKT) Criteria, Optimization Methods, Barrier Method, Interior-Points Method, Lagrange Multipliers, Stopping Criteria, Feasible Region, Feasible Solution, Local and Global optimal solutions.

This Thesis used the *DYNAMIC OPTIMIZATION BY DIRECT COLLOCATION*, which was proposed by (MAGNUSSON; ÅKESSON, 2012), as support for the implementation of automated numerical simulations for solving non-compliant OCPs. The method is summarized in Appendix A.

The review proceeded with the use of more recent sources which presented a discussion on nonconvexity, (CESARI, 1974), (ANCONA, 2008), (CLARKE, 1983), (CLARKE, 2011), (CELLINA; COLOMBO, 1990), (MARICONDA, 1992), (EKELAND; TEMAM, 1976) and (ZASLAVSKI, 2007).

Next, Existence Theorems (CESARI, 1983, p. 367-472) to ensure the optimal solution were studied. Some of them consider slow and monotonic growth of the performance index and convexity to classify a OCP as solvable.

This review has highlighted a relevant fact that, for an eventual non-compliant OCP, for example, nonconvex, the theoretical existence of optimal solution can still be mathematically defined. Zaslavski shows in (ZASLAVSKI, 2013, p. 2) the existence of a solution for OCP set under the unique condition of characteristics of the objective function with monotonic non-decreasing growth (CESARI, 1983, p. 104).

After the cited initial reviews, a survey was performed in some databases available on the Internet, searching for articles on the topics of the Thesis.

Searches were carried out in Scopus and Web of Science from the Institute for Scientific

Information (ISI)¹. The first search was performed using the keywords "Control Problems" and "Numerical Simulations". In the second search, the expressions "Optimal Control" and "Numerical Simulations" were used. In both cases, the results were filtered for the fields: "Computer Science", "Mathematics" and "Engineering".

The results are presented in Table 1.

Table 1 – Results of Searches in Databases

Search	Scopus	ISI Web of Science
1	4034	125
2	4034	536

Source: The author.

After inspection of the titles and abstracts of the papers obtained in the searches, with most of them standing out of the scope of this Thesis, a detailed study of the most cited articles was made, with a minimum of 20 citations. The articles were sorted by topics and type of contribution, considering if they were about a theoretical development or an application. A summary of this analysis is presented in the Table 2.

¹At <https://www.webofknowledge.com/>, accessed on may, 6th, 2016.

Table 2 – Searches in Databases: Most Cited Articles on the Topic

#	Article	Citations	Remarks
1	(TODOROV, 2005)	127	development of theory stochastic optimal control coordinate-descendants algorithm
2	(RAFIKOV; BALTHAZAR, 2008)	121	development of theory nonlinear systems chaos synchronization linear feedback controller
3	(LUO, 2005)	106	application tracking control problem
4	(CHESI, 2003)	94	development of theory
5	(JOO, 1999)	91	development of theory digital control of chaotic systems model-based fuzzy controller
6	(SHAO, 2007)	78	application
7	(LIN, 1999)	77	application
8	(BANI-HANI; GHABOUSSI, 1998)	69	development of theory neural networks nonlinear control
9	(BEWLEY, 2000)	66	application linear and nonlinear problems
10	(LI, 2007)	60	development of theory
11	(HOUSKA, 2011)	59	application NMPC ^a
12	(ILZHOEFER, 2007)	59	application NMPC ^a
13	(CIMEN; BANKS, 2004)	57	development of theory ASRE ^b
14	(LIU, 2012)	56	application MAS ^c
15	(CHOI, 2004)	53	application
16	(ZHOU, 1995)	47	application
16	(ERKUS, 2002)	45	application
18	(DING, 2004)	40	application
18	(BONNARD, 2009)	39	application
20	(LENCI; REGA, 2003)	35	development of theory

^a Nonlinear Model Predictive Control^b Approximating Sequence Riccati Equations^c Multi-agent Systems

Source: Survey data.

After this study, it was found that article (RAFIKOV; BALTHAZAR, 2008) is the work which is more closely related to the scope of this Thesis, presenting an approach to the treatment of nonlinearities with linear feedback control.

It was also evidenced that there is a gap about the treatment of the problem that is the focus of the Thesis, which means, not compliant with the conditions and classical methods.

Another important point about the bibliographical review includes the characteristics of the solutions obtained by numerical simulations. Thus, concepts such as integration of trajectories, integration steps, single-step algorithm, multi-step algorithm, rounding error, truncation error and numerical stability are presented in Chapter 4 of (PARKER; CHUA, 1991).

2.2 THE SCIENTIFIC PROBLEM

The scientific problem can be summarized within the following research question:

How to treat, through automated numerical simulations, and obtain an acceptable solution for Optimal Control Problems (OCPs) that, despite being interesting, have characteristics of non-compliance (e.g., nonconvexity, nonlinearity) with the classical theory and with the current main solution methods?

An algorithm to treat non-compliant OCPs was proposed in Section 3.1, as an answer to the research question

2.3 THE HYPOTHESIS

The hypothesis of this doctoral research is defined by:

Null hypothesis (H0): The algorithm to treat non-compliant OCPs presented in Section 3.1, when applied in a particular context in which the OCP studied is consistently defined in relation to the theory and to the restrictions of the environment for numeric simulations and optimizations, is able to regularly present a median of effectiveness of correction of solutions classifications equal to or greater than 70%.

The alternative hypothesis, that will refute the Thesis, is defined by:

Alternative hypothesis (H1): The algorithm to treat non-compliant OCPs presented in Section 3.1, when applied in a particular context in which the OCP studied is consistently defined in relation to the theory and to the restrictions of the environment for numeric simulations and optimizations, regularly presents a median of effectiveness of correction of solutions classifications lower than 70%.

2.4 THE REFUTATION METHOD

The problem to evaluate and compare solutions from several numerical simulations, in face of the diversity of possible integration algorithms and simulation parameters, is about how to obtain the reference solution of the problem to be compared with the provided solutions.

If the OCP in question allows an analytical or exact solution, this solution will be the reference for the comparison.

This was the approach found in (PARKER; CHUA, 1991, p. 83), in which various solutions are provided by several integration methods applied to the same problem of a known exact solution. Therefore, in (PARKER; CHUA, 1991, p. 83), the algorithms assessment took place through the comparison of its numerical solution with the exact solution.

Obviously, the algorithm proposed in this Thesis does not restrict the OCP in relation to the condition of having a known analytical solution, neither sets restrictions to the objective function.

On the contrary, as any other algorithm to be applied with the assistance of numerical simulations, the possible attainment of solutions to generic OCPs with no additional restrictions is precisely its applicability advantage.

However, in the context of corroboration of the hypothesis and the obligation to evaluate the solutions provided by the algorithm, one can resort to, as an analogy to that was carried out in (PARKER; CHUA, 1991, p. 83), a comparison technique with the analytical or exact solution.

Another consideration is the randomness introduced to the process by variation of parameters performed in steps 2 and 3 of the algorithm, as well as by the diversity of possibilities of parametrically modeling of a given OCP (step 1).

Thus, it is considered to be not practical to develop an algorithm with effectiveness of 100% correct classifications with 100% confidence.

Therefore, it becomes necessary to elaborate a refutation method that, based on metric indexes (such as *Normalized Root-Mean-Squared Error - NRMSE*), makes use of statistical methods to test the hypothesis and estimate the median of the effectiveness distribution of the algorithm.

The following steps are suggested as the refutation method:

1. Choose an OCP² with characteristics of non-compliance and that has, in any possible way, an exact known solution;
2. When applying the algorithm presented in Section 3.1, and without loss of generality, associate each *region* defined on step 4 of the proposed algorithm, chosen by the user based on his or her requirements, to a single *context* of simulations, as Definition 3.4;
3. When considering the simulations in each context, calculate the effectiveness of correct classifications made by the algorithm in that context. This is done by comparing the NRMSE indexes of rejected and candidate solutions against the canonical analytical solution of the OCP, settling a certain error tolerance level to achieve a correctly classified solution as candidate. Solutions classified as candidates with indexes lower than the tolerance and the ones classified as rejected with indexes higher than the tolerance are considered to be correct classifications. The effectiveness in the context is given by
$$\eta = \frac{N_{\text{Correct Classified Solutions}}}{N_{\text{Total of Solutions}}};$$
4. The corroboration will occur when, after calculating the effectiveness of the algorithm in various contexts, the **Null Hypothesis (H0)** defined in Section 2.3 is corroborated by a statistical method, that is, the estimation of the median of the effectiveness is equal to or greater than 70%; and

²In fact, it is recommended to choose several OCPs, as done in this work.

5. The rebuttal will occur when, after systematic and consistent testing, the **Null Hypothesis (H0)** gets refuted, raising the evidence of the **Alternative Hypothesis (H1)**, with the median estimation of the effectiveness of the algorithm lower than 70%.

It should be highlighted that the exclusion of solutions performed by the solutions classification method is a technical non-subjective procedure.

On the other hand, defining whether a recommended solution is acceptable or not is a subjective evaluation procedure and may not be used in the verification process.

However, once a solution is obtained, and after its classification as candidate or rejected, its confrontation against the exact solution in its context via NRMSE is a quantitative assessment perfectly compatible with the objective criteria to calculate the effectiveness of the algorithm.

2.5 STATISTICAL ANALYSIS AND TOOLS TO VERIFY THE HYPOTHESIS

The Statistics Science is primarily applied in biological and human sciences (TRIOLA, 1999) in the study of processes involving random variables, which are usually non-numerical. However, its application in engineering is also common (LEVINE, 2000).

In this Thesis, statistical methods were used with the help of specific tools to estimate the effectiveness of the proposed algorithm applicability.

It is not scope of this study to address, in depth, statistical concepts. However, a brief consideration about the problem of the statistical evaluation of an algorithm is necessary.

Therefore, the following assumptions were considered:

- With the parametric variation introduced in steps 2 and 3, as well as the possible existing solutions in a nonconvex OCP, there will probably be a diversity of numerical solutions as high as the nonconvexity characteristics, the greater the parametric variations. This diversity of solutions and the limitations of the automatic classification algorithm suggest a

distribution of the effectiveness of correct classification of solutions for various simulations in several contexts.

- It is considered that the algorithm has limited effectiveness at the minimum of 0% and maximum of 100%. It is also supposed an unknown distribution shape, as well as unknown mean and median.
- It is necessary to be cautious and to have computational experience to efficiently apply an algorithm. A definition of an unfeasible OCP or inappropriate and unreasonable choice of parameters may lead to biased results that make it difficult to use the algorithm with efficiency. On the other hand, it is expected that, with the appropriate configuration and definition of the OCP, the algorithm is effective in the proper classification of numerous solutions obtained in an automated manner, saving considerable user time and effort in the individual classification of solutions.

Given the assumptions above, and specially because one cannot affirm about the shape of the distribution, the Sign Test was chosen, as described in (TRIOLA, 1999, p.321) and (GIBBONS, 2003), to estimate the median of the effectiveness.

This method was chosen for being "non-parametric", therefore, it does not require Normal distribution.

The method was applied by own coding in Python language and using the routine "signtest()" from the MATLAB® *Statistics* Toolbox.

In this Thesis, 4 nonconvex OCPs were studied and a total of 972 simulations were optimized and analyzed in an automated way. It was possible, with the help of a second statistical method, to infer more information about the effectiveness distribution of the algorithm application in these OCPs.

The second method is known as Kruskal-Wallis Test, (DANIEL, 1990). This test is also non-parametric and does not assume Normal distribution. However, the method has the following requirements (TRIOLA, 1999, p. 329):

- Minimum of 3 samples (there are 4 studied OCPs);
- The test verifies the null hypothesis in which the samples derive from the same population or identical populations. In numerical terms it means to have the same median and the same distribution;
- Each sample must have a minimum of five observations. In this Thesis there were 9 observations of effectiveness (one for each context in each OCP); and
- The different samples cannot have very different variances.

With the results of the simulations, the Kruskal-Wallis Test was applied with the aid of three different instruments:

1. By the function “`scipy.stats.mstats.kruskalwallis()`” of Python language;
2. By the function “`kruskalwallis()`” from MATLAB[®] *Statistics* Toolbox; and
3. By using the application IBM[®] SPSS[®] *Statistics*, which results are presented in appendix C.

The first two procedures for the Kruskal-Wallis Test resulted in the non-refutation of the Null Hypothesis (H_0) in which the four samples had the same median. The latter presented as additional results an estimation of this median and the confirmation that the samples present the same distribution.

The results, as detailed in Chapter 4, THE HYPOTHESIS VERIFICATION, were consistent.

The next chapter presents the contribution of the Thesis, primarily by the proposition of an algorithm to treat nonconvex OCPs and, secondarily, by the structure ASAF - Automated Simulation and Analysis Framework, a set of *Python* scripts for automated simulations of OCPs.

— 3 —

CONTRIBUTIONS

Here, the main contributions of this doctoral Thesis are presented.

Initially, the details of the algorithm to treat nonconvex OCPs are presented as the main result of the Thesis. Next, there is a description of JModelica, IPOPT and other tools used as support for the research. Last, the ASAF - Automated Simulation and Analysis Framework is presented as secondary result.

3.1 ALGORITHM TO TREAT NONCONVEX OPTIMAL CONTROL PROBLEMS

The treatment of nonconvex OCPs by automated numerical simulations can be carried out following the steps defined in Algorithm 1:

Algorithm 1: Algorithm to Treat Nonconvex OCPs

Data: 1) A problem that generates an OCP model;

2) Candidate solution requirements $(\alpha, \varepsilon, \rho)^a$;

Result: 1) Classification of numerically viable solutions (feasible OCP);

2) Recommended solution (feasible OCP);

repeat

The Modeling Phase;

1 model the system, as well as the performance index, with fidelity to the real needs of the real problem and within acceptable domains^b:
 $(\mathcal{D}_{X_i} = [x_{i,L}, x_{i,U}], i = 1 \dots n, \mathcal{D}_{u_j} = [u_{j,L}, u_{j,U}], j = 1 \dots m)$;

;

- 2 make use of parameterization resources of the modeling and simulation tools to change the conditions of the model (system and performance index) within the limits defined in step 1;

The Phase of Automated Obtaining of Solutions;

- 3 solve, in an automated manner, the OCP for parametric variation allowed by the modeling language, by the simulation and optimization tools and by the computational cost;

The Phase of Automated Classification of Solutions;

- 4 analyze the results in an automated way for the previous parametric variation, define and distinguish *regions*, depending on the values of the parameters, in which the results of the OCP solutions represent a similar behavior;
- 5 exclude solutions, although they are numerically viable, that are inadequate from the practical applicability point of view, e.g., trivially null solutions;
- 6 exclude solutions that clearly do not comply with the restrictions of the classic theorems of existence (convexity^c, non-decreasing monotonicity^d and smooth growth of the performance index). The remainder solutions are called *candidate solutions*;

foreach *Solution* $S_v \in$ *Solution Set* $\mathcal{S}$ **do**

$r[0] \leftarrow 0; mon \leftarrow 0; conv_x \leftarrow 0; conv_u \leftarrow 0;$

for $i \leftarrow 0$ **to** $l - 1$ (*where* $l =$ *number of discretization points*) **do**

$r[i + 1] = \frac{dJ}{dt}[i + 1] / abs(J[i + 1])$;

if $(r[i + 1] < 0) \vee (r[i + 1] > \rho)$ **then** $mon++$;

for $j \leftarrow 1$ **to** n **do**

$\lfloor$ **if** $(x_j[i + 1] - x_{j,L} < \varepsilon) \vee (x_{j,U} - x_j[i + 1] < \varepsilon)$ **then** $conv_x++$;

for $j \leftarrow 1$ **to** m **do**

$\lfloor$ **if** $(u_j[i + 1] - u_{j,L} < \varepsilon) \vee (u_{j,U} - u_j[i + 1] < \varepsilon)$ **then** $conv_u++$;

if

$(mon * 100 / l > \alpha) \vee (conv_x * 100 / (n * l) > \alpha) \vee (conv_u * 100 / (m * l) > \alpha)$

then Rejected Solution;

else Candidate Solution;

The Choosing of the Recommended Solution;

- 7 among the remaining regions, choose the region that best suits the reality of the problem modeled by the OCP. The best candidate solution in this region, the recommended for this region, will be the recommended solution for the OCP;

until *The Remodeling Phase is exhausted with infeasible OCP for the User's requirements or the recommended solution found is acceptable;*

^aThese parameters are essential, however, they are not exhaustive in the implementation of the algorithm. Eventually, the user may reset them and adapt them to the characteristics of his or her definitions of simulations and contexts.

^bDo not to worry in advance if the resulting model does not comply with the conditions of linearity, convexity or monotonicity criteria of classical existence theorems.

^cThis is done by observing that the solution point (t, x) should be *interior-point* (CESARI, 1983, p. 107). In practical terms, the control and state variables cannot reach the boundaries of the OCP.

^dThis is done by direct inspection of performance index behavior.

In fact, the expression "*interior-point*"¹ is related to a specific solution point in time. Perhaps for all solution points a better expression could be "*Interior-Trajectory*".

Remark 3.1. The symbolologies $[\bullet]_L$ and $[\bullet]_U$ stand for lower and upper limits, respectively.

Remark 3.2. The symbology $\text{conv}_{[\bullet]}$ denotes a nonconvexity counter.

Remark 3.3. The variable mon denotes a non-compliant monotonicity counter.

The notations in the algorithm, state vector $\mathbf{x}$ and control vector $\mathbf{u}$, are used according the Optimal Control Theory and the definition of OCP presented in Appendix A.

In Algorithm 1, the value of variable n represents the dimension of state vector $\mathbf{x}$, the value of variable m represents the dimension of control vector $\mathbf{u}$ and the value of variable l represents the length of the numerical solutions vectors defined by the time discretization.

Therefore, the variable l corresponds to the total number of collocation points.

In this Thesis, for the practical implementation of the Algorithm 1, the following parameters were used:

Table 3 – Parameters for the Solution Classification as Candidate

Parameter	Description	Value
α	Maximum acceptable index for non-compliance (nonconvexity, decreasing monotonicity, not slow growth of the performance index)	0,3(%)
ε	Minimum acceptable slack to the limits of the variables (indicates the saturation of the variable)	1e-4
ρ	Maximum acceptable growth rate of the Performance Index	0,1

Source: The author.

The sequence of actions taken in the use of the algorithm is shown in Figure 2.

¹The understanding of this expression here is *similar*, however, not the same, as in a classical static optimization, in a Nonlinear Program (NLP). It is similar to force the constraint that solution should not present active inequality constraints (LUENBERGER; YE, 2008, p. 322), more precisely, the solution should not be at the frontier of the feasibility region (LUENBERGER; YE, 2008, p. 322)

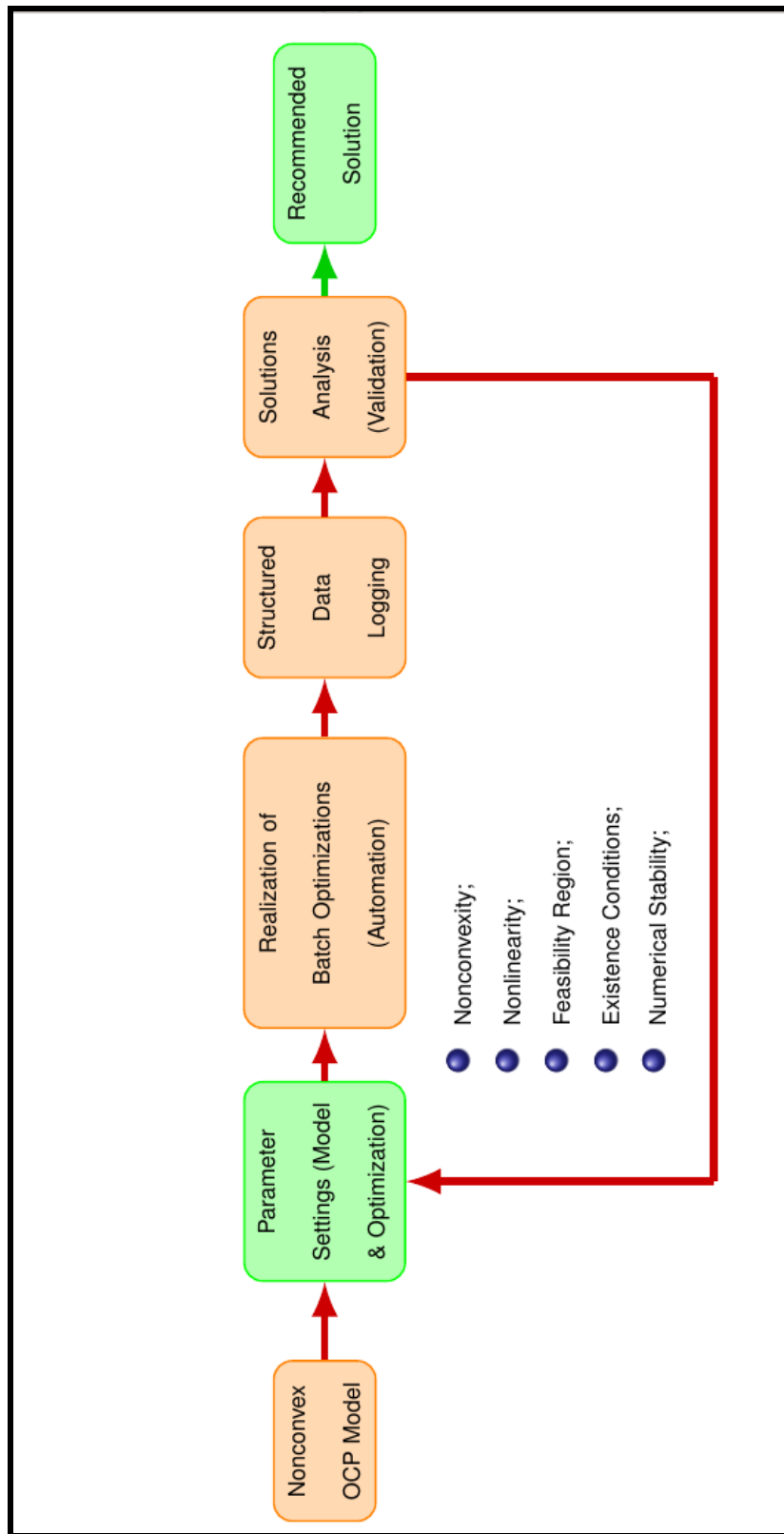


Figure 2 – Sequence for the Application of the Algorithm. Source: The author.

It is understood that numerical simulations algorithms, in which parameters present some randomness or arbitrary choice², carry this sensitivity to the possible solutions that were reached.

Therefore, depending on the definition of the problem, the choices of the parameters, and the methods used in numerical simulation, it may not converge, converge to a specific solution and, once the convergence is reached, the solution may be acceptable or not.

The proposed algorithm presents a quantitative dimension during the phase when the candidate solutions are defined. However, the choice of acceptable solution among the various recommended ones is made in a subjective manner by the user.

The quantitative assessment of the solutions can be easily incorporated in the automation process, however, the qualitative phase, consisted by the definition about the acceptability or non-acceptability of the recommended solution, cannot and should not be automated, since it is a result of the judgment of qualitative merit made by the human element who made the initial parametric choices.

It was verified that the initial freedom the user has to make parametric choices can affect three very specific and delimited groups of parameters:

²As in the mutation factor in genetic algorithms, or the definition of the network structure, number of neurons, layers and feedback of an artificial neural network.

Definition 3.1. Type-I Parameters Group:

It constitutes the group of parameters that affect the equations that model the system in study, including those related to the law of control³.

Definition 3.2. Type-II Parameters Group:

They are related to the essential and specific parameters of simulation and optimization tools⁴, such as number of discretization elements, steps, tolerances and stopping criteria. The definition of algorithms and heuristics used by the tools can be found in this group.

Definition 3.3. Type-III Parameters Group:

Contains the parameters determined by the characteristics of the OCP Design and Constraints. They are restrictions of lower or upper limits to the state variables, to the control variables, the initial conditions of these variables, if fixed or free, the parameters that affect the performance index and, eventually, other, if they exist^{5,6}.

Starting time and Ending time of the simulations are examples of parameters of this group.

It should be noted that the parameters variation procedure performed in the initial phase of the implementation of the algorithm shown in Figure 2, allows a diversity of solutions and, in most situations, various contexts.

In this Thesis and specifically to perform the verification of the hypothesis, context is defined as:

Definition 3.4. Context:

³Without loss of generality, but specifically to the tools chosen in this Thesis, the model parameters of the system are the ones that affect the section of equations defined after the keyword *equation* of Modelica language, at the file of the system model.

⁴Actually, all the parameters, including the type-I and type-III, are set in the tools, but one can understand as type-II parameters the ones that do not affect the system model neither the design of the OCP, they are specific and unique of the tools and their algorithms, such as tolerances, steps and stopping criteria.

⁵Eventually the control may require that a variable have limits for its derivative.

⁶Similarly, for the tools used based on modeling in Modelica language, typical examples are the constraints inserted in the model after the keyword *constraint*.

It is a specific and unique choice of values for type-I and type-III parameters. All parameters that are able to modify the general settings of the OCP, either by the change in the system model, via state equations with the law of control, or by change in the objective function, or by restriction or delimitation of the possible solution sets of state and control variables.

The definition of *Context* is required to make the objective comparison of solutions consistent, regardless if they are rejected or candidate, under a same numeric scale, such as the NRMSE. It is a definition necessary only in the process of validation of the hypothesis.

In the general scope of the algorithm application, this definition is unnecessary because there is no formal application of metric scale to candidate solutions.

Figure 3 presents a graphical visualization of the use of the algorithm and the delimitation of contexts and regions.

It is appropriate to note that the proposed algorithm has a generic characteristic and does not depend on a specifically determined tool or context for its application.

In fact, the final choice of the recommended solution is a qualitative judgment of merit by the user, in consonance with the parametric choices, that mirror his or her real requirements for the solution of the OCP.

However, in the scope of verification of the Hypothesis, the definition of the concept of Context is necessary and essential.

Another important note is that the term *region* used in the algorithm is more generic and is related to the perspective the user has about the results of the simulations. In fact, a region could be a group of solutions, a unique and isolated context or even a group of contexts.

Next, the details about the integrated simulation environment that was used to perform the optimizations that supported the Thesis are presented.

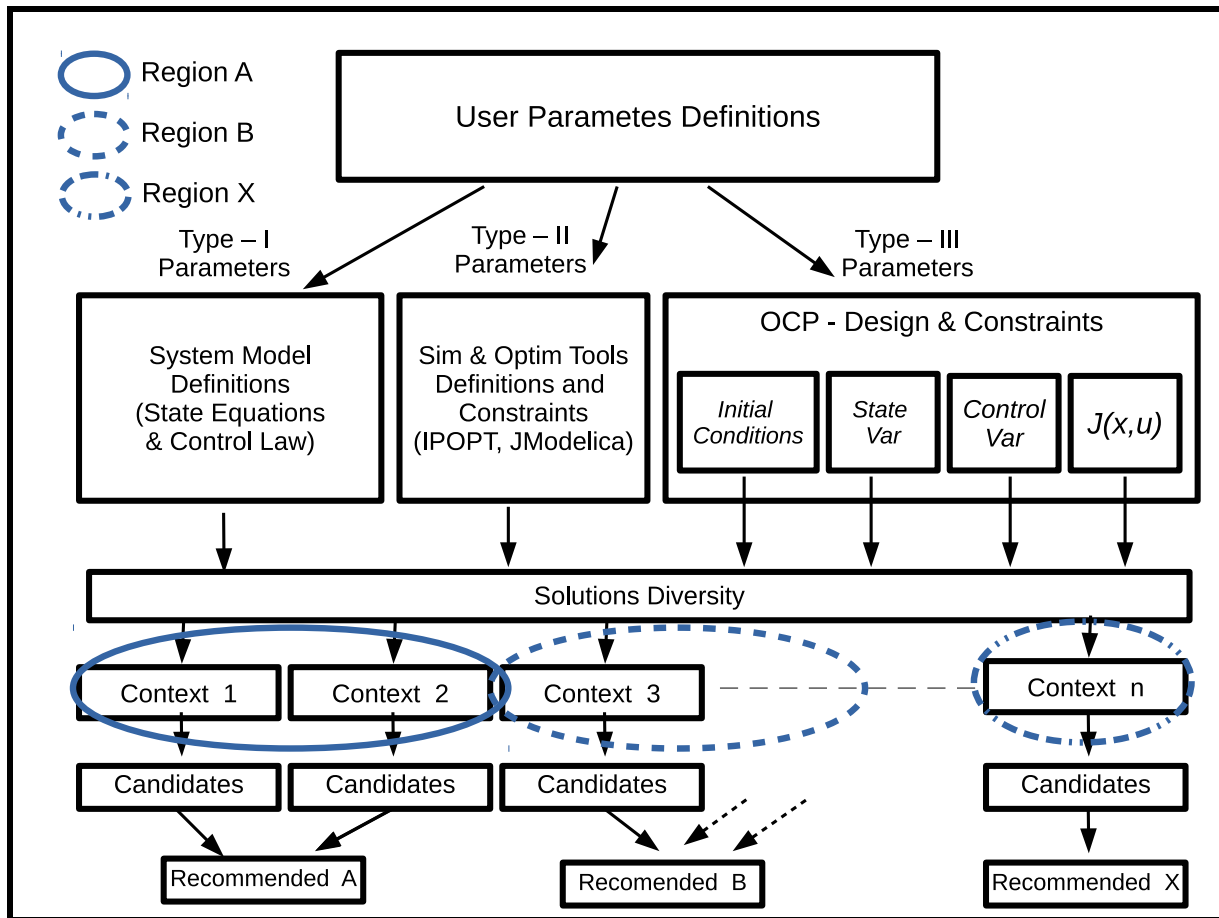


Figure 3 – Description of the Process of Algorithm Application. Source: The author.

3.2 THE INTEGRATED AND AUTOMATED ENVIRONMENT FOR MODELING, SIMULATION AND OPTIMIZATION

The proposed algorithm tries to make better use of the parameterization capabilities and automation of modern computational tools. It is well known that nonlinear and nonconvex systems, in most cases, have different solutions and behaviors due to parameterization and initial conditions.

To analyze such systems, researchers use classical instruments such as Attractors, Basins of Attraction and Bifurcation, Phase Plane Analysis and Phase Portraits, (THOMPSON; STEWART, 2002). However, (SLOTINE; LI, 1991, p.17) affirms the natural complexity of the graphic study of systems of order 3 and greater.

Thus, the application of the proposed algorithm demanded the implementation of a computational environment that deals with this matter by holding hundreds, even thousands,

of optimizations and simulations with the generation of huge amount of raw data for further analysis.

Then, it was carried out the development of a code in Python scripts specific to the Thesis, defined by the *ASAF-Automated Simulation and Analysis Framework* structure, presented in Section 3.3.

The *ASAF-Automated Simulation and Analysis Framework* has its execution based on the following tools.

3.2.1 *JModelica.org*

The JModelica.org tool (<http://jmodelica.org/>) (ÅKESSON, 2010) is an integrated environment of modeling, simulation and optimization of dynamic systems. This is the result of a research held by Department of Automatic Control at the University of Lund, Sweden, that currently has its development and maintenance made by the Swedish company Modelon AB.

A description of the JModelica.org integration with other tools, especially the flow of data and commands, is shown in Figure 4.

This simulation environment was deployed following the tutorials provided by maintainers of the tools, (ÅKESSON, 2010), (ANDERSSON, 2011; MAGNUSSON; ÅKESSON, 2012), (WÄCHTER; BIEGLER, 2006), in a more evolved form than what was presented in Sections 4.2 to 4.5 of (RESENDE, 2015).

The main update was regarding the operating system, which currently is the Ubuntu LTS Desktop 16.04, corresponding to the version with support available until the year 2021 (*Long-Term Support*).

The current versions of the tools are presented in the Table 4.

Through the observation of Figure 4, one can verify that the user calls directly the JModelica.org routines, which communicates with other tools through internal routines. For example, the user does not interact directly with the IPOPT optimizer, but one can, by calling

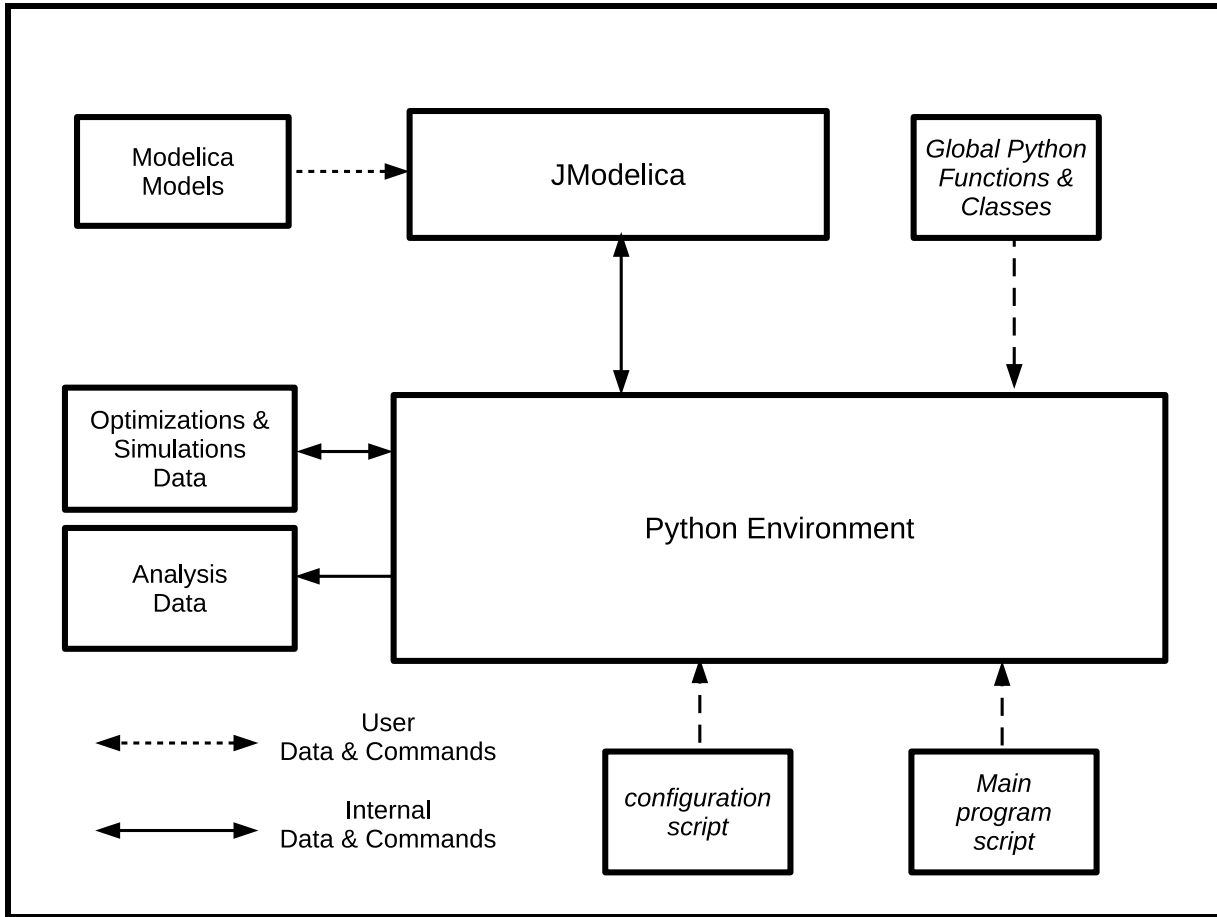


Figure 4 – Automated Structure to Solve Nonlinear and Nonconvex OCPs. Source: The author.

the JModelica.org routine, set the tolerances and other parameters of the optimizer.

Analogously, after the optimization, IPOPT does not report the results directly to the user by saving the values of variables, but rather through calls to routines and objects of the JModelica.org.

The main feature of JModelica.org in this environment is to interpret an OCP coded in Modelica language, translating the OCP to a Nonlinear Program (NLP) to be optimized by IPOPT.

3.2.2 IPOPT

The IPOPT tool, (WÄCHTER; BIEGLER, 2006), is compiled separately from the JModelica.org, however, in this environment, its operation is integrated to JModelica.org. Once these tools are compiled, the user only has to set the optimization options, in case of different

values from the default ones, and execute the optimization.

The interesting part about this integrated environment is that all options available in the IPOPT User Manual can be set via JModelica.org: The ones that affect the stopping mechanisms, the speed and convergence of iterations, including options concerned to the barrier method, KKT conditions and others. It is evidenced that the good choice of these options can determine the success⁷ and the speed of the optimization.

It is important to emphasize that the IPOPT performs a purely static optimization. It is up to the JModelica.org to transform the intrinsic dynamism of the original OCP to a static NLP, as described in appendix A.

The IPOPT has an integrated library called *sIPOPT* that allows the execution of sensitivity analysis.

3.2.3 HSL Routines

The HSL routines, according to (Science and Technology Facilities Council, 2015), constitute a set of solvers for large-scale linear problems. These routines are provided under the “HSL ACADEMIC LICENSE VERSION 1.1 JANUARY 2011” license, and can be compiled in an integrated manner to IPOPT or dynamically loaded.

Since there are several routines, once compiled, the usage is made by setting the option “solver” of the IPOPT.

3.2.4 Python and other tools

The JModelica.org, IPOPT and HSL routines constitute the core of the modeling, optimization and simulation environment. However, other tools are required both in the installation process of the environment as well as in its operation.

The Python language interpreter itself must be installed, (ROSSUM, 1995). In addition,

⁷An important factor that directly affects the optimization is the consistent definition of the OCP constraints. Eventually, the IPOPT will not converge due to the simple fact that the OCP has been formulated so restrictively that it became an infeasible problem.

it is necessary to have a compiler for the C programming language and the Java Development Kit (JDK) installed for the compilation and operation of the JModelica.org, IPOPT and HSL tools.

If the environment is *Microsoft*[®] *Windows*[®], one can install the binaries downloaded from the JModelica.org Web site.

This Thesis, however, has used the Linux environment⁸ with the tools in Table 4.

Table 4 – Tools Used in this Thesis.

Tool	Version	License
JModelica.org	1.17, 2.0, 2.1 and 2.2	GPL v3
IPOPT	3.12	Eclipse Public License - v 1.0
HSL	2014.01.10	HSL ACADEMIC LICENSE VERSION 1.1 JANUARY 2011
Python	2.7.12	PYTHON SOFTWARE FOUNDATION LICENSE VERSION 2
Java (OpenJDK)	1.8.0_131	GPL v2+
gcc	5.4.0	GPL v3

Source: Data provided publicly in the tools documentation.

3.3 ASAF - AUTOMATED SIMULATION AND ANALYSIS FRAMEWORK

Here, the specific Python and Modelica codes developed for the Thesis are presented as secondary result of the research.

The ASAF - Automated Simulation and Analysis Framework is limited to the Python scripts and the Modelica model required to perform the optimizations and simulations.

The scripts that comprise the *ASAF* structure are:

- *<SystemName>Pack.mop* - This is an entry file to the JModelica.oirg, containing the system modeled in Modelica language;
- *asaf_config_<SystemName>.py* - This is the configuration file that sets the main parameters for simulations and optimizations;
- *asaf_pgm.py* - This is the script that is responsible to perform the simulations and optimizations;

⁸The operating system used in this Thesis was the Ubuntu LTS Desktop 16.04

- *asaf.py* - This is the file with the code of Python classes that are specific to the ASAF structure. It is a generic code used to treat dynamic systems;
- *asaf_global.py* - This file contains global functions called by the main script; and
- *asaf_graph.py* - This file contains graphics functions necessary to get the results of the analysis, as plots of state and control signals.

All these files constitute a framework for studying nonlinear dynamical systems based on the environment described in Section 3.2. To use this structure, users have to edit only two files: *<SystemName>Pack.mop* and *asaf_config_<SystemName>.py*.

The procedure to use this environment to study a specific system starts by modeling the system in Modelica language directly into the file *<SystemName>Pack.mop*. Next, comes the editing of the *asaf_config_<SystemName>.py* configuration file according to the model, to set the parameters for simulations/optimizations.

The script *asaf_config_<SystemName>.py* is executed and all the configuration variables are stored in the file *asaf.dict*, which will be read by the main script *asaf_pgm.py*.

After that, the main script *asaf_pgm.py* is executed. It will generate all the data and figures resulting from the simulations/optimizations. The numerical results are written in files type “.h5” (Hierarchical Data Format - HDF) and the figures are stored in graphic formats defined in the configuration (e.g. “.jpg” and “.eps”).

An important consequence of the use of integrated and automated environment is that, once installed the tools presented in Table 4, the operation of the ASAF structure, with the use of the respective configuration and the Modelica model files for a given system/OCP, allows total and complete reproduction of the numerical results and graphics, not only of this Thesis, but for any OCP for which one has stored the model and configuration files.

3.3.1 Main Features of the ASAF Structure

The main features of the *ASAF* application can be used by setting two of its most important configuration items: The *Running_Type* and the *Running_Procedure* variables.

The current options for *Running_Type* are:

- *Optimization*: to perform optimizations - in this case the system model, jointly defined by *model_name* and *Opt_Label* variables, must be coded with Optimica extension of Modelica language;
- *Simulation*: to perform simulations;
- *Initialization*: to attempt to find equilibrium points of a system; and
- *Analysis*: to perform analysis, generating graphics and animations from raw data calculated in previous simulations or optimizations.

The current options for *Running_Procedure* are:

- *Default*: to carry out an analysis, simulation or optimization of a model without parametric variation;
- *Cases*: to carry out an analysis, optimizations or simulations of a model with parametric variation corresponding to specific cases of vectors of different parameters defined in a group;
- *Parameters*: to carry out an analysis, optimizations or simulations of a model with parametric variation corresponding to specific cases of vectors of predefined parameters. Each parameter has its vector of values and every simulation or optimization occurs after a change, from the model with default values, of a single parameter value at a time, therefore covering all the vector of values of each parameter;
- *Phase_Portrait*: for the generation of Phase Portrait with simulations of a model;

- *Mixed_Cases_and_Parameters*: jointly *Cases* and *Parameters*;
- *Mixed_Phase_Portrait_and_Parameters*: jointly *Phase_Portrait* and *Parameters*; and
- *Mixed_Phase_Portrait_and_Cases*: jointly *Phase_Portrait* and *Cases*.

The features described above are mature, stable and can be fully applied in the studies of dynamical systems and solutions of OCPs. However, other features can and will be under development, such as Basins of Attraction, Poincaré maps, Lyapunov Exponents calculation and 0-1 Test (GOTTWALD; MELBOURNE, 2009).

A project of graphical user interface for the ASAF structure is under development:

Basic Config	Sim Config	Optim Config	Other Config	Graphics	Python	Model Edition	Load File	Choose File	Save	Run	Exit																								
Load&Save&Run: (BAS_02) Model: (BAS_04) Running #: (BAS_06) Data Gen: asaf_default.dict Ex4 1 Data_Record Package: Ext Label: Proj Name: Config Label: Ex4Pack _Opt Tese_Prova L_11																																			
Reset Dir: False Grid: True																																			
Time Definition Start Time: Final Time: Piecewise Factor: 0.0 1.0 0.1 Discretization Elements # (n_e): Collocation Pts # (n_cp): Linear Solver: 200 10 ma57																																			
Running Type: Optimization Running Procedure: Default																																			
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#	State Var 1	description	label	unit	lower	upper	initial																												
1	State Var 1		x	*	-20	200	0																												
1	State Var 2		y	*	-20	200	0																												

Figure 5 – Development of the Graphical User Interface. Source: The author

Figure 5 shows the configuration used for simulations with OCP#4. The development of this easiness to the user aims to minimize the chances of problems and mistakes in parametric definitions, allowing direct and objective choice of parameters, as well as to ensure the proper registration on file for all Simulations/Optimizations performed.

The importance of this feature is evidenced by the fact that the parametric definition (such as the steps and *barrier parameter*) is essential for an algorithm to achieve numerical stability (PARKER; CHUA, 1991, p. 97) and can determine whether or not the convergence of iterations will occur, and if the solution will be achieved.

As application of this structure in the study of dynamical systems, in (RESENDE, 2017) it was presented numerical simulations of a generalized Lorenz system with four dimensions and hyperchaotic behavior, according to the model presented in (MACEK; STRUMIK, 2010) and (MACEK; STRUMIK, 2014).

Next, in the following chapter it will be presented the entire procedure performed for an initial verification of the hypothesis.

— 4 —

THE HYPOTHESIS VERIFICATION

The main caution that should be taken in an attempt to test the **Null Hypothesis (H0)** using the method defined in Section 2.4, is related to the choice of OCPs to be used. The chosen OCPs must contain two very specific features: It must be “non-compliant” with the classical theory, which means that it must be nonconvex, and it must have an exact solution known.

These conditions for the OCPs that are candidates to be used in the verification of the hypothesis makes it very difficult to find appropriate OCPs for the examination.

The following sections present the OCPs chosen for the Hypothesis Verification. The analytical solutions, necessary to establish a comparison reference for the numerical solutions provided by the application of the algorithm, are also presented.

4.1 FIRST OCP USED IN THE HYPOTHESIS VERIFICATION

After initial research, the following problem was found in (MEZIAT, 2007, p.156):

minimize:

$$J[\mathbf{x}, \mathbf{u}, t] = \int_0^1 (x - 10 * t)^2 dt \quad (1)$$

subject to:

$$\dot{x} = u^2 - u * x + x \quad (2)$$

$$x(0) = 0 \quad (3)$$

By observing the OCP defined above, one may verify that it is nonlinear and nonconvex. It can be verified by direct inspection of the performance index, that it is possible to infer the state optimal trajectory as $\mathbf{x}^* \equiv x(t) = 10 * t$.

By substituting the expression for $x(t)$ in the equation (2), the following possibilities for the control vector $\mathbf{u}^*$ are obtained:

$$u_1^*(t) = \frac{10 * t - \sqrt{100 * t^2 - 4 * (10 * t - 10)}}{2} \quad (4)$$

$$u_2^*(t) = \frac{10 * t + \sqrt{100 * t^2 - 4 * (10 * t - 10)}}{2} \quad (5)$$

The curves of the respective solutions are presented, respectively in Figures 6 and 7:

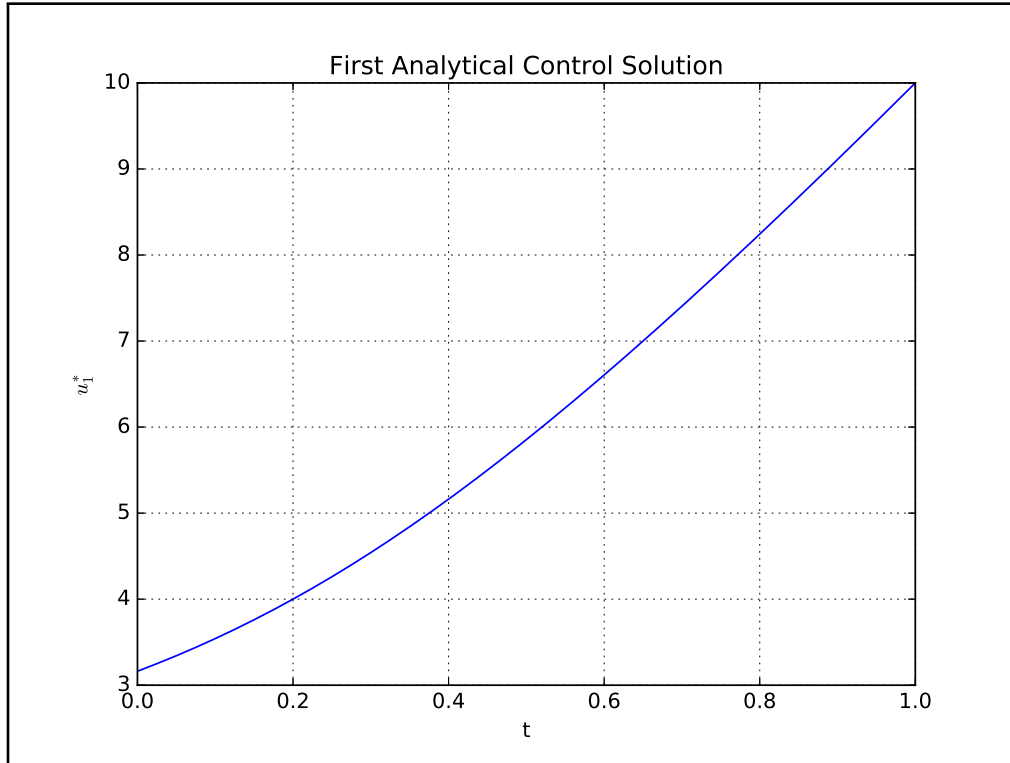


Figure 6 – First Analytical Solution of OCP#1. Source: The author.

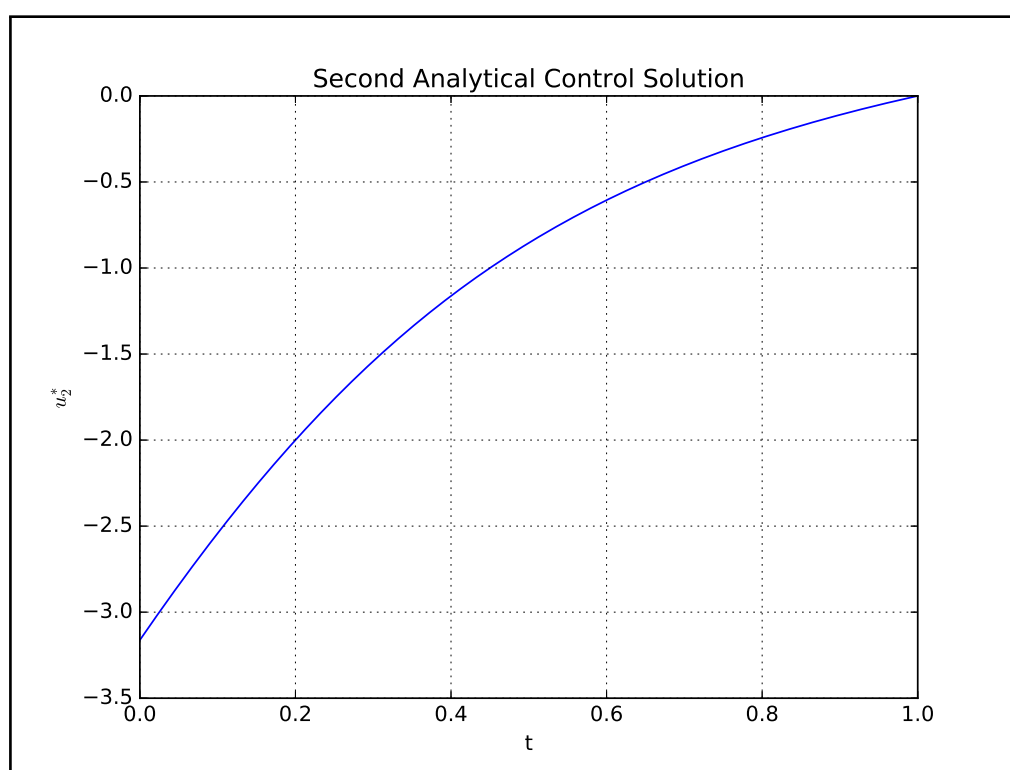


Figure 7 – Second Analytical Solution of OCP#1. Source: The author.

4.2 SECOND OCP USED IN THE HYPOTHESIS VERIFICATION

After research, the following problem was found in (MEZIAT, 2007, p.157):

minimize:

$$J[\mathbf{x}, \mathbf{u}, t] = \int_0^{10} (x - t^2)^2 dt \quad (6)$$

subject to:

$$\dot{x} = u^2 - u * x + x \quad (7)$$

$$x(0) = 0 \quad (8)$$

Again, it is possible to infer the state optimal trajectory as $\mathbf{x}^* \equiv x(t) = t^2$ and the following possibilities for the control vector $\mathbf{u}^*$:

$$u_1^*(t) = \frac{t^2 + \sqrt{t^4 - 4 * (t^2 - 2 * t)}}{2} \quad (9)$$

$$u_2^*(t) = \frac{t^2 - \sqrt{t^4 - 4 * (t^2 - 2 * t)}}{2} \quad (10)$$

The curves of the solutions are:

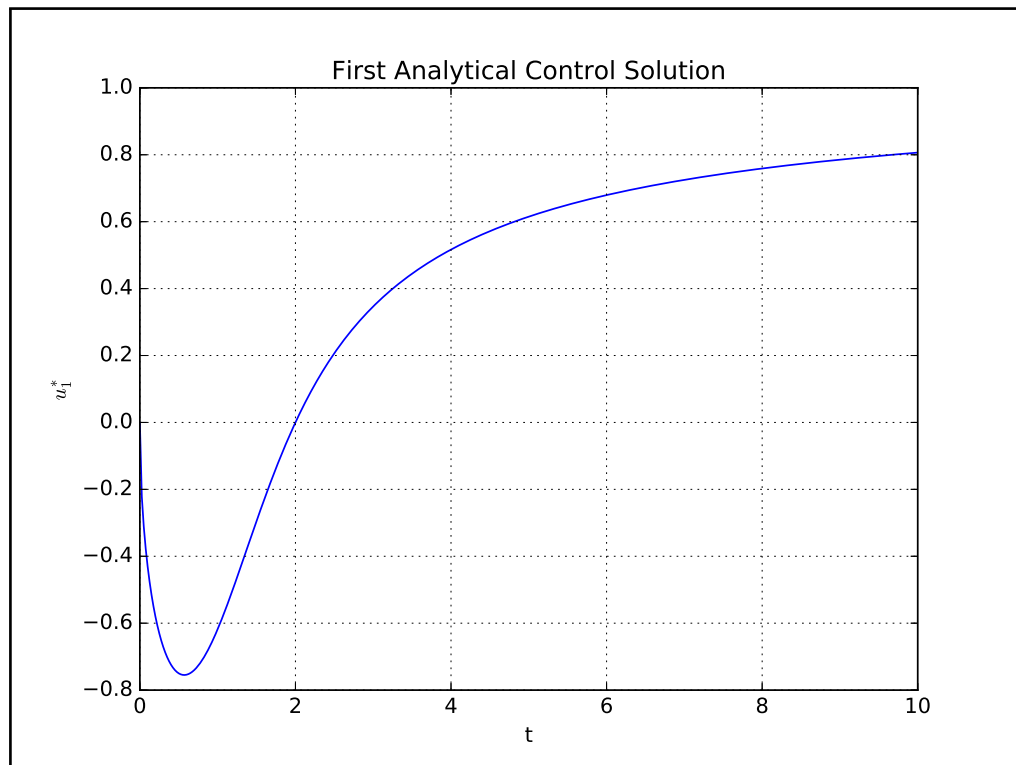


Figure 8 – First Analytical Solution for OCP#2. Source: The author.

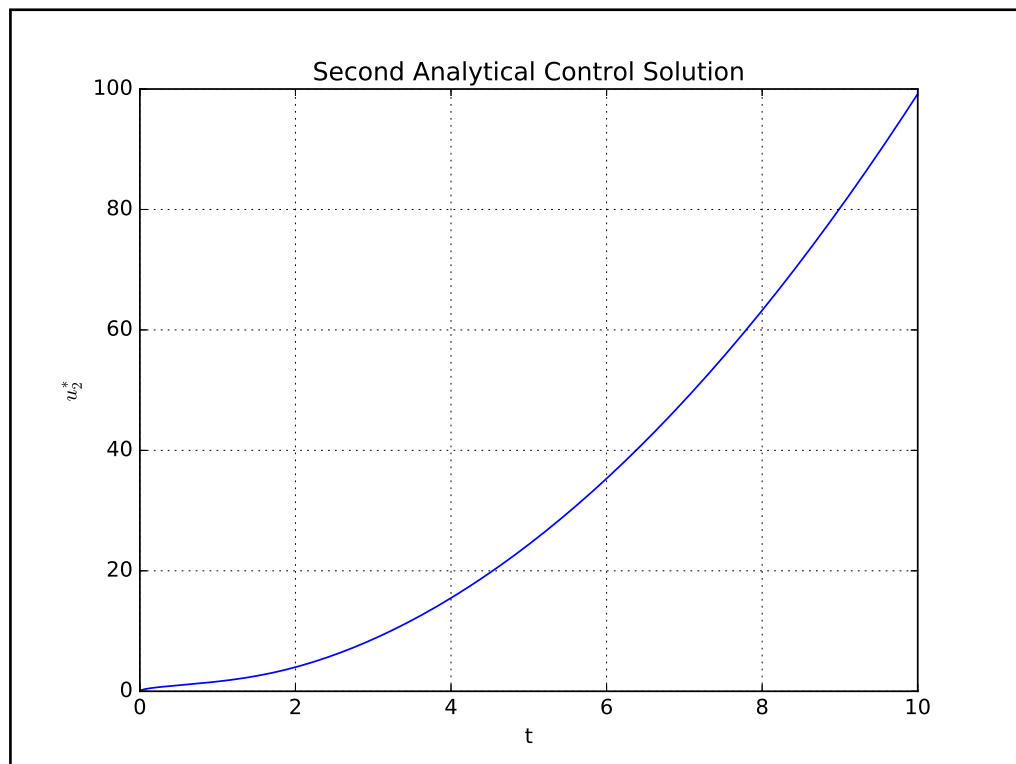


Figure 9 – Second Analytical Solution for OCP#2. Source: The author.

4.3 THIRD OCP USED IN THE HYPOTHESIS VERIFICATION

With the aim to amplify the possibilities of verifying the hypothesis, a third OCP was chosen from (MEZIAT, 2007, p. 158):

minimize:

$$J[\mathbf{x}, \mathbf{u}, t] = \int_0^1 ((x-t)^2 + (y-t)^2) dt \quad (11)$$

subject to:

$$\dot{x} = u \quad (12)$$

$$\dot{y} = (1 - u^2)^2 + x^2 \quad (13)$$

$$x(0) = 0; y(0) = 0 \quad (14)$$

Once again, it can be observed by direct inspection of the performance index that it is possible to infer the optimal trajectory as $\mathbf{x}^* \equiv (x(t), y(t)) = (t, t)$. This does not mean that this ideal trajectory is feasible by the system. For this OCP, however, even if it is possible to find the state ideal solution, an analytical solution for the control vector $\mathbf{u}^*$ was not obtained.

4.4 FOURTH OCP USED IN THE HYPOTHESIS VERIFICATION

Finally, the problem presented in (MEZIAT, 2007, p. 160) was chosen as fourth OCP:

minimize:

$$J[\mathbf{x}, \mathbf{u}, t] = \int_0^1 ((x-t)^2 + (y-t)^2) dt \quad (15)$$

subject to:

$$\dot{x} = u \quad (16)$$

$$\dot{y} = (1 - u)^2 * (2 - u)^2 + x^2 \quad (17)$$

$$x(0) = 0; y(0) = 0 \quad (18)$$

Once again, it is possible to infer the optimal trajectory as $\mathbf{x}^* \equiv (x(t), y(t)) = (t, t)$. This does not mean that this ideal trajectory is feasible by the system. For this OCP, as in the previous OCP, an analytical solution for the control vector $\mathbf{u}^*$ was not obtained.

4.5 RESULTS

Table 5 presents a summary of the canonical solutions for the used OCPs.

Table 5 – Summary of the Canonical Solutions of the Used OCPs

	OCP #1	OCP #2	OCP #3	OCP #4
u_1^*	$\frac{10*t - \sqrt{100*t^2 - 4*(10*t - 10)}}{2}$	$\frac{t^2 + \sqrt{t^4 - 4*(t^2 - 2*t)}}{2}$	-	-
u_2^*	$\frac{10*t + \sqrt{100*t^2 - 4*(10*t - 10)}}{2}$	$\frac{t^2 - \sqrt{t^4 - 4*(t^2 - 2*t)}}{2}$	-	-
x^*	$10 * t$	t^2	t	t
y^*	-	-	t	t

Source: The author.

As an attempt to verify the research hypothesis, optimizations with the default parameters of the IPOPT optimizer defined in version 3.12 were performed, except for the changes defined in Table 6:

Table 6 – Optimization Parameters Used in All the Optimizations

Parameter	Tool	Description	Value
n_c	JModelica.org	As defined in List of Symbols	10
linear_solver	IPOPT ^a	Linear Solver	ma57
max_iter ^b	IPOPT	Maximum number of iterations	3e4
mu_max ^b	IPOPT	Maximum value for the barrier parameter	2
mu_min ^b	IPOPT	Minimum value for the barrier parameter	1e-300

^a According to Section C, *Options Reference*, from the User's Guide *Introduction to Ipopt: A tutorial for downloading, installing, and using Ipopt*, of the IPOPT 3.12 tool.

^b Has effect only if the option “*mu_strategy*” chosen is “*adaptive*”.

Source: The author.

The definition of each context was made by choosing two parameters, setting the duo:

$$C = \{PC_1, PC_2\} \quad (19)$$

Where PC_1 and PC_2 were determined by the conditions presented in Table 7.

Table 7 – Parameters Used in the Definitions of the Contexts

Index	Parameter (s) to Set	Possible Values	OCP ^a			
			#1	#2	#3	#4
PC_1	Interval $[u_{min}, u_{max}]$	I	[2, 12]	[-2, 2]	[0, 2]	[-2, 10]
		II	[-5, 0]	[-1, 110]	[0, 7, 5]	[-5, 1]
		III	[-2, 8]	[0, 64]	[-1, 0, 7]	[1, 10]
PC_2	$period^b$	I ^c	0	0	0	0
		II	0,05	0,5	0,05	0,05
		III	0,1	1	0,1	0,1

^a Specific values for each OCP

^b A parameter that sets the “*blocking_factors*” option of the JModelica.org tool, used for piecewise constant controls.

^c With “*period*” = 0, here is no restriction of constant controls in periods.

Source: The author

Therefore, with the possibilities of adjustments on Table 7, nine contexts are defined for each OCP.

The definition of each simulation was made by choosing four parameters, setting the tuple:

$$S = \{PS_1, PS_2, PS_3, PS_4\} \quad (20)$$

Where each PS_i was determined by Table 8.

Therefore, with the possibilities of adjustments on Table 8, twenty-seven different simulations are defined, that remain the same for each context of each OCP.

Table 9 presents a summary of the scope of the simulations.

The parameters of Table 6 were kept the same in all the optimizations.

The parameters in Tables 7 and 8 were adjusted accordingly for each specific simulation in a given context.

Next, the obtained results are presented.

Table 8 – Parameters Used in the Definitions of the Simulations

Index	Parameter to Set	Possible Values	Set
PS_1	n_e^a	I	40
		II	200
		III	1000
PS_2	$\mu_strategy^b$	I	adaptive
		II	monotone
PS_3	$adaptive_mu_globalization^c$	I	kkt-error
		II	obj-constr-filter
PS_4	$\alpha_for_y^d$	I	safer-min-dual-infeas
		II	primal
		III	full

^a The parameter that sets the option “ n_e ” of the JModelica.org tool, used to determine the number of elements in the discretization.

^b The parameter that sets the option “ $\mu_strategy$ ” of the IPOPT tool.

^c The parameter that sets the option “ $adaptive_mu_globalization$ ” of IPOPT tool. Has effect only if the option “ $\mu_strategy$ ” chosen is “ $adaptive$ ”.

^d The parameter that sets the option “ α_for_y ” of the IPOPT tool.

Source: The author.

Table 9 – Scope of Simulations

Total of OCPs	Contexts per OCP	Simulations per Context	Total of Simulations
4	9	27	972

Source: The author.

4.5.1 Optimizations Results for OCP#1

Here, the results of the optimizations are presented:

Table 10 – Optimizations Results for OCP#1

Context	Candidates	Rejected	Not Solved	Correct Classifications	Effectiveness (%)
A	21	6	0	21	78
B	25	2	0	27	100
C	27	0	0	27	100
J	27	0	0	18	67
K	27	0	0	27	100
L	27	0	0	0	0
S	13	9	5	15	68
T	9	16	2	17	68
U	14	13	0	25	93
Average	-	-	-	-	74.89

Source: The author.

4.5.1.1 Analysis of the Optimizations Results for OCP#1

It is observed, in context A, that because of the definition of the control variable limits, the results were similar to the first canonical solution as shown in the figures of the simulation A-1:

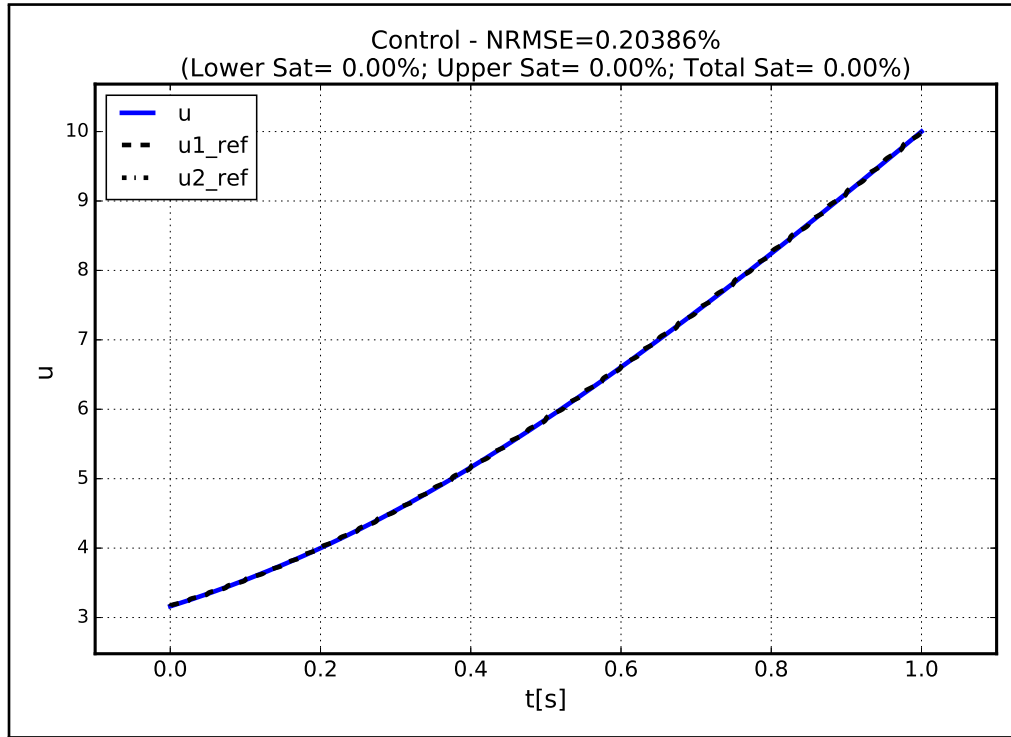


Figure 10 – Candidate Solution for OCP#1 - Context A: Control. Source: The author.

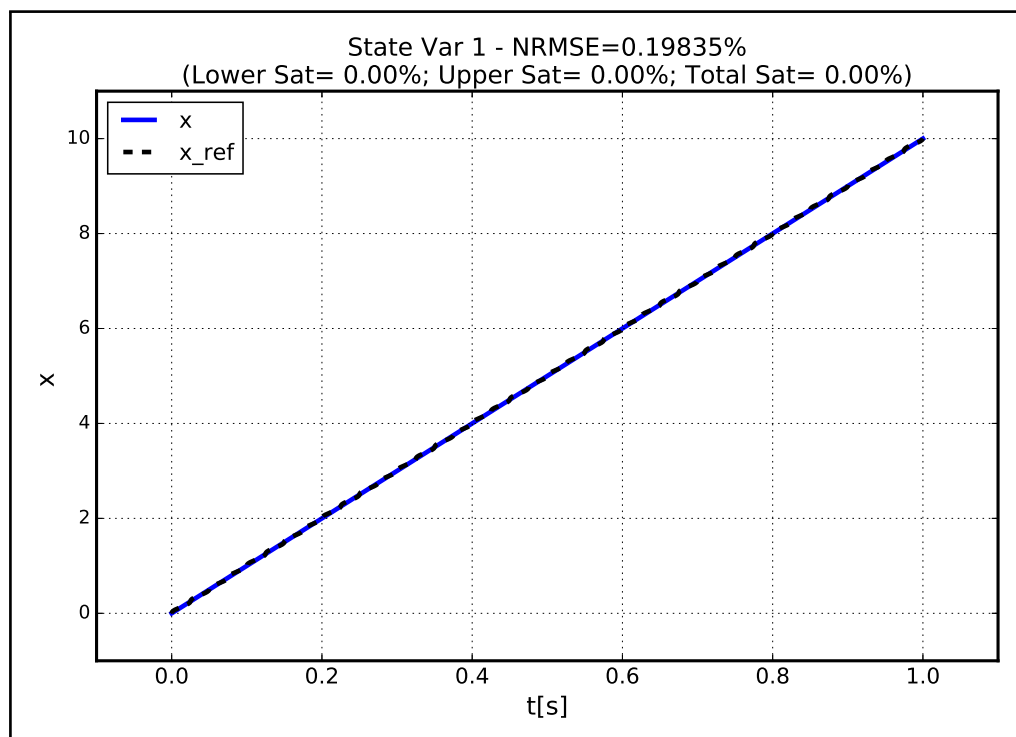


Figure 11 – Candidate Solution for OCP#1 - Context A: State. Source: The author.

Diversely, the solution was rejected in the simulation A-4. The rejection did not occur due to an inadequacy of the found state, but rather due to the high NRMSE index of the control variable:

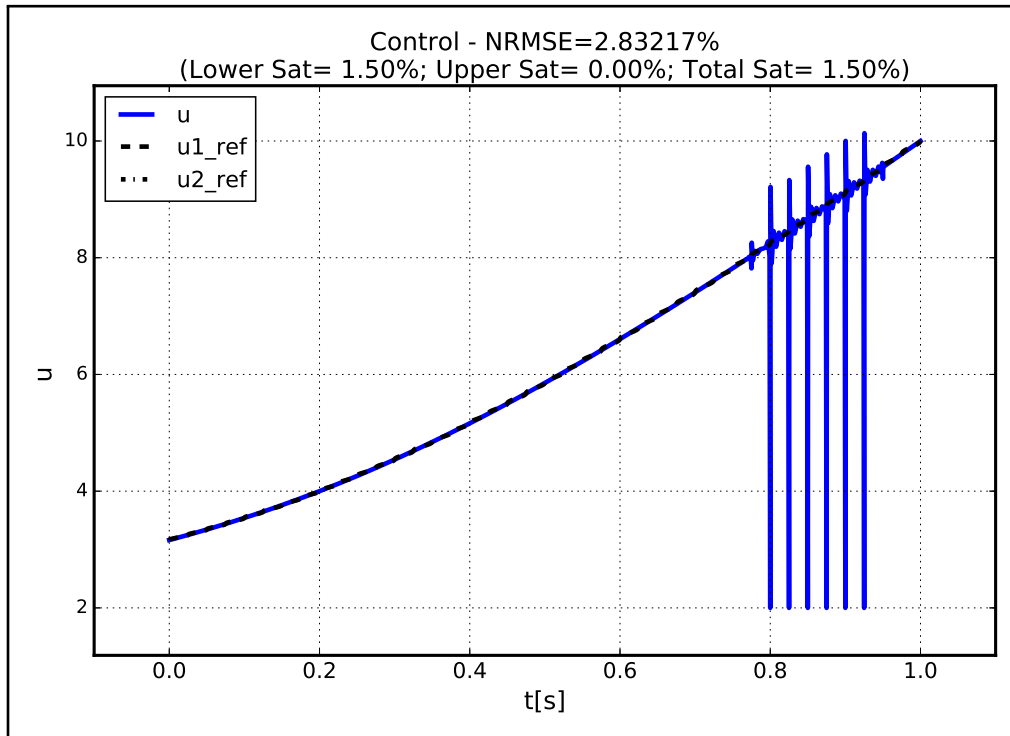


Figure 12 – Rejected Solution for OCP#1 - Context A: Control. Source: The author.

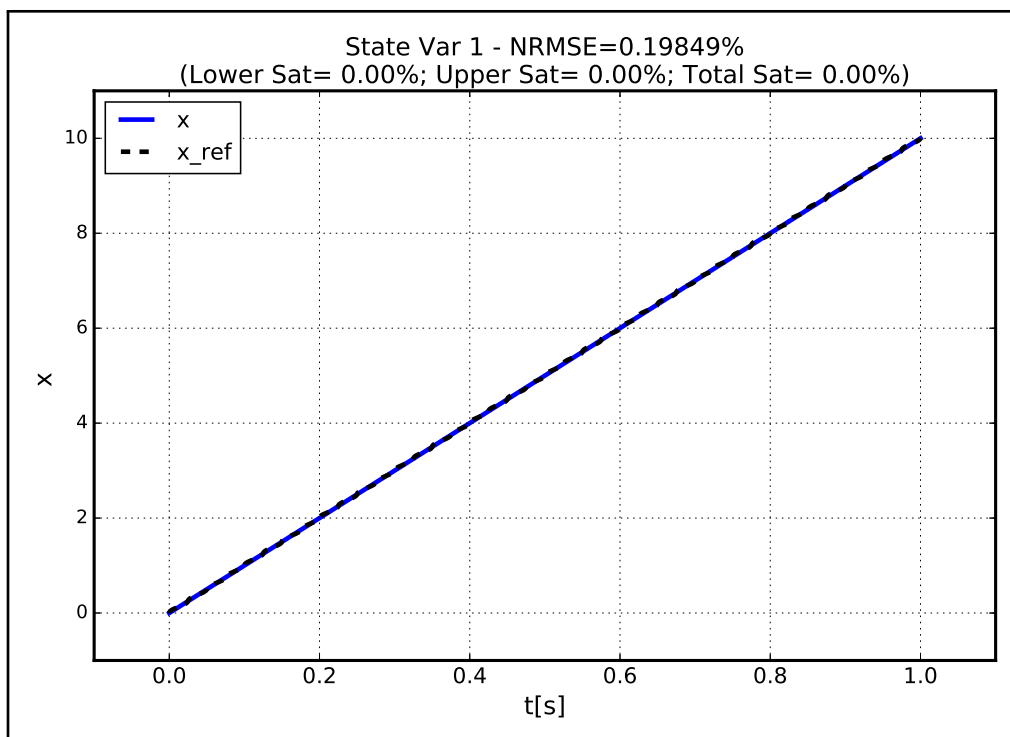


Figure 13 – Rejected Solution for OCP#1 - Context A: State. Source: The author.

The solution A-17 was numerically classified as a candidate by the algorithm. However, it is observed in Figure 14 that the solution is not qualitatively interesting. This wrong classification was correctly identified in the verification of the hypothesis, when performing the automated confrontation against the ideal analytical solution.

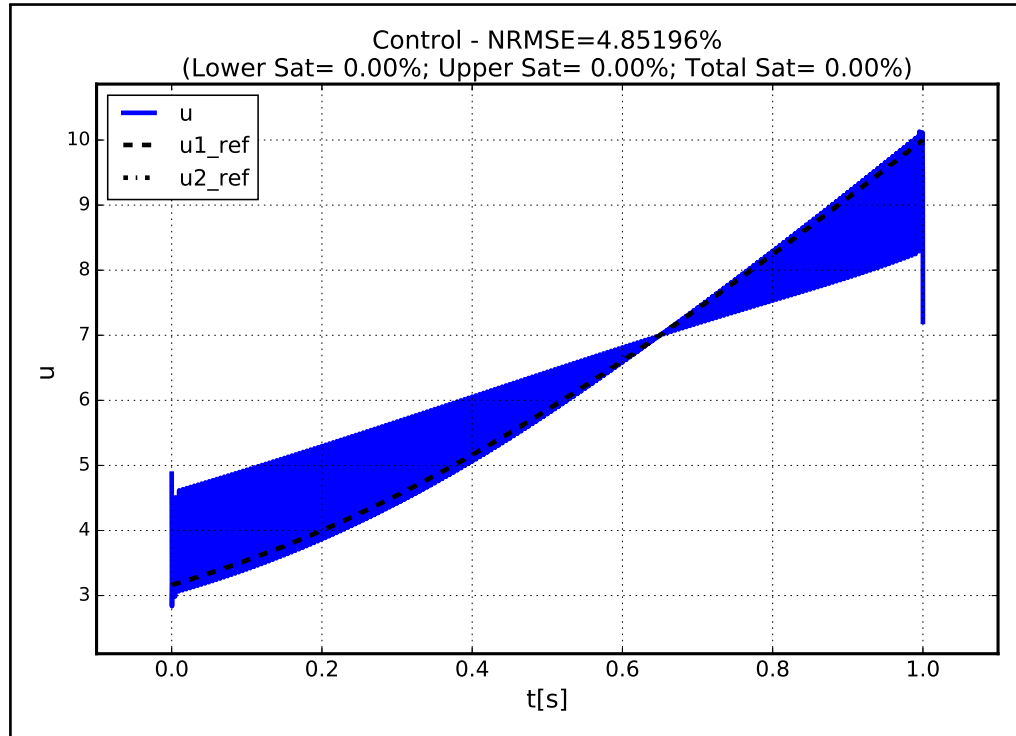


Figure 14 – Solution Improperly Classified as Candidate for OCP#1 - Context A: Control.
Source: The author.

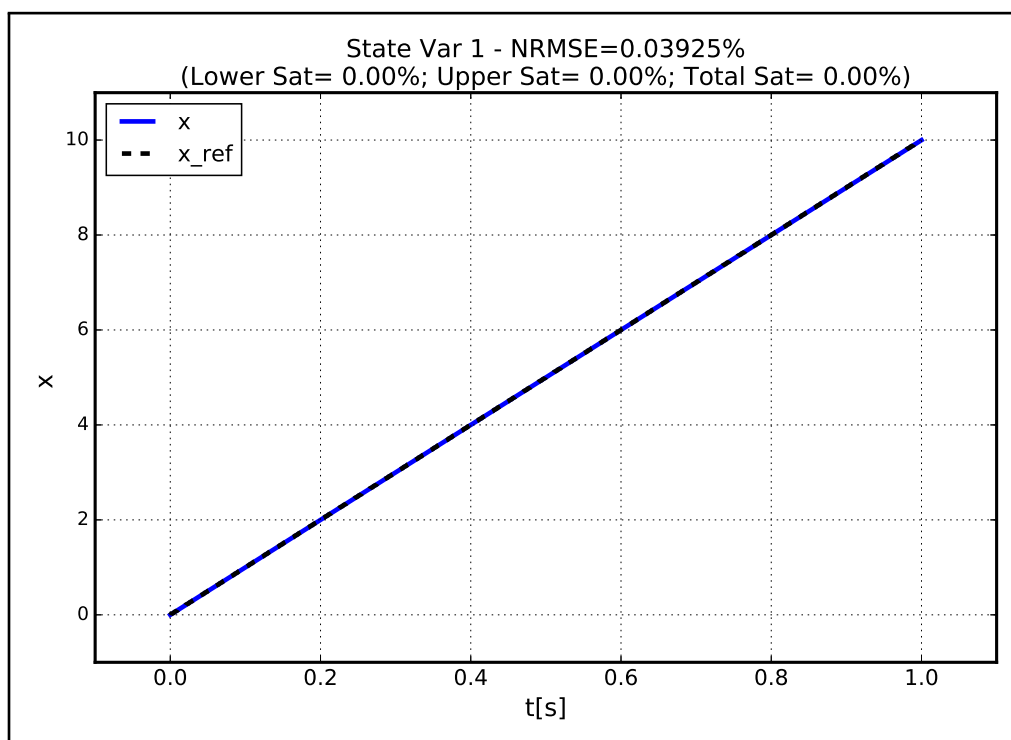


Figure 15 – Solution Improperly Classified as Candidate for OCP#1- Context A: State. Source: The author.

Similar situations have occurred in all other simulations of this context.

Context B also presented solutions that leaned towards to the first canonical solution, respecting the piecewise control restriction, as shown by the figures of simulation B-2, as follows:

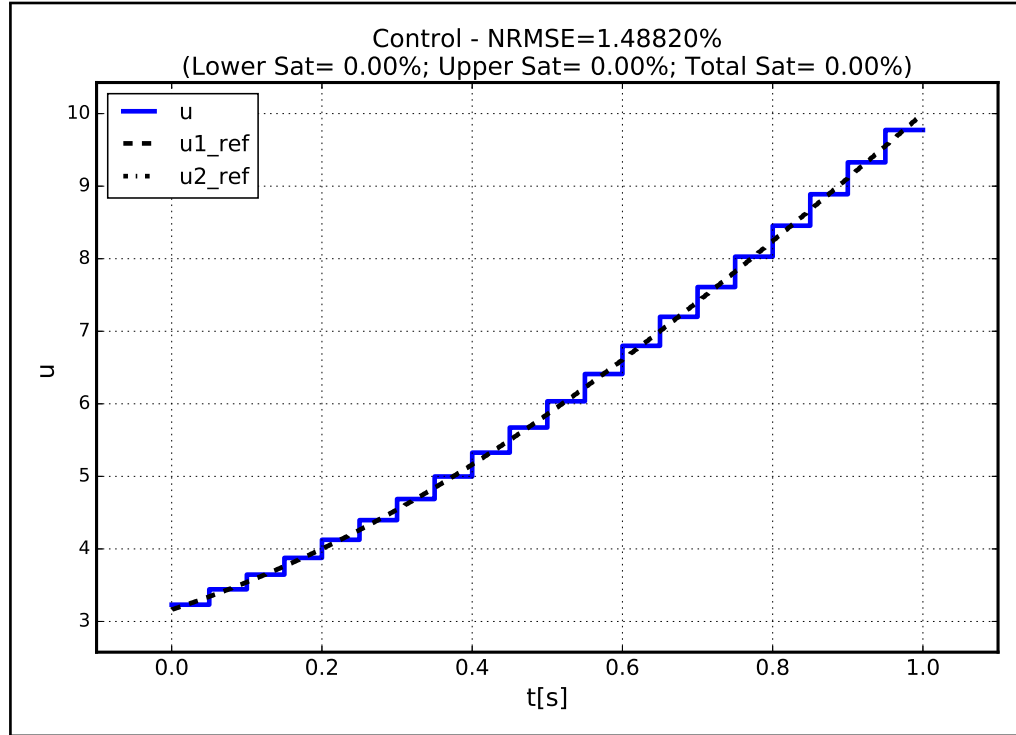


Figure 16 – Candidate Solution for OCP#1 - Context B: Control. Source: The author.

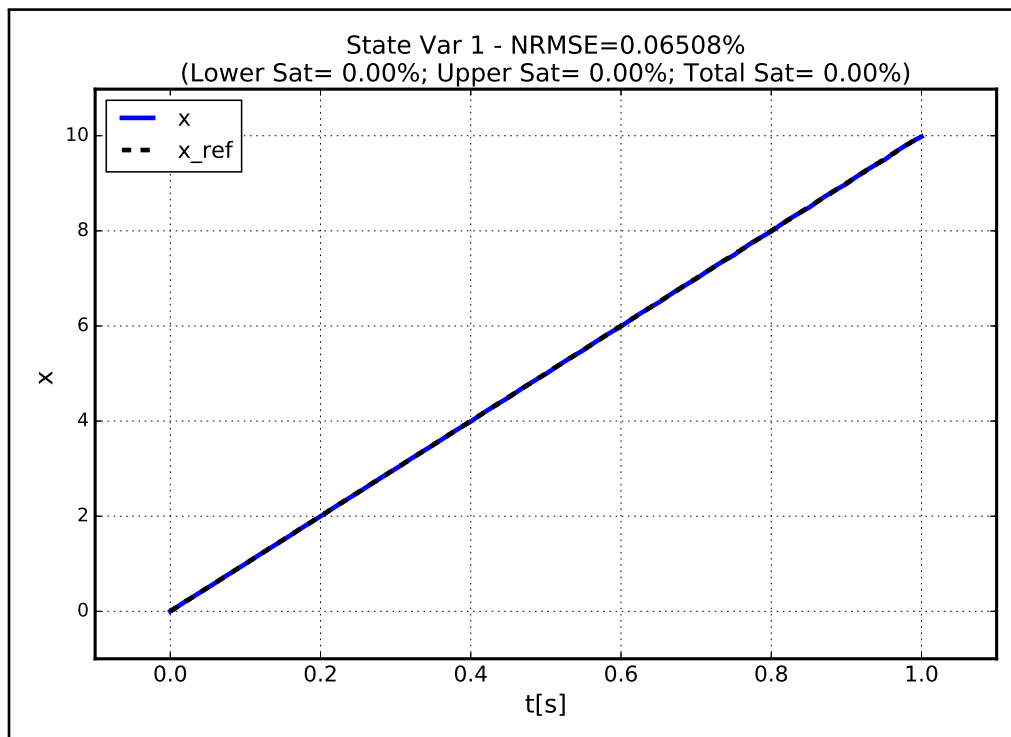


Figure 17 – Candidate Solution for OCP#1 - Context B: State. Source: The author.

Simulation B-1 was rejected due to the high level of saturation index of the control variable:

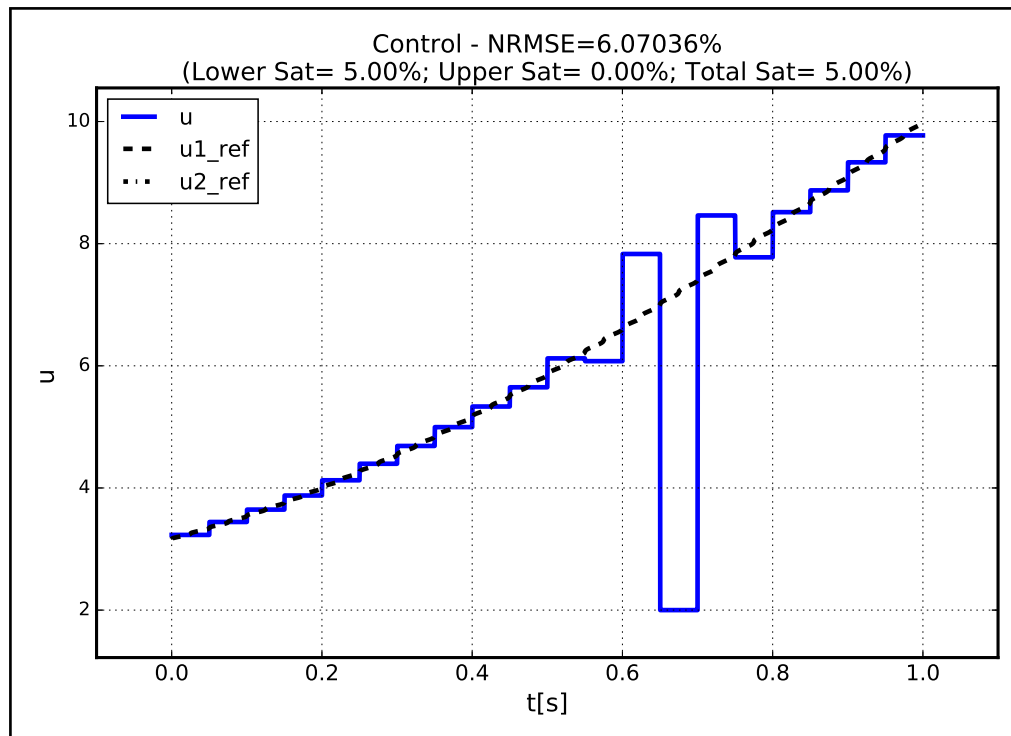


Figure 18 – Rejected Solution for OCP#1 - Context B: Control. Source: The author.

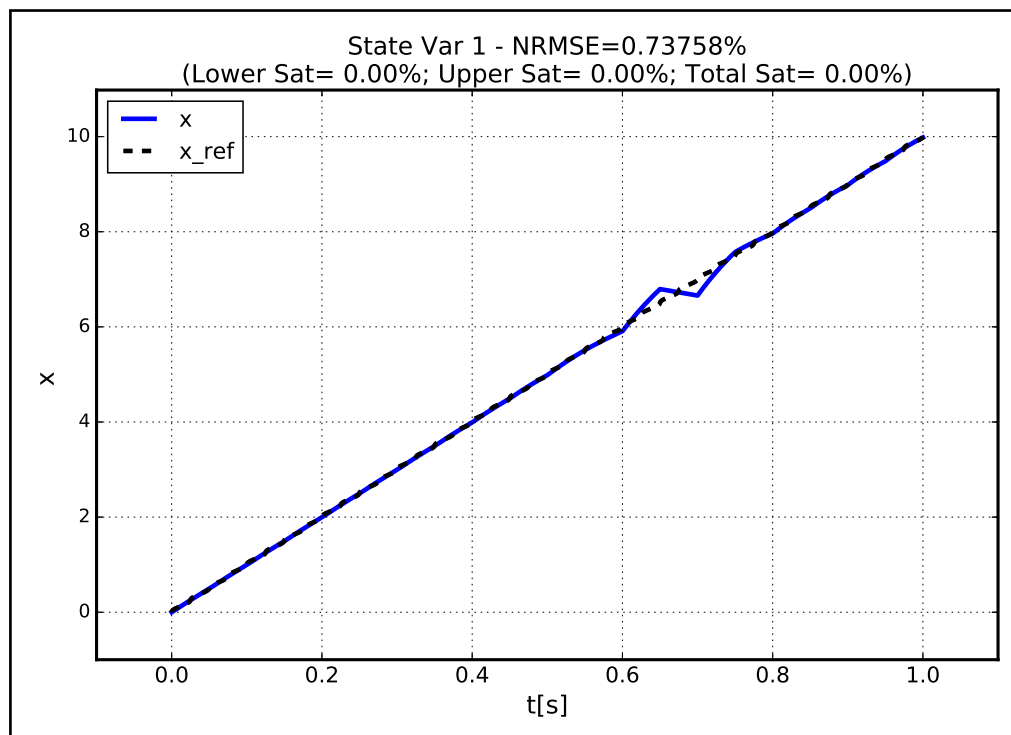


Figure 19 – Rejected Solution for OCP#1 - Context B: State. Source: The author.

Context C also presented solutions that leaned towards to the first canonical solution:

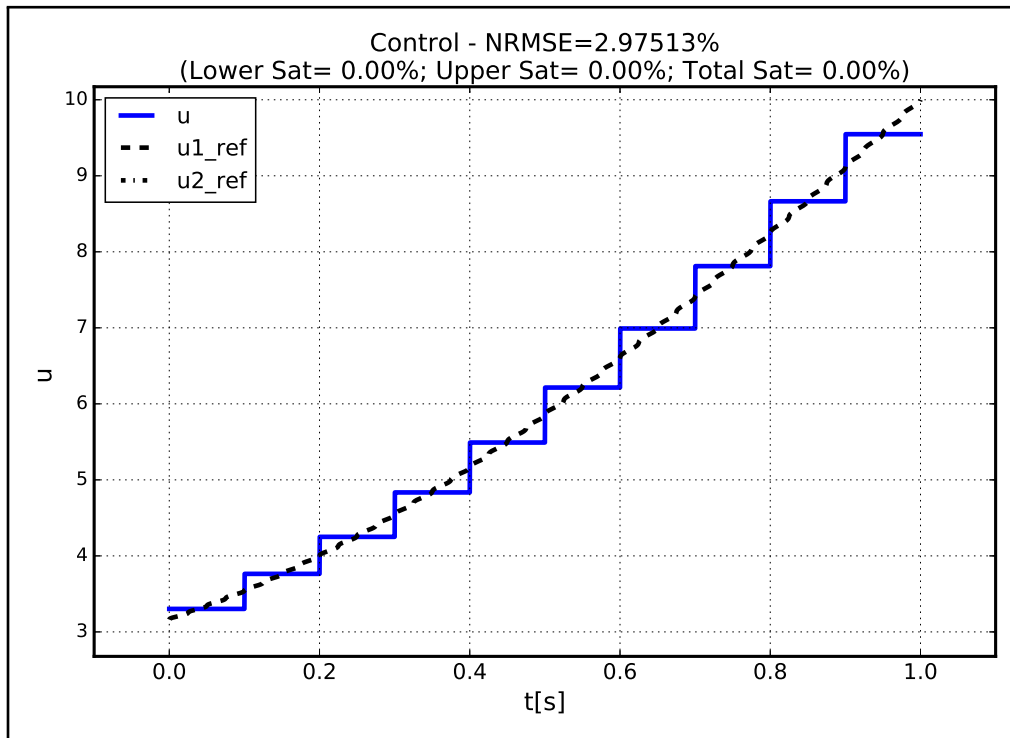


Figure 20 – Candidate Solution for OCP#1 - Context C: Control. Source: The author.

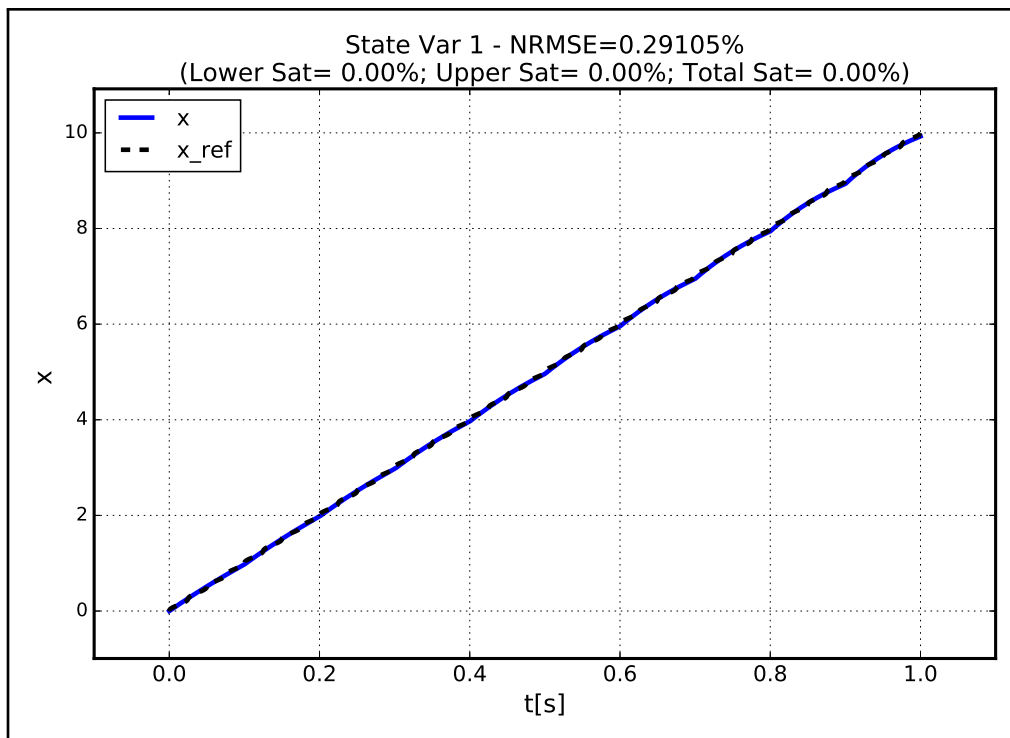


Figure 21 – Candidate Solution for OCP#1 - Context C: State. Source: The author.

Without presenting further figures, a similar phenomenon was observed for the J, K and L contexts, this time, in the range of the second canonical solution.

However, something really interesting happened in contexts S, T and U, where the range of the control partially encompassed both canonical solutions.

Thus, it was found in many simulations the occurrence of numerical solution with oscillation points between the canonical solutions, as shown in the following figures.

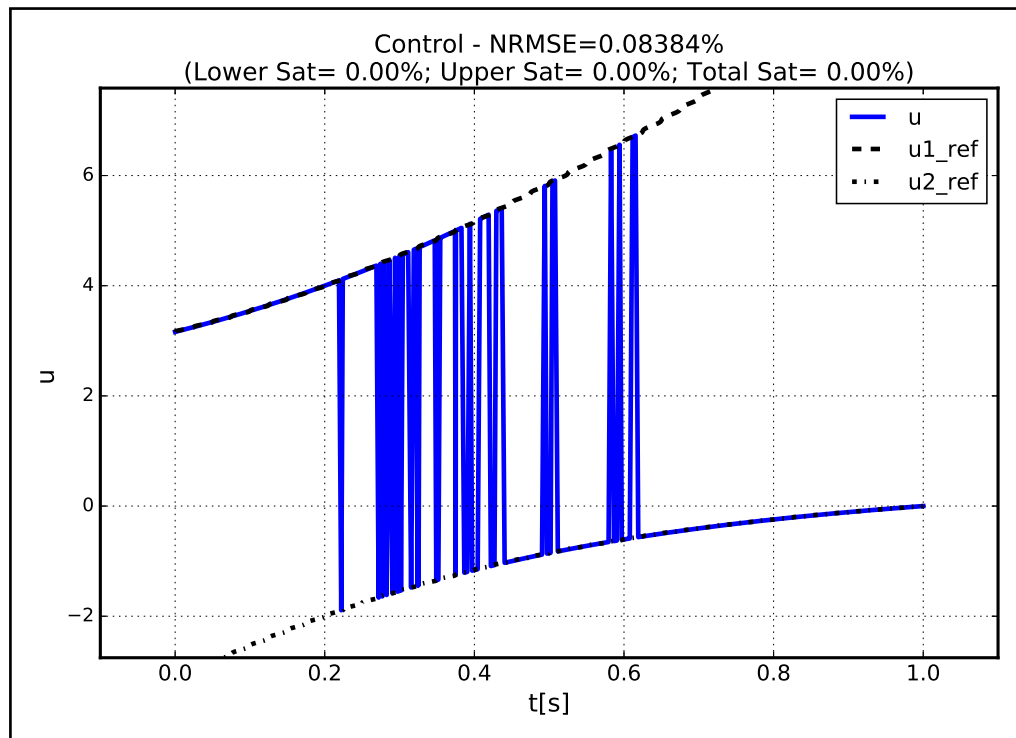


Figure 22 – Candidate Solution for OCP#1 - Context S: Control. Source: The author.

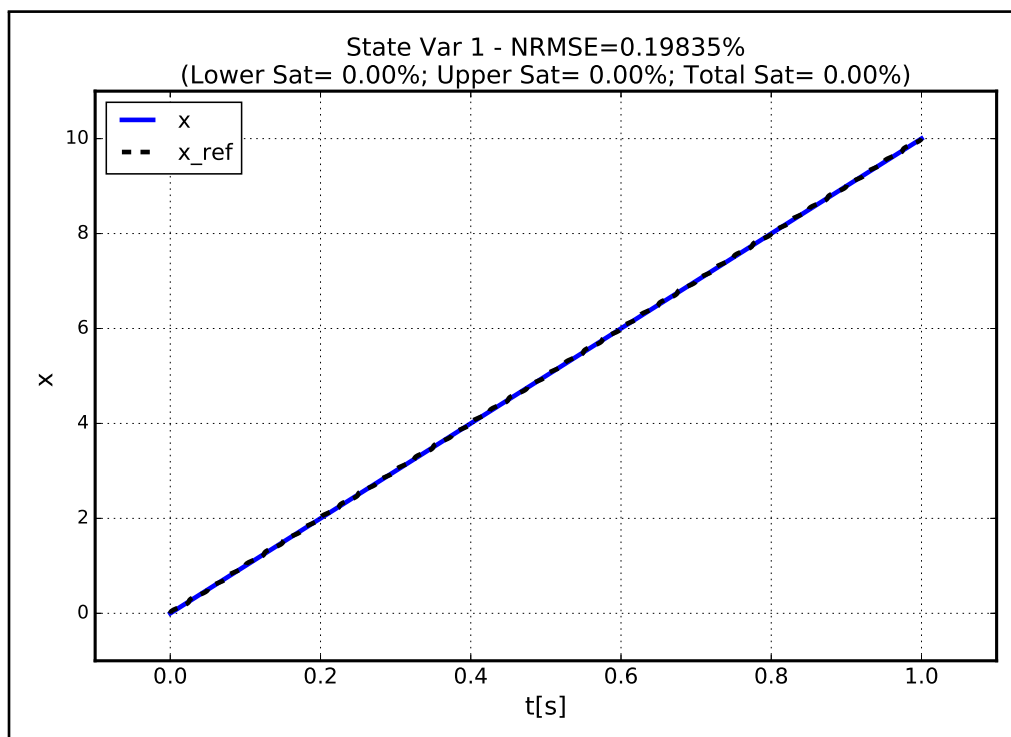


Figure 23 – Candidate Solution for OCP#1 - Context S: State. Source: The author.

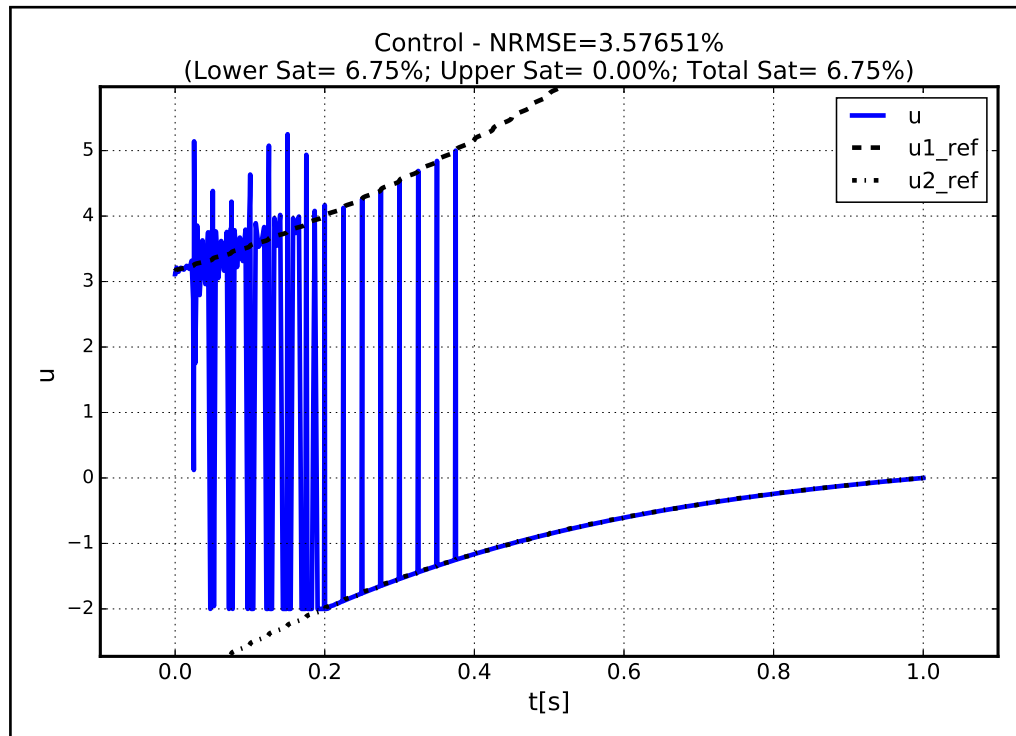


Figure 24 – Rejected Solution for OCP#1 - Context S: Control. Source: The author.

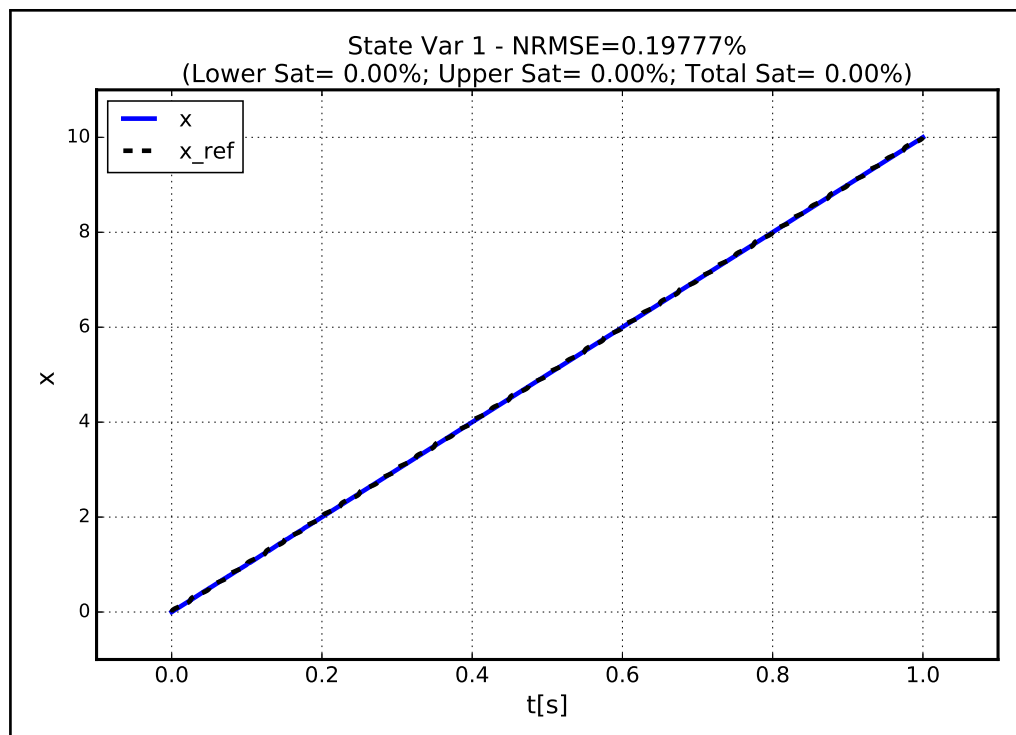


Figure 25 – Rejected Solution for OCP#1 - Context S: Control. Source: The author.

Similarly, it was observed in context T:

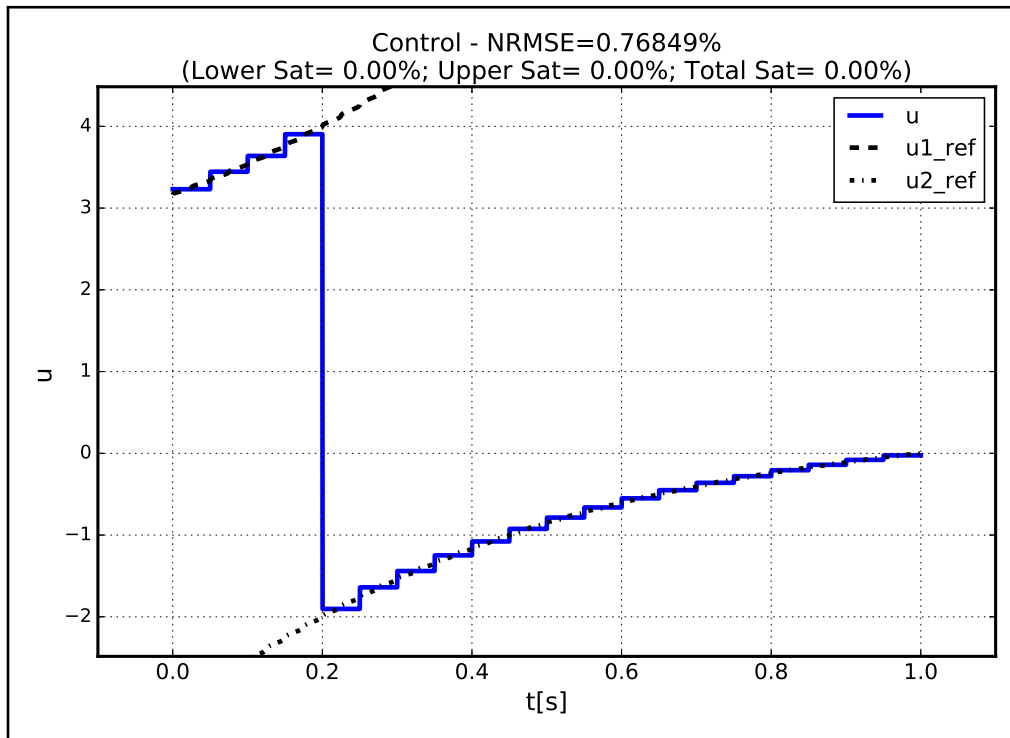


Figure 26 – Candidate Solution for OCP#1 - Context T: Control. Source: The author.

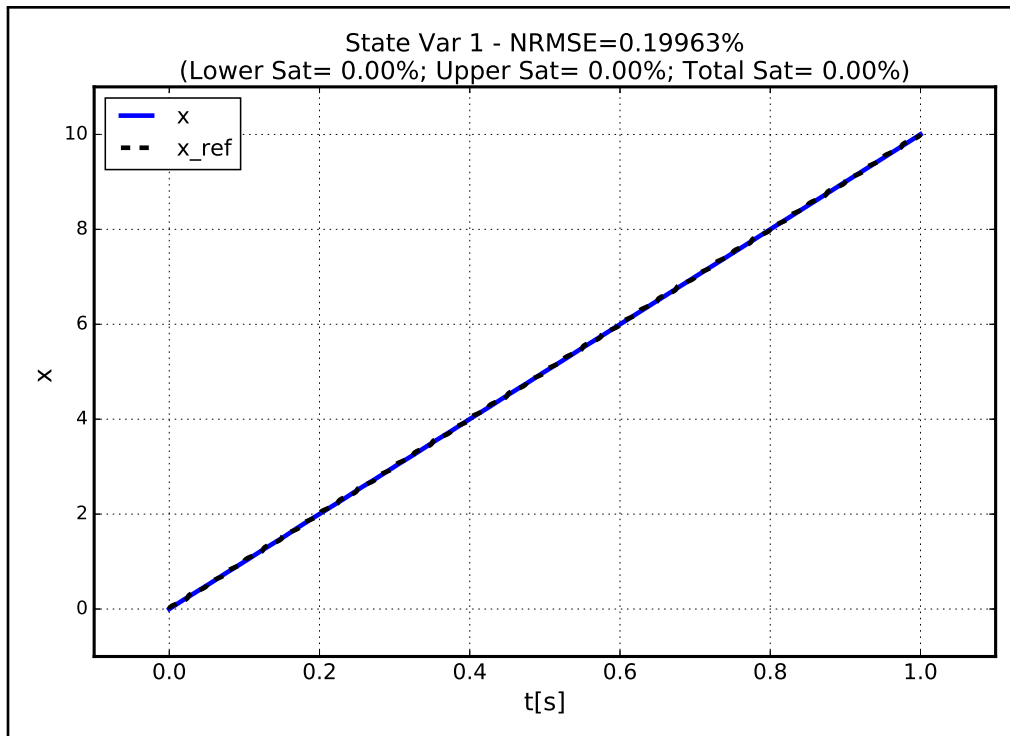


Figure 27 – Candidate Solution for OCP#1 - Context T: State. Source: The author.

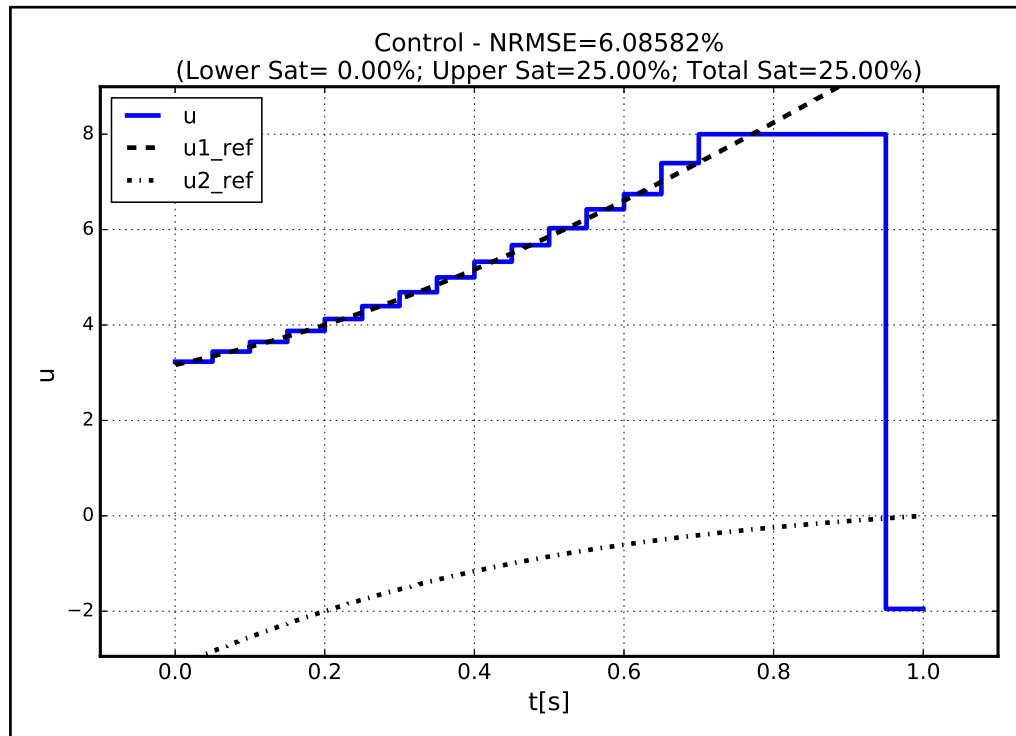


Figure 28 – Rejected Solution for OCP#1 - Context T: Control. Source: The author.

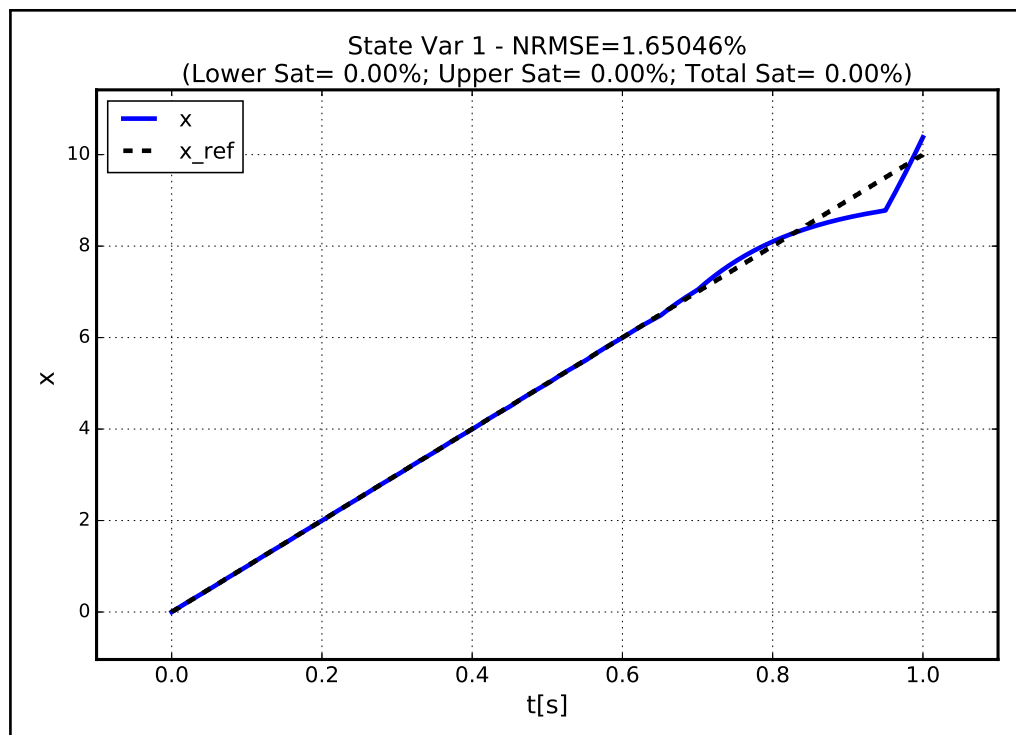


Figure 29 – Rejected Solution for OCP#1 - Context T: State. Source: The author.

Similarly, it was observed in context U:

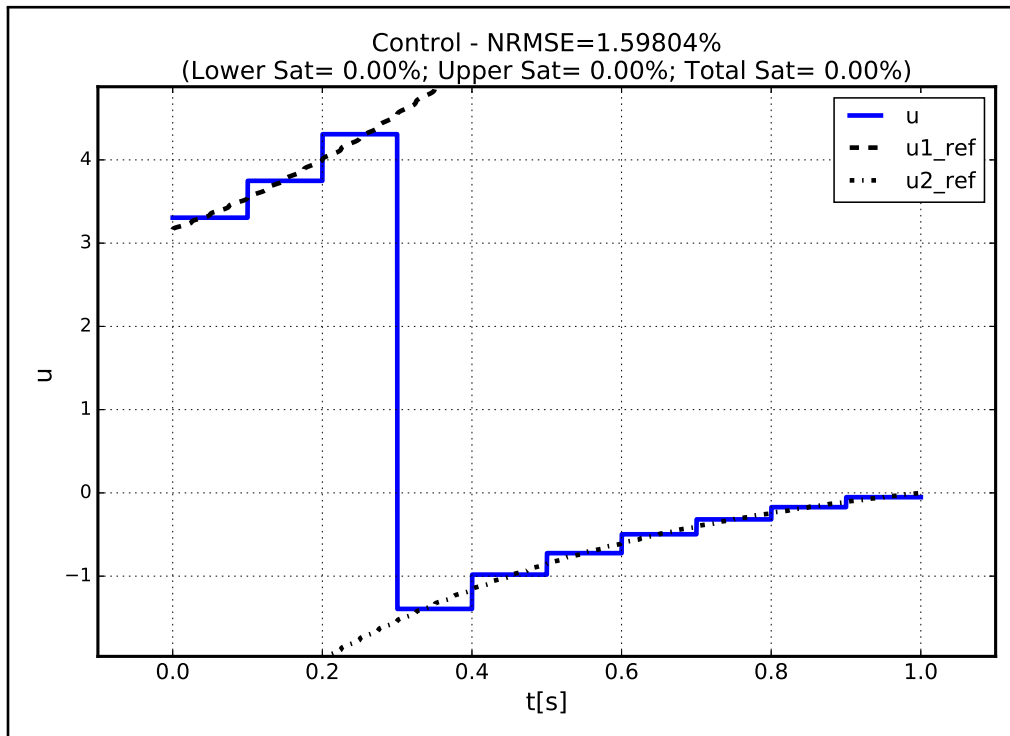


Figure 30 – Candidate Solution for OCP#1 - Context U: Control. Source: The author.

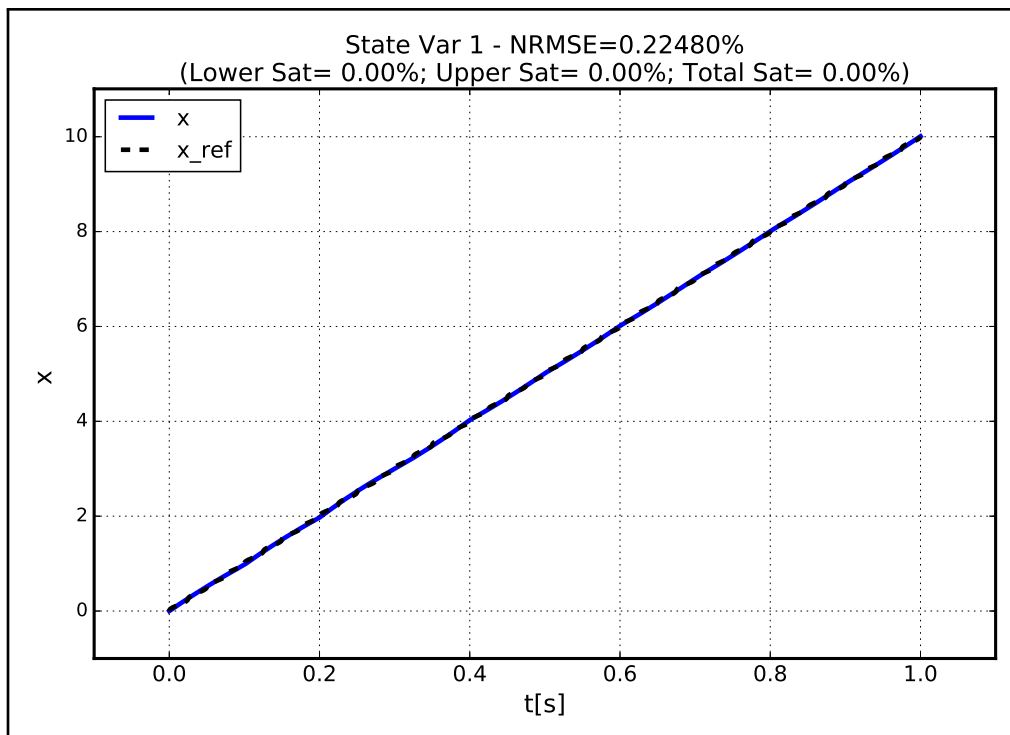


Figure 31 – Candidate Solution for OCP#1 - Context U: State. Source: The author.

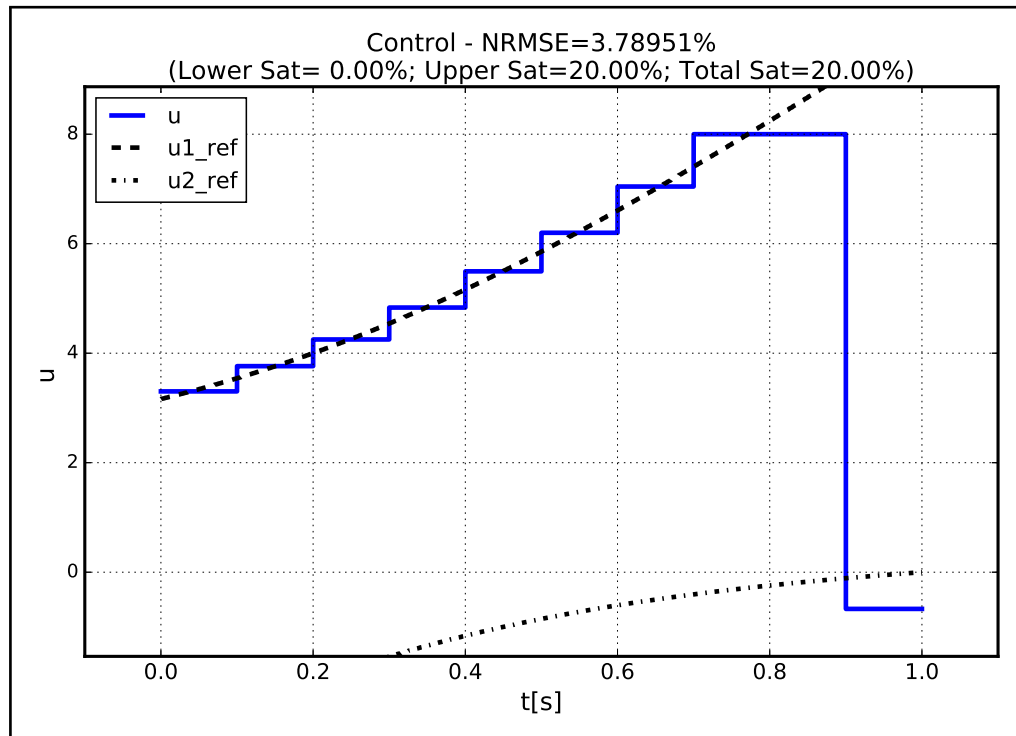


Figure 32 – Rejected Solution for OCP#1 - Context U: Control. Source: The author.

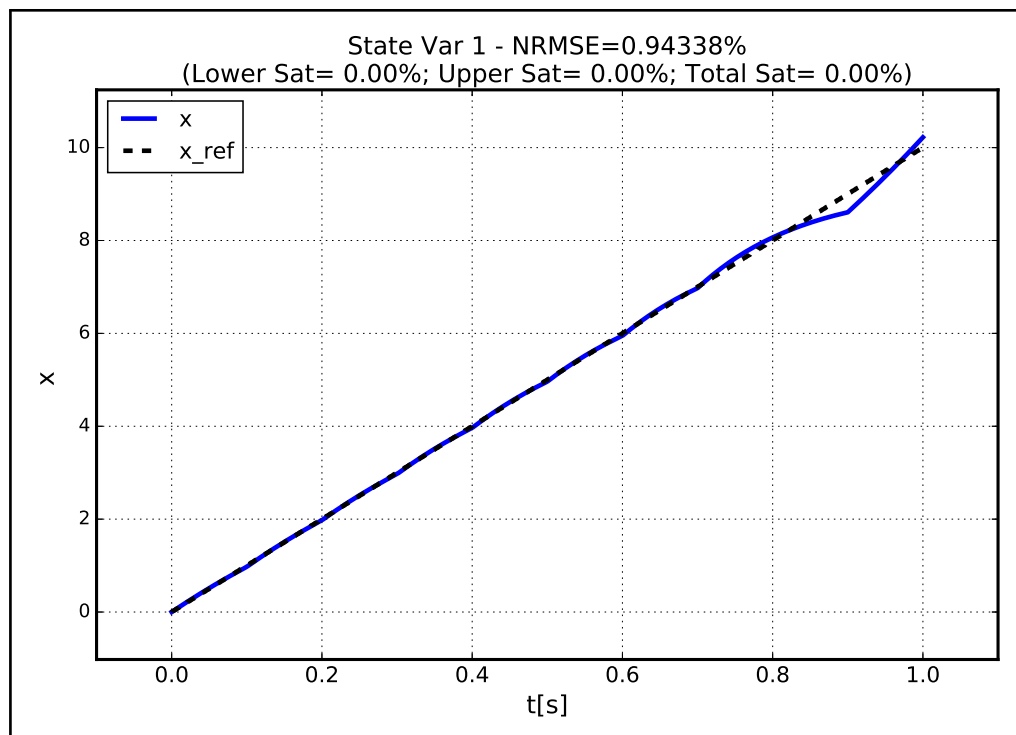


Figure 33 – Rejected Solution for OCP#1 - Context U: State. Source: The author.

The simulations of OCPs from 2 to 4 presented a similar outcome. The detailed numerical report is presented in Appendix B.

A brief summary of the results is presented in Table 11.

Table 11 – Estimation of the Effectiveness of the Algorithm: Summary of the Results

OCP#	Effectiveness Average (%)
	JM1.17, JM2.0, JM2.1 and JM2.2
1	74.89
2	72.78
3	95.56
4	87.22
Global Average of the Effectiveness	
	82.61

Source: The author.

The simulations were performed in four different versions of JModelica.org. The results were numerically the same in all versions.

4.6 THE HYPOTHESIS VERIFICATION BY STATISTICAL TESTS

The *Signal Test* was performed in MATLAB® with the following code:

4.6.1 The MATLAB® Code for the Hypothesis Verification

```

1 %-----
2 clear all; close all; clc;
3
4 clear a;
5 a=[78, 100, 100, 67, 100, 0, 68, 68, 93;...
6 100, 63, 45, 100, 0, 64, 100, 83, 100;...
7 100, 100, 93, 100, 89, 78, 100, 100, 100;...
8 100, 100, 63, 67, 88, 92, 75, 100, 100
9 ];
10
11 a=round(a)
12 [p,tbl,stats] = kruskalwallis(a')
13 figure

```

```

14 multcompare( stats )
15 [p1,h1,stats] = signtest(a(1,:),70,'tail','right','alpha',0.001,'method','exact')
16 [p2,h2,stats] = signtest(a(2,:),70,'tail','right','alpha',0.001,'method','exact')
17 [p3,h3,stats] = signtest(a(3,:),70,'tail','right','alpha',0.001,'method','exact')
18 [p4,h4,stats] = signtest(a(4,:),70,'tail','right','alpha',0.001,'method','exact')
19
20
21
22 [p1,h1,stats] = signtest(a(1,:),90,'tail','right','method','exact')
23 [p2,h2,stats] = signtest(a(2,:),90,'tail','right','method','exact')
24 [p3,h3,stats] = signtest(a(3,:),90,'tail','right','method','exact')
25 [p4,h4,stats] = signtest(a(4,:),90,'tail','right','method','exact')

```

The Signal Test results corroborated the **Null Hypothesis** in which the median estimate of the effectiveness of the algorithm is equal to or greater than 70%. On the other hand, the Kruskal-Wallis Test, (DANIEL, 1990), corroborated the hypothesis that the four samples related to the four studied OCPs present the same median of effectiveness of the algorithm implementation, also being originated from the same population.

Further detailing of the Kruskal-Wallis Test was achieved with the application IBM® SPSS® *Statistics*. The results can be found in Appendix C, where the median estimate for all samples was calculated at 93%, with significance of 0.05 (95%).

Therefore, within the scope defined and based on the statistical tests performed, the hypothesis of this doctoral research is corroborated.

The next chapter presents a brief discussion on the results of the research and its implications for the study and solution of nonconvex OCPs by means of automated numerical simulations.

— 5 —

DISCUSSION OF THE RESULTS

The discussion of the results of this Thesis starts by evoking the experience lived by Umberto Eco, described in (ECO, 2007, p. 112)¹, which resulted in his guidance on Scientific Humility:

“the fact is that we must respectfully listen everyone, without ceasing to express value judgments...”

In this Thesis, parallelly to the concept originally defined for bibliographic references, an attempt was made to apply Scientific Humility in the scope of the possible context contributions and their numerical solutions provided by the algorithm.

Hence, one should not exclude in advance the project of possible contexts and, consequently, their numerical solutions for a given OCP. It is recommended that, for the efficient application of the proposed algorithm, one should vary parameters, define various contexts without bias, seek solutions in an automated manner, classify solutions with criteria, reject the inappropriate and, among the remainder, choose a recommended solution, if acceptable.

With the continuous advancement of computational capacity and metaprogramming techniques, the approach by automated numerical simulations is an important resource that can, at first, perform numerous parametric variations to generate a diversity of numerical solutions for non-compliant problems, that cannot be solved by conventional methods.

¹Text in Portuguese

This Thesis explores exactly these new computational possibilities and features a criteria and a rational method, without excluding a context or a solution in advance, to provide a recommended solution to a nonconvex OCP, object of the algorithm.

The proposed algorithm is generic enough to not depend on current computational tools and can be applied with other tools and computational techniques. Parallel computation or analysis of dynamical systems with the aid of virtual reality to study the results by 3D visualization, could be exploited jointly to the use of the algorithm.

In the future, emerging technologies such as quantum computing may become viable and open new frontiers for the application of this algorithm. In fact, (BERNSTEIN, 2009, p. 19) states that the interest in quantum computation is least related to the algorithms and programs, but rather mostly to the ability of processing of data in a parallel way.

Thus, the new technologies, whether they are conventional or emerging, point to a increasingly lower computational cost to perform automated numerical simulations and the respective processing of data.

The algorithm of Section 3.1 stands out with the main result of the Thesis, however, the computational structure presented in Section 3.3 represents a very important secondary accessory contribution, since it made it possible to perform optimizations in an automated manner that, for now, corroborates the hypothesis.

Usually, the academic study of dynamical systems encompasses the specific coding to solve each problem that is presented. The framework developed in this Thesis has the functionality to treat OCPs in a generic way and with very little coding work.

The tools used by the framework have free or academic licenses and the user has full access to all features of the programs. The framework and tools used are updated to the most recent versions released by the developers.

Any researcher who install the tools following their tutorials, can achieve the complete reproduction of all results of the simulations and optimizations by the automated execution of

the configuration codes specifically developed for this Thesis.

In this Thesis, it is observed that in every simulation process, classifications of solutions and even of hypothesis verification, there was a high degree of automation and parameterization.

It was considered an error threshold allowed for the NRMSE for a candidate solution over the ideal solution.

In contexts without constant controls restrictions ($PC_1 = I$), this threshold was 1%, in intermediate-constrained contexts ($PC_1 = II$), this threshold was 2%, and in other contexts ($PC_1 = III$), this threshold was 3%.

Parametric variations which, at the stage of OCP modeling and optimizations are the origin of the diversity of the numerical solutions found, at the stage of classifications and verification of the hypothesis can lead to biases that may interfere in the results.

Thus, it is important to have criteria to define them, especially the analysis parameters.

It is also noted that the strictly numerical automated evaluation can differ from a qualitative individual assessment. Individual qualitative assessments of the 972 simulations were not performed.

The relevance of this Thesis is in the implementation of the proposed algorithm to OCPs for which there are no known exact and analytical solutions.

For these situations, the algorithm, with or without the aid of the ASAF structure, might provide a viable approach for obtaining acceptable candidate options to be recommended as a solution for the OCP², saving considerable time and effort of the user by making efficient use of automation features.

Therefore, the algorithm is presented as an important decision assistance tool for choosing an acceptable solution among many numeric possibilities found for a particular OCP.

²It is important to note that the algorithm does not guarantee optimal solution. In the case of the simulations present candidate solutions, the proposed algorithm does provide a recommended solution that could be acceptable or not, according to the decision of the user. Eventually, if an acceptable solution is not obtained in a number of contexts, the user may define new contexts to search for new solutions.

The next chapter presents the conclusion of this doctoral Thesis, focusing on the importance of the obtained results.

— 6 —

CONCLUSION

This Thesis presented the results of the research on the method to treat nonconvex optimal control problems by automated numerical simulations.

After the study of the main references and theoretical concepts related to *Optimization* and *Optimal Control*, an algorithm to obtain numerical solutions recommended for nonconvex OCPs was proposed in Section 3.1. Furthermore, it was presented in Section 3.3, the *ASAF* structure, developed based on modern modeling, simulation and optimization tools, JModelica, (ANDERSSON, 2011; MAGNUSSON; ÅKESSON, 2012), IPOPT, (WÄCHTER; BIEGLER, 2006), and HSL routines, (Science and Technology Facilities Council, 2015), which was useful in the implementation of this algorithm.

The verification of this Thesis hypothesis was performed through the study of four different nonconvex OCPs, in a total of 972 automated simulations. The resulting solutions were classified, verified and the application effectiveness was assessed, also, in an automated way.

Thus, the algorithm was tested in a methodological manner using classical statistical methods that were able to estimate the median of its application effectiveness of 93%, with significance of 0.05 (confidence of 95%).

Future works may explore the contexts that have already been defined in this research, performing additional simulations as an attempt to refute the hypothesis and, consequently, improve the proposed algorithm, or even develop new contexts for new OCPs of interest.

The research has impact when considering the range of OCPs for which it presents a viable approach to achieve solutions.

The applications of optimal control theory in real physical problems, regardless if they are related to engineering, or biological systems, most frequently deal with the characteristics of non-compliance such as nonconvexity and nonlinearity, than with other problems that are already treated by the conventional theory.

Therefore, with the results of this Thesis, there is an expansion of the possibilities for solving nonconvex OCP, which means an important and an original contribution to the subject.

Other strong and interesting possibilities are related to the development of simulation, optimization and modeling tools, as well as computational techniques. Eventually, the application of the proposed method with the help of emerging technologies, when viable, may result in greater ease of the algorithm in obtaining acceptable recommended solutions.

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APPENDIX A

SUMMARY OF THE *DYNAMIC OPTIMIZATION BY DIRECT COLLOCATION*

This Appendix summarizes the *dynamic optimization by direct collocation method* demonstrated by (MAGNUSSON; ÅKESSON, 2012), in an integrated environment for modeling, simulation and optimization of models coded in Modelica language.

According to (MAGNUSSON; ÅKESSON, 2012), *optimal control* and *parameter estimation* are the direct applications of the referenced method.

The application as *optimal control* aims to achieve the trajectories of the control variables, normally denoted by the vector $\mathbf{u} \in \mathbb{R}^r$, which minimize certain resources to perform a specific action.

The goal of the application as *parameter estimation* is to obtain the values of the unknown parameters of the system model that enable it to behave according to a set of previously provided data.

Optimal control problems, usually, are solved by direct application of Pontryagin Maximum (or Minimum) Principle or by dynamic programming based on the Bellman Optimality Principle. As stated in (MAGNUSSON; ÅKESSON, 2012), these approaches present troubles with large-scale OCPs, specially with inequality constraints.

The direct methods¹ are based on the technique that transform an infinite OCP, continuous in time, in a finite Nonlinear Program (NLP), discretized in time, with an approximate solution.

The main reference, (MAGNUSSON; ÅKESSON, 2012), points *multiple shooting* and *direct collocation* as possible approaches in these new techniques.

The mathematic formulation provided in (MAGNUSSON; ÅKESSON, 2012) for the *direct collocation method* starts with the dynamic system object of the optimization process, that

¹In consonance with (MAGNUSSON; ÅKESSON, 2012), the non-direct methods approach the OCP by variational calculus.

is defined by:

$$F(t, \dot{\mathbf{x}}(t), \mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t), \mathbf{p}) = 0, \quad (21)$$

where $t \in \mathbb{R}$, time, is the unique independent variable, $\mathbf{x} \in \mathbb{R}^{n_x}$ is the state vector, $\mathbf{u} \in \mathbb{R}^{n_u}$ is the control vector to be optimized, in *optimal control*.

There are, also, $\mathbf{w} \in \mathbb{R}^{n_w}$, which is the algebraic variable vector and $\mathbf{p} \in \mathbb{R}^{n_p}$, which is the parameters vector.

The initial conditions are defined by:

$$F_0(\dot{\mathbf{x}}(t_0), \mathbf{x}(t_0), \mathbf{u}(t_0), \mathbf{w}(t_0), \mathbf{p}) = 0, \quad (22)$$

where t_0 is the *starting time*.

All time-dependent variables were grouped into a single variable $\mathbf{z}$:

$$\mathbf{z} \equiv (\dot{\mathbf{x}}, \mathbf{x}, \mathbf{u}, \mathbf{w}). \quad (23)$$

The system dynamics is summarized, according to (MAGNUSSON; ÅKESSON, 2012), by:

$$F(t, \mathbf{z}(t), \mathbf{p}) = 0, \quad \forall t \in [t_0, t_f], \quad (24)$$

$$F_0(\mathbf{z}(t_0), \mathbf{p}) = 0, \quad (25)$$

where t_f is the *final time*, and

$$F: \mathbb{R}^{n_z} \times \mathbb{R}^{n_p} \rightarrow \mathbb{R}^{n_x + n_w}, \quad (26)$$

$$F_0: \mathbb{R}^{n_z} \times \mathbb{R}^{n_p} \rightarrow \mathbb{R}^{n_x}, \quad (27)$$

$$n_z = 2n_x + n_u + n_w. \quad (28)$$

Therefore, the general OCP object of the *direct collocation method* is presented by (MAGNUSSON; ÅKESSON, 2012) as:

$$\text{Minimize:} \quad f(t_0, t_f, \mathbf{z}, \mathbf{p}), \quad (29a)$$

$$\text{with respect to:} \quad t_0, t_f, \mathbf{z}, \mathbf{p},$$

$$\text{subject to:} \quad F(t, \mathbf{z}(t), \mathbf{p}) = \mathbf{0}, \quad (29b)$$

$$F_0(\mathbf{z}(t_0), \mathbf{p}) = \mathbf{0}, \quad (29c)$$

$$\mathbf{z}_L \leq \mathbf{z}(t) \leq \mathbf{z}_U, \quad (29d)$$

$$\mathbf{p}_L \leq \mathbf{p} \leq \mathbf{p}_U, \quad (29e)$$

$$\mathbf{g}_e(t_0, t_f, t, \mathbf{z}(t), \mathbf{p}) = \mathbf{0}, \quad (29f)$$

$$\mathbf{g}_i(t_0, t_f, t, \mathbf{z}(t), \mathbf{p}) \leq \mathbf{0}, \quad (29g)$$

$$\mathbf{G}_e(t_0, t_f, \mathbf{Z}_e, \mathbf{p}) = \mathbf{0}, \quad (29h)$$

$$\mathbf{G}_i(t_0, t_f, \mathbf{Z}_i, \mathbf{p}) \leq \mathbf{0}, \quad (29i)$$

$$\forall t \in [t_0, t_f].$$

where (29a), in optimal control, may be described, as in (MAGNUSSON; ÅKESSON, 2012), as:

$$f(t_0, t_f, \mathbf{z}, \mathbf{p}) = \Phi(t_0, \mathbf{z}(t_0), t_f, \mathbf{z}(t_f), \mathbf{p}) + \int_{t_0}^{t_f} L(t, \mathbf{z}(t), \mathbf{p}) dt, \quad (30)$$

with $\Phi : \mathbb{R} \times \mathbb{R}^{n_z} \times \mathbb{R} \times \mathbb{R}^{n_z} \times \mathbb{R}^{n_p} \rightarrow \mathbb{R}$ known as Mayer term, and $L : \mathbb{R} \times \mathbb{R}^{n_z} \times \mathbb{R}^{n_p} \rightarrow \mathbb{R}$ known as Lagrange integrand or Lagrangian.

This formulation may be analyzed in relation to the contributions of each equation.

Thus, (29b) and (29c) are related to the system dynamics and the initial conditions.

The variables constraints are implemented by (29d) e (29e).

Remark A.1. The symbolologies $[\bullet]_L$ and $[\bullet]_U$ in (29d) to (29e) stand for lower and upper limits, respectively.

Remark A.2. The symbolologies $[\bullet]_e$ and $[\bullet]_i$ in (29f) to (29i) stand for equality and inequality constraints, respectively.

The *equality constraints* are defined by (29f) and the *inequality constraints* are defined by (29g).

At last, the *point equality constraints* are defined by (29h) and the *point inequality constraints* are defined by (29i). Both are valid for a specific instant in time.

The vectors $\mathbf{Z}_e$ and $\mathbf{Z}_i$ are the values of the variables used with these constraints:

$$\mathbf{Z}_e = (\mathbf{Z}(T_1), \mathbf{Z}(T_2), \dots, \mathbf{Z}(T_m)), \quad (31)$$

where T_i is the instant in time when the constraint is applied.

Another important condition reported by (MAGNUSSON; ÅKESSON, 2012) is that $\mathbf{g}_e, \mathbf{g}_i, \mathbf{G}_e$ e $\mathbf{G}_i$ must be two times continuously differentiable.

A.1 COLLOCATION POLYNOMIALS

Here, a short foundation on *Collocation Polynomials*, based in (MAGNUSSON; ÅKESSON, 2012), is presented.

The *direct collocation method* presented by (MAGNUSSON; ÅKESSON, 2012) approaches the OCP described in (29a) to (29i) with a discretization in the time horizon in n_e elements. Inside each time element i , the time-dependent variable $\mathbf{z}(t)$ is approximated by interpolation of the exact values $\mathbf{z}_i = (\dot{\mathbf{x}}(t), \mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t), \mathbf{p})$ by the called *collocation polynomials*.

Within time element i , time is normalized as follows:

$$\tilde{t}_i(\tau) = t_{i-1} + h_i \cdot (t_f - t_0) \cdot \tau, \quad \tau \in [0, 1], \quad \forall i \in [1, \dots, n_e], \quad (32)$$

where t_i is the instant at end of time element i and h_i is the duration of element i .

Since all elements have the same duration:

$$h_i = \frac{t_f - t_0}{n_e}. \quad (33)$$

The *collocation polynomials* are implemented from the choice of n_c collocation points, that are the same inside each time element. The notation $\tau_{i,k}$ is used to represent the collocation point “ k ” inside the time element i . The notation $\mathbf{z}_{i,k}$ is the value of $\mathbf{z}(\tau_{i,k})$. It is necessary to have an additional interpolation point at the start of each time element, that is denoted by $\tau_{i,0} \equiv 0$.

Therefore, (MAGNUSSON; ÅKESSON, 2012) defines the *collocation polynomials* as:

$$\mathbf{x}_i(\tau) = \sum_{k=0}^{n_c} \mathbf{x}_{i,k} \tilde{\ell}_k(\tau), \quad (34a)$$

$$\mathbf{u}_i(\tau) = \sum_{k=1}^{n_c} \mathbf{u}_{i,k} \ell_k(\tau), \quad (34b)$$

$$\mathbf{w}_i(\tau) = \sum_{k=1}^{n_c} \mathbf{w}_{i,k} \ell_k(\tau), \quad (34c)$$

where $\tilde{\ell}_k$ and ℓ_k are the Lagrange basis polynomials, respectively, with and without interpolation at point $\tau_{i,0}$:

$$\tilde{\ell}_k(\tau) = \prod_{l \in [0, \dots, n_c] \setminus \{k\}} \frac{\tau - \tau_l}{\tau_k - \tau_l}, \quad \forall k \in [0, \dots, n_c], \quad (35a)$$

$$\ell_k(\tau) = \prod_{l \in [1, \dots, n_c] \setminus \{k\}} \frac{\tau - \tau_l}{\tau_k - \tau_l}, \quad \forall k \in [1, \dots, n_c], \quad (35b)$$

A.2 THE TRANSFORMATION OF THE OCP

In this Section, based in (MAGNUSSON; ÅKESSON, 2012), a brief explanation on *The Transformation of the OCP* is presented.

The solution suggested by (MAGNUSSON; ÅKESSON, 2012) to solve the infinite-dimensional OCP defined by (29a) to (29i) is to transform it into a finite Nonlinear Program (NLP) with the

aid of previously presented polynomials. The main idea is that the time-continuous variable z may be represented by a finite number of values: The values at the collocation points².

A new variable Z for the NLP is defined by (MAGNUSSON; ÅKESSON, 2012), comprised by the collocation points $z_{i,k}$, the state values at start of each time element i , $x_{i,0}$, and the parameters vector p . Furthermore, the initial conditions of $z_{1,0}$, the start time, t_0 , and the final time t_f were also considered.

$$\begin{aligned}
 Z = & (z_{1,0}, z_{1,1}, z_{1,2}, \dots, z_{1,n_c}, \\
 & (x_{2,0}, z_{2,1}, z_{2,2}, \dots, z_{2,n_c}, \\
 & (x_{3,0}, z_{3,1}, z_{3,2}, \dots, z_{3,n_c}, \\
 & \vdots \quad \dots \\
 & (x_{n_e,0}, z_{n_e,1}, z_{n_e,2}, \dots, z_{n_e,n_c}, p, t_0, t_f).
 \end{aligned} \tag{36}$$

As result of this transformation, the NLP is described as:

$$\text{Minimize:} \quad \tilde{f}(Z), \tag{37a}$$

$$\text{with respect to:} \quad Z \in \mathbb{R}^{n_z},$$

$$\text{subject to:} \quad F(t_{i,k}, z_{i,k}, p) = 0, \tag{37b}$$

$$F_0(z_{1,0}, p) = 0, \tag{37c}$$

$$u_{1,0} - \sum_{k=1}^{n_c} u_{1,k} \cdot \ell_k(0) = 0, \tag{37d}$$

$$z_L \leq z_{i,k} \leq z_U, \tag{37e}$$

$$p_L \leq p \leq p_U, \tag{37f}$$

$$g_e(t_{i,k}, z_{i,k}, p) = 0, \tag{37g}$$

$$g_i(t_{i,k}, z_{i,k}, p) \leq 0, \tag{37h}$$

$$G_e(Z_e) = 0, \tag{37i}$$

$$G_i(Z_i) \leq 0, \tag{37j}$$

²The type of the *collocation* is related to how the collocation points are spread inside each time element i . (MAGNUSSON; ÅKESSON, 2012) has used the *Radau* collocation. More on discretization of OCP can be found in (HAGER, 2000).

$$\begin{aligned}
& \forall (i, k) \in \{(1, 0)\} \cup ([1..n_e] \times [1..n_c]), \\
& \dot{\mathbf{x}}_{j,l} = \frac{1}{h_j(t_f - t_0)} \cdot \sum_{m=0}^{n_c} \mathbf{x}_{j,m} \cdot \frac{d\tilde{\ell}_m}{d\tau}(\tau_l), \\
& \forall (j, l) \in [1..n_e] \times [1..n_c], \\
& \mathbf{x}_{n,n_c} = \mathbf{x}_{n+1,0}, \quad \forall n \in [1..n_e - 1].
\end{aligned} \tag{37k}$$

where

$$n_Z = (1 + n_e \cdot n_c) \cdot n_z + (n_e - 1) \cdot n_x + n_p + 2 \tag{38}$$

is the number of variables of the NLP and $t_{i,k} \equiv \tilde{t}_i(\tau_k)$ stands for the “ k ” collocation point inside time element i .

Obviously, the objective and Lagrangian functions are similarly transformed.

The main conclusion to point out is that (MAGNUSSON; ÅKESSON, 2012) shows that the solution of the NLP defined in (37a) to (37k) results in an approximated solution of the original OCP defined by (29a) to (29i).

APPENDIX B

STATISTICAL ANALYSIS OF THE SIMULATIONS

This analysis was performed by using own code in Python language, which assessed in an automated manner the classifications of the solutions by the proposed algorithm.

```

1  Análise Estatística dos Resultados das Simulações com JModelica.org - Versão 2.2
2  Initial Definitions
3  Null Hypothesis (H0):
4  *****
5  The algorithm, when applied in a particular context in which the OCP under study is consistently
6  defined with respect to the theory and to the restrictions of the used environment for numeric
7  simulations and optimizations, is capable of regularly present median of effectiveness of correction
8  of solutions classifications equal to or greater than 70.0%.
9  *****
10 Statistical Method Used: Sign Test. [p. 321] (Triola, 1999)
11 Type of Method: non-parametric (does not require Normal Distribution).
12 Application: Information about the median of a property of one single population.
13 Reference:
14 Mário F. Triola, Introdução à Estatística, Sétima Edição, Editora LTC, Rio de Janeiro, 1999.
15 ISBN: 85-216-1154-4
16
17 Results for Test#1 (OCP#1 - Context A):
18 x_NRMSE_max=1.000e+00%
19 u_NRMSE_max=1.000e+00%
20 Sim#| return_status | objective | JOK | con | u_Energy | u_RMS | x_SAT_IND(%) | u_SAT_IND(%) | x_NRMSE(%) | u_NRMSE(%) | x_IR | x_IC | u_IR | u_IC | IR | IC | solution |
21 A-1|Solve_Succeeded| -3.13e-23 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 1.98e-01 | 2.04e-01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
22 A-2|Solve_Succeeded| -4.06e-20 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 3.97e-02 | 4.09e-02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
23 A-3|Solve_Succeeded| -8.50e-23 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 1.98e-03 | 8.18e-03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
24 A-4|Solve_Succeeded| 4.47e-08 | 1 | 0 | 4.10e+01 | 6.40e+00 | 0.00e+00 | 0.00e+00 | 1.50e+00 | 2.83e+00 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | Rejected |
25 A-5|Solve_Succeeded| 6.04e-17 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 3.97e-02 | 4.09e-02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
26 A-6|Solve_Succeeded| -1.31e-24 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 7.94e-03 | 8.18e-03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
27 A-7|Solve_Succeeded| 1.02e-09 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 1.98e-01 | 6.06e-01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
28 A-8|Solve_Succeeded| 2.05e-08 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 3.93e-02 | 4.85e+00 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | Candidate |
29 A-9|Solve_Succeeded| 6.05e-08 | 1 | 1 | 4.13e+01 | 6.42e+00 | 0.00e+00 | 0.00e+00 | 7.76e-03 | 9.42e+00 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | Candidate |
30 A-10|Solve_Succeeded| 2.33e-08 | 1 | 0 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 1.99e-01 | 2.69e+00 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | Rejected |
31 A-11|Solve_Succeeded| 8.37e-03 | 0 | 0 | 3.53e+01 | 5.94e+00 | 0.00e+00 | 0.00e+00 | 9.08e-01 | 6.64e+00 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | Rejected |
32 A-12|Solve_Succeeded| -3.93e-23 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 7.94e-03 | 8.18e-03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
33 A-13|Solve_Succeeded| -2.70e-18 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 7.84e-01 | 6.40e+00 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | Rejected |
34 A-14|Solve_Succeeded| 6.25e-03 | 0 | 0 | 3.61e+01 | 6.01e+00 | 0.00e+00 | 0.00e+00 | 7.94e-03 | 8.18e-03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
35 A-15|Solve_Succeeded| -9.21e-25 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 7.94e-03 | 8.18e-03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
36 A-16|Solve_Succeeded| 1.02e-09 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 1.98e-01 | 6.06e-01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
37 A-17|Solve_Succeeded| 2.05e-08 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 3.93e-02 | 4.85e+00 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | Candidate |
38 A-18|Solve_Succeeded| 6.05e-08 | 1 | 1 | 4.13e+01 | 6.42e+00 | 0.00e+00 | 0.00e+00 | 7.76e-03 | 9.42e+00 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | Candidate |
39 A-19|Solve_Succeeded| -8.72e-22 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 1.98e-01 | 2.04e-01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
40 A-20|Solve_Succeeded| 4.94e-03 | 0 | 0 | 3.69e+01 | 6.09e+00 | 0.00e+00 | 0.00e+00 | 7.94e-01 | 6.28e+00 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | Rejected |
41 A-21|Solve_Succeeded| -1.26e-34 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 7.94e-03 | 8.18e-03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
42 A-22|Solve_Succeeded| -1.56e-26 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 1.98e-01 | 2.04e-01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
43 A-23|Solve_Succeeded| 5.58e-03 | 0 | 0 | 3.69e+01 | 6.09e+00 | 0.00e+00 | 0.00e+00 | 7.45e-01 | 6.41e+00 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | Rejected |
44 A-24|Solve_Succeeded| -9.87e-40 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 7.94e-03 | 8.18e-03 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
45 A-25|Solve_Succeeded| 1.02e-09 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 1.98e-01 | 6.06e-01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Candidate |
46 A-26|Solve_Succeeded| 2.05e-08 | 1 | 1 | 4.13e+01 | 6.43e+00 | 0.00e+00 | 0.00e+00 | 3.93e-02 | 4.85e+00 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | Candidate |
47 A-27|Solve_Succeeded| 6.05e-08 | 1 | 1 | 4.13e+01 | 6.42e+00 | 0.00e+00 | 0.00e+00 | 7.76e-03 | 9.42e+00 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | Candidate |
48 Results for Test#1 (OCP#1 - Context A):
49 Solutions: 27
50 Candidates: 21
51 Rejected: 6
52 Solutions Improperly Accepted as Candidate (IC): 6
53 Solutions Improperly Rejected (IR): 0
54 Effective Classifications: 21
55 Wrong Classifications: 6
56 Effectiveness in Context= 78.00%
57
58 Results for Test#2 (OCP#1 - Context B):
59 x_NRMSE_max=2.000e+00%

```

60	u_NRMSE_max=2.000e+00%															
61	Sim#	return_status	objective	JOK	con	u_Energy	u_RMS	x_SAT_IND(%)	u_SAT_IND(%)	x_NRMSE(%)	u_NRMSE(%)	x_IR	x_IC	u_IR	u_IC	IR_IC
62	B-1	Solve_Succeeded	4.64e-03	0	0	4.00e+01	6.32e+00	0.00e+00	5.00e+00	7.38e-01	6.07e+00	1	0	0	0	0
63	B-2	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	6.51e-02	1.49e+00	0	0	0	0	0
64	B-3	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	5.19e-02	1.49e+00	0	0	0	0	0
65	B-4	Solve_Succeeded	1.02e+00	0	0	3.38e+01	5.82e+00	0.00e+00	5.50e+01	1.02e+01	2.23e+01	0	0	0	0	0
66	B-5	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	6.51e-02	1.49e+00	0	0	0	0	0
67	B-6	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	5.19e-02	1.49e+00	0	0	0	0	0
68	B-7	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	2.08e-01	1.49e+00	0	0	0	0	0
69	B-8	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	6.51e-02	1.49e+00	0	0	0	0	0
70	B-9	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	5.19e-02	1.49e+00	0	0	0	0	0
71	B-10	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	2.08e-01	1.49e+00	0	0	0	0	0
72	B-11	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	6.51e-02	1.49e+00	0	0	0	0	0
73	B-12	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	5.19e-02	1.49e+00	0	0	0	0	0
74	B-13	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	2.08e-01	1.49e+00	0	0	0	0	0
75	B-14	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	6.51e-02	1.49e+00	0	0	0	0	0
76	B-15	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	5.19e-02	1.49e+00	0	0	0	0	0
77	B-16	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	2.08e-01	1.49e+00	0	0	0	0	0
78	B-17	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	6.51e-02	1.49e+00	0	0	0	0	0
79	B-18	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	5.19e-02	1.49e+00	0	0	0	0	0
80	B-19	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	2.08e-01	1.49e+00	0	0	0	0	0
81	B-20	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	6.51e-02	1.49e+00	0	0	0	0	0
82	B-21	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	5.19e-02	1.49e+00	0	0	0	0	0
83	B-22	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	2.08e-01	1.49e+00	0	0	0	0	0
84	B-23	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	6.51e-02	1.49e+00	0	0	0	0	0
85	B-24	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	5.19e-02	1.49e+00	0	0	0	0	0
86	B-25	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	2.08e-01	1.49e+00	0	0	0	0	0
87	B-26	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	6.51e-02	1.49e+00	0	0	0	0	0
88	B-27	Solve_Succeeded	2.64e-05	1	1	4.13e+01	6.42e+00	0.00e+00	0.00e+00	5.19e-02	1.49e+00	0	0	0	0	0
89	Results for Test#2 (OC#1 - Context B):															
90	Solutions: 27															
91	Candidates: 25															
92	Rejected: 2															
93	Solutions Improperly Accepted as Candidate (IC): 0															
94	Solutions Improperly Rejected (IR): 0															
95	Effective Classifications: 27															
96	Wrong Classifications: 0															
97	Effectiveness in Context= 100.00%															
98																
99	Results for Test#3 (OC#1 - Context C):															
100	u_NRMSE_max=3.000e+00%															
101	Sim#	return_status	objective	JOK	con	u_Energy	u_RMS	x_SAT_IND(%)	u_SAT_IND(%)	x_NRMSE(%)	u_NRMSE(%)	x_IR	x_IC	u_IR	u_IC	IR_IC
102	C-1	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.91e-01	2.98e+00	0	0	0	0	0
103	C-2	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.09e-01	2.98e+00	0	0	0	0	0
104	C-3	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.06e-01	2.98e+00	0	0	0	0	0
105	C-4	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.91e-01	2.98e+00	0	0	0	0	0
106	C-5	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.09e-01	2.98e+00	0	0	0	0	0
107	C-6	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.06e-01	2.98e+00	0	0	0	0	0
108	C-7	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.91e-01	2.98e+00	0	0	0	0	0
109	C-8	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.09e-01	2.98e+00	0	0	0	0	0
110	C-9	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.06e-01	2.98e+00	0	0	0	0	0
111	C-10	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.91e-01	2.98e+00	0	0	0	0	0
112	C-11	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.09e-01	2.98e+00	0	0	0	0	0
113	C-12	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.06e-01	2.98e+00	0	0	0	0	0
114	C-13	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.91e-01	2.98e+00	0	0	0	0	0
115	C-14	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.09e-01	2.98e+00	0	0	0	0	0
116	C-15	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.06e-01	2.98e+00	0	0	0	0	0
117	C-16	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.91e-01	2.98e+00	0	0	0	0	0
118	C-17	Solve_Succeeded	4.23e-04	1	1	4.11e+01	6.41e+00	0.00e+00	0.00e+00	2.09e-01	2.98e+00	0	0	0	0	0

178	Wrong Classifications: 9
179	Effectiveness in Context= 67.00%
180	
181	Results for Test#5 (OCP#1 - Context K):
182	x_NRMSE_max=2.000e+00%
183	u_NRMSE_max=2.000e+00%
184	Sim# return_status objective JOK con u_Energy u_RMS x_SAT_IND(%) u_SAT_IND(%) x_NRMSE(%) u_NRMSE(%) x_IR x_IC u_IR u_IC IR IC solution
185	K-1 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.00e-01 1.67e+00 0 0 0 0 0 0 Candidate
186	K-2 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 4.67e-02 1.66e+00 0 0 0 0 0 0 Candidate
187	K-3 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.52e-02 1.66e+00 0 0 0 0 0 0 Candidate
188	K-4 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.00e-01 1.67e+00 0 0 0 0 0 0 Candidate
189	K-5 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 4.67e-02 1.66e+00 0 0 0 0 0 0 Candidate
190	K-6 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.52e-02 1.66e+00 0 0 0 0 0 0 Candidate
191	K-7 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.00e-01 1.67e+00 0 0 0 0 0 0 Candidate
192	K-8 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 4.67e-02 1.66e+00 0 0 0 0 0 0 Candidate
193	K-9 Solve_Succeeded 5.80e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.52e-02 1.66e+00 0 0 0 0 0 0 Candidate
194	K-10 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.00e-01 1.67e+00 0 0 0 0 0 0 Candidate
195	K-11 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 4.67e-02 1.66e+00 0 0 0 0 0 0 Candidate
196	K-12 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.52e-02 1.66e+00 0 0 0 0 0 0 Candidate
197	K-13 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.00e-01 1.67e+00 0 0 0 0 0 0 Candidate
198	K-14 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 4.67e-02 1.66e+00 0 0 0 0 0 0 Candidate
199	K-15 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.52e-02 1.66e+00 0 0 0 0 0 0 Candidate
200	K-16 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.00e-01 1.67e+00 0 0 0 0 0 0 Candidate
201	K-17 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 4.67e-02 1.66e+00 0 0 0 0 0 0 Candidate
202	K-18 Solve_Succeeded 5.80e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.52e-02 1.66e+00 0 0 0 0 0 0 Candidate
203	K-19 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.00e-01 1.67e+00 0 0 0 0 0 0 Candidate
204	K-20 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 4.67e-02 1.66e+00 0 0 0 0 0 0 Candidate
205	K-21 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.52e-02 1.66e+00 0 0 0 0 0 0 Candidate
206	K-22 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.00e-01 1.67e+00 0 0 0 0 0 0 Candidate
207	K-23 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 4.67e-02 1.66e+00 0 0 0 0 0 0 Candidate
208	K-24 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.52e-02 1.66e+00 0 0 0 0 0 0 Candidate
209	K-25 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.00e-01 1.67e+00 0 0 0 0 0 0 Candidate
210	K-26 Solve_Succeeded 5.66e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 4.67e-02 1.66e+00 0 0 0 0 0 0 Candidate
211	K-27 Solve_Succeeded 5.80e-06 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 2.52e-02 1.66e+00 0 0 0 0 0 0 Candidate
212	Results for Test#5 (OCP#1 - Context K):
213	Solutions: 27
214	Candidates: 27
215	Rejected: 0
216	Solutions Improperly Accepted as Candidate (IC): 0
217	Solutions Improperly Rejected (IR): 0
218	Effective Classifications: 27
219	Wrong Classifications: 0
220	Effectiveness in Context= 100.00%
221	
222	Results for Test#6 (OCP#1 - Context L):
223	x_NRMSE_max=3.000e+00%
224	u_NRMSE_max=3.000e+00%
225	Sim# return_status objective JOK con u_Energy u_RMS x_SAT_IND(%) u_SAT_IND(%) x_NRMSE(%) u_NRMSE(%) x_IR x_IC u_IR u_IC IR IC solution
226	L-1 Solve_Succeeded 1.10e-04 1 1 2.03e+00 1.42e+00 0.00e+00 0.00e+00 2.29e-01 3.38e+00 0 0 0 0 0 0 Candidate
227	L-2 Solve_Succeeded 1.10e-04 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 1.13e-01 3.36e+00 0 0 0 0 0 0 Candidate
228	L-3 Solve_Succeeded 1.10e-04 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 1.05e-01 3.36e+00 0 0 0 0 0 0 Candidate
229	L-4 Solve_Succeeded 1.10e-04 1 1 2.03e+00 1.42e+00 0.00e+00 0.00e+00 2.29e-01 3.38e+00 0 0 0 0 0 0 Candidate
230	L-5 Solve_Succeeded 1.10e-04 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 1.13e-01 3.36e+00 0 0 0 0 0 0 Candidate
231	L-6 Solve_Succeeded 1.10e-04 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 1.05e-01 3.36e+00 0 0 0 0 0 0 Candidate
232	L-7 Solve_Succeeded 1.10e-04 1 1 2.03e+00 1.42e+00 0.00e+00 0.00e+00 2.29e-01 3.38e+00 0 0 0 0 0 0 Candidate
233	L-8 Solve_Succeeded 1.10e-04 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 1.13e-01 3.36e+00 0 0 0 0 0 0 Candidate
234	L-9 Solve_Succeeded 1.10e-04 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 1.05e-01 3.36e+00 0 0 0 0 0 0 Candidate
235	L-10 Solve_Succeeded 1.10e-04 1 1 2.03e+00 1.42e+00 0.00e+00 0.00e+00 2.29e-01 3.38e+00 0 0 0 0 0 0 Candidate
236	L-11 Solve_Succeeded 1.10e-04 1 1 2.02e+00 1.42e+00 0.00e+00 0.00e+00 1.13e-01 3.36e+00 0 0 0 0 0 0 Candidate

473	u_NRMSE_max=3.000e+00%																		
474	Sim#	return_status	objective	JOK	con	u_Energy	u_RMS	x_SAT_IND(%)	u_SAT_IND(%)	x_NRMSE(%)	u_NRMSE(%)	x_IR	x_IC	u_IR	u_IC	IR	IC	solution	
475	C-1	Solved_To_Accept	2.00e+04	0	1	1.06e-35	1.03e-18	0.00e+00	0.00e+00	4.45e+01	3.95e+01	0	0	0	0	0	0	Null Sol	
476	C-2	Solve_Succeeded	8.77e-02	1	1	4.41e+00	6.64e-01	0.00e+00	0.00e+00	1.05e-01	6.37e+00	0	0	0	1	0	0	1Candidate	
477	C-3	Maximum_Iterati	-1.76e+05	-1	-1	0.00e+00	0.00e+00	0.00e+00	0.00e+00	-1.00e+00	-1.00e+00	0	0	1	0	0	0	Not Solv	
478	C-4	Solve_Succeeded	8.77e-02	1	1	4.41e+00	6.64e-01	0.00e+00	0.00e+00	2.49e-01	6.65e+00	0	0	0	1	0	1	1Candidate	
479	C-5	Solve_Succeeded	4.90e-02	1	1	3.80e+00	6.16e-01	0.00e+00	0.00e+00	8.39e-02	5.33e+00	0	0	0	1	0	1	1Candidate	
480	C-6	Solve_Succeeded	1.04e+00	0	0	8.68e+00	9.32e-01	0.00e+00	1.00e+01	3.23e-01	8.64e+00	1	0	0	0	0	0	Rejected	
481	C-7	Solve_Succeeded	2.00e+04	0	1	3.31e-32	5.75e-17	0.00e+00	0.00e+00	4.45e+01	3.95e+01	0	0	0	0	0	0	Null Sol	
482	C-8	Maximum_Iterati	-1.44e+08	-1	-1	0.00e+00	0.00e+00	0.00e+00	0.00e+00	-1.00e+00	-1.00e+00	0	0	1	0	0	0	Not Solv	
483	C-9	Maximum_Iterati	-1.18e+08	-1	-1	0.00e+00	0.00e+00	0.00e+00	0.00e+00	-1.00e+00	-1.00e+00	0	0	1	0	0	0	Not Solv	
484	C-10	Solve_Succeeded	2.00e+04	0	1	7.02e-33	2.65e-17	0.00e+00	0.00e+00	4.45e+01	3.95e+01	0	0	0	0	0	0	Null Sol	
485	C-11	Solve_Succeeded	8.77e-02	1	1	4.41e+00	6.64e-01	0.00e+00	0.00e+00	1.05e-01	6.37e+00	0	0	0	1	0	1	1Candidate	
486	C-12	Solve_Succeeded	8.77e-02	1	1	4.41e+00	6.64e-01	0.00e+00	0.00e+00	9.41e-02	6.32e+00	0	0	0	0	1	0	1Candidate	
487	C-13	Solved_To_Accept	2.00e+04	0	1	1.94e-27	1.39e-14	0.00e+00	0.00e+00	4.45e+01	3.95e+01	0	0	0	0	0	0	Null Sol	
488	C-14	Solve_Succeeded	4.90e-02	1	1	3.80e+00	6.16e-01	0.00e+00	0.00e+00	8.39e-02	5.33e+00	0	0	0	1	0	0	1Candidate	
489	C-15	Solve_Succeeded	1.04e+00	0	0	8.68e+00	9.32e-01	0.00e+00	1.00e+01	3.23e-01	8.64e+00	1	0	0	0	0	0	Rejected	
490	C-16	Solve_Succeeded	2.00e+04	0	1	5.23e-33	2.29e-17	0.00e+00	0.00e+00	4.45e+01	3.95e+01	0	0	0	0	0	0	Null Sol	
491	C-17	Solve_Succeeded	4.90e-02	1	1	3.80e+00	6.16e-01	0.00e+00	0.00e+00	8.39e-02	5.33e+00	0	0	0	1	0	0	1Candidate	
492	C-18	Maximum_Iterati	-4.24e+05	-1	-1	0.00e+00	0.00e+00	0.00e+00	0.00e+00	-1.00e+00	-1.00e+00	0	0	1	0	0	0	Not Solv	
493	C-19	Solve_Succeeded	2.00e+04	0	1	7.02e-33	2.65e-17	0.00e+00	0.00e+00	4.45e+01	3.95e+01	0	0	0	0	0	0	Null Sol	
494	C-20	Solve_Succeeded	8.77e-02	1	1	4.41e+00	6.64e-01	0.00e+00	0.00e+00	1.05e-01	6.37e+00	0	0	0	1	0	1	1Candidate	
495	C-21	Restoration_Fail	1.86e+04	-1	-1	0.00e+00	0.00e+00	0.00e+00	0.00e+00	-1.00e+00	-1.00e+00	0	0	1	0	0	0	Not Solv	
496	C-22	Solve_Succeeded	4.90e-02	1	1	3.80e+00	6.16e-01	0.00e+00	0.00e+00	2.41e-01	5.52e+00	0	0	0	1	0	1	1Candidate	
497	C-23	Solve_Succeeded	8.77e-02	1	1	4.41e+00	6.64e-01	0.00e+00	0.00e+00	1.05e-01	6.37e+00	0	0	0	0	1	0	1Candidate	
498	C-24	Solve_Succeeded	8.77e-02	1	1	4.41e+00	6.64e-01	0.00e+00	0.00e+00	9.41e-02	6.32e+00	0	0	0	1	0	1	1Candidate	
499	C-25	Solve_Succeeded	2.00e+04	0	1	8.68e-31	2.95e-16	0.00e+00	0.00e+00	4.45e+01	3.95e+01	0	0	0	0	0	0	Null Sol	
500	C-26	Solve_Succeeded	4.90e-02	1	1	3.80e+00	6.16e-01	0.00e+00	0.00e+00	8.39e-02	5.33e+00	0	0	0	1	0	1	1Candidate	
501	C-27	Solve_Succeeded	6.22e+02	0	0	2.40e+01	1.55e+00	0.00e+00	5.00e+01	7.89e+00	2.89e+01	0	0	0	0	0	0	Rejected	
502	Results for Test#12 (OCP#2 - Context C):																		
503	Solutions: 22																		
504	Candidates: 12																		
505	Rejected: 10																		
506	Solutions Improperly Accepted as Candidate (IC): 12																		
507	Solutions Improperly Rejected (IR): 0																		
508	Effective Classifications: 10																		
509	Wrong Classifications: 12																		
510	Effectiveness in Context= 45.00%																		
511																			
512	Results for Test#13 (OCP#2 - Context J):																		
513	x_NRMSE_max=1.000e+00%																		
514	u_NRMSE_max=1.000e+00%																		
515	Sim#	return_status	objective	JOK	con	u_Energy	u_RMS	x_SAT_IND(%)	u_SAT_IND(%)	x_NRMSE(%)	u_NRMSE(%)	x_IR	x_IC	u_IR	u_IC	IR	IC	solution	
516	J-1	Solve_Succeeded	-2.17e-18	1	1	4.95e+03	2.22e+01	0.00e+00	0.00e+00	2.28e-01	1.73e-01	0	0	0	0	0	0	1Candidate	
517	J-2	Solve_Succeeded	-1.41e-21	1	1	6.20e+03	2.49e+01	0.00e+00	0.00e+00	4.58e-02	3.68e-02	0	0	0	0	0	0	1Candidate	
518	J-3	Maximum_Iterati	1.69e+04	-1	-1	0.00e+00	0.00e+00	0.00e+00	0.00e+00	-1.00e+00	-1.00e+00	0	0	1	0	0	0	Not Solv	
519	J-4	Solve_Succeeded	-1.20e-21	1	1	3.72e+03	1.93e+01	0.00e+00	0.00e+00	2.28e-01	1.39e-01	0	0	0	0	0	0	1Candidate	
520	J-5	Solve_Succeeded	-1.25e-23	1	1	7.88e+03	2.81e+01	0.00e+00	0.00e+00	4.58e-02	4.30e-02	0	0	0	0	0	0	1Candidate	
521	J-6	Maximum_Iterati	1.87e+04	-1	-1	0.00e+00	0.00e+00	0.00e+00	0.00e+00	-1.00e+00	-1.00e+00	0	0	1	0	0	0	Not Solv	
522	J-7	Solve_Succeeded	1.87e-10	1	1	8.44e+03	2.91e+01	0.00e+00	0.00e+00	2.28e-01	1.76e-01	0	0	0	0	0	0	1Candidate	
523	J-8	Solve_Succeeded	2.15e-09	1	1	7.35e+02	8.58e+00	0.00e+00	0.00e+00	4.58e-02	3.20e-02	0	0	0	0	0	0	1Candidate	
524	J-9	Solve_Succeeded	8.98e-08	1	1	2.86e+03	1.69e+01	0.00e+00	0.00e+00	9.17e-03	1.94e-01	0	0	0	0	0	0	1Candidate	
525	J-10	Solve_Succeeded	5.30e-18	1	1	7.34e+03	2.71e+01	0.00e+00	0.00e+00	2.28e-01	2.06e-01	0	0	0	0	0	0	1Candidate	
526	J-11	Solve_Succeeded	-1.05e-21	1	1	7.11e+03	2.67e+01	0.00e+00	0.00e+00	4.58e-02	4.24e-02	0	0	0	0	0	0	1Candidate	
527	J-12	Restoration_Fail	1.91e+04	-1	-1	0.00e+00	0.00e+00	0.00e+00	0.00e+00	-1.00e+00	-1.00e+00	0	0	1	0	0	0	Not Solv	
528	J-13	Solve_Succeeded	-7.28e-24	1	1	8.25e+02	9.08e+00	0.00e+00	0.00e+00	2.28e-01	4.99e-02	0	0	0	0	0	0	1Candidate	
529	J-14	Solve_Succeeded	8.33e-20	1	1	7.66e+03	2.77e+01	0.00e+00	0.00e+00	4.58e-02	4.25e-02	0	0	0	0	0	0	1Candidate	
530	J-15	Solved_To_Accept	1.44e-15	1	1	1.37e+04	3.70e+01	0.00e+00	0.00e+00	9.17e-03	9.13e-03	0	0	0	0	0	0	1Candidate	
531	J-16	Solve_Succeeded	1.98e-10	1	1	3.75e+03	1.94e+01	0.00e+00	0.00e+00	2.28e-01	1.00e-01	0	0	0	0	0	0	1Candidate	

709	Candidates: 0
710	Rejected: 24
711	Solutions Improperly Accepted as Candidate (IC): 0
712	Solutions Improperly Rejected (IR): 4
713	Effective Classifications: 20
714	Wrong Classifications: 4
715	Effectiveness in Context= 83.00%
716	
717	Results for Test#18 (OCP#2 - Context U):
718	x_NRMSE_max=3.000e+00%
719	u_NRMSE_max=3.000e+00%
720	Sim# return_status objective JOK con u_Energy u_RMS x_SAT_IND(%) u_SAT_IND(%) x_NRMSE(%) u_NRMSE(%) x_IR x_IC u_IR u_IC IRI IC solution
721	U-1 Solve_Succeeded 6.71e+02 0 0 4.28e+03 2.07e+01 0.00e+00 0.00e+00 8.04e+00 8.19e+00 0 0 0 0 0 0 0 0 Rejected
722	U-2 Maximum_Iterati -2.67e+08 -1 -1 0.00e+00 0.00e+00 0.00e+00 0.00e+00 -1.00e+00 -1.00e+00 0 0 1 0 0 0 0 0 Not Solv
723	U-3 Solve_Succeeded 2.43e+01 0 1 1.54e+03 1.24e+01 0.00e+00 0.00e+00 1.56e+00 4.50e+00 1 0 0 0 0 0 0 0 Rejected
724	U-4 Solve_Succeeded 6.88e+02 0 0 5.21e+03 2.28e+01 0.00e+00 2.00e+01 8.14e+00 8.25e+00 0 0 0 0 0 0 0 0 Rejected
725	U-5 Solve_Succeeded 6.88e+02 0 0 5.50e+03 2.34e+01 0.00e+00 1.00e+01 8.27e+00 8.43e+00 0 0 0 0 0 0 0 0 Rejected
726	U-6 Solve_Succeeded 6.27e+00 0 1 2.06e+02 4.54e+00 0.00e+00 0.00e+00 7.92e-01 5.51e+00 1 0 0 0 0 0 0 0 Rejected
727	U-7 Solve_Succeeded 7.87e+02 0 0 1.15e+04 3.39e+01 0.00e+00 2.00e+01 8.78e+00 8.93e+00 0 0 0 0 0 0 0 0 Rejected
728	U-8 Solve_Succeeded 1.87e+02 0 0 7.80e+03 2.79e+01 0.00e+00 1.00e+01 4.33e+00 4.23e+00 0 0 0 0 0 0 0 0 Rejected
729	U-9 Solve_Succeeded 2.10e+00 0 1 5.24e+01 2.29e+00 0.00e+00 0.00e+00 4.58e-01 6.12e+00 1 0 0 0 0 0 0 0 Rejected
730	U-10 Solve_Succeeded 7.73e+02 0 0 9.71e+03 3.12e+01 0.00e+00 2.00e+01 8.69e+00 8.85e+00 0 0 0 0 0 0 0 0 Rejected
731	U-11 Solve_Succeeded 7.62e+02 0 0 8.68e+03 2.95e+01 0.00e+00 2.00e+01 8.71e+00 8.85e+00 0 0 0 0 0 0 0 0 Rejected
732	U-12 Solve_Succeeded 7.94e+02 0 0 1.45e+04 3.80e+01 0.00e+00 2.00e+01 8.91e+00 9.12e+00 0 0 0 0 0 0 0 0 Rejected
733	U-13 Solve_Succeeded 7.94e+02 0 0 1.43e+04 3.78e+01 0.00e+00 2.00e+01 8.81e+00 9.01e+00 0 0 0 0 0 0 0 0 Rejected
734	U-14 Solve_Succeeded 7.70e+02 0 0 9.54e+03 3.09e+01 0.00e+00 3.00e+01 8.76e+00 8.84e+00 0 0 0 0 0 0 0 0 Rejected
735	U-15 Solve_Succeeded 7.94e+02 0 0 1.45e+04 3.80e+01 0.00e+00 2.00e+01 8.91e+00 9.12e+00 0 0 0 0 0 0 0 0 Rejected
736	U-16 Solve_Succeeded 1.10e+01 0 1 4.47e+02 6.69e+00 0.00e+00 0.00e+00 1.06e+00 3.77e+00 1 0 0 0 0 0 0 0 Rejected
737	U-17 Solve_Succeeded 3.96e+01 0 1 3.32e+03 1.82e+01 0.00e+00 0.00e+00 2.00e+00 4.14e+00 1 0 0 0 0 0 0 0 Rejected
738	U-18 Solve_Succeeded 2.10e+00 0 1 5.24e+01 2.29e+00 0.00e+00 0.00e+00 4.58e-01 6.12e+00 1 0 0 0 0 0 0 0 Rejected
739	U-19 Solve_Succeeded 2.98e-01 0 1 1.14e+01 1.07e+00 0.00e+00 0.00e+00 2.90e-01 6.87e+00 1 0 0 0 0 0 0 0 Rejected
740	U-20 Restoration_Fai -5.22e+04 -1 -1 0.00e+00 0.00e+00 0.00e+00 0.00e+00 -1.00e+00 -1.00e+00 0 0 1 0 0 0 0 0 Not Solv
741	U-21 Solve_Succeeded 6.67e+02 0 0 4.15e+03 2.04e+01 0.00e+00 1.00e+01 8.16e+00 8.34e+00 0 0 0 0 0 0 0 0 Rejected
742	U-22 Solve_Succeeded 7.94e+02 0 0 1.43e+04 3.78e+01 0.00e+00 2.00e+01 8.81e+00 9.01e+00 0 0 0 0 0 0 0 0 Rejected
743	U-23 Solve_Succeeded 7.87e+02 0 0 1.15e+04 3.39e+01 0.00e+00 2.00e+01 8.85e+00 9.01e+00 0 0 0 0 0 0 0 0 Rejected
744	U-24 Solve_Succeeded 6.78e+02 0 0 4.71e+03 2.17e+01 0.00e+00 1.00e+01 8.23e+00 8.41e+00 0 0 0 0 0 0 0 0 Rejected
745	U-25 Restoration_Fai -2.05e+08 -1 -1 0.00e+00 0.00e+00 0.00e+00 0.00e+00 -1.00e+00 -1.00e+00 0 0 1 0 0 0 0 0 Not Solv
746	U-26 Solve_Succeeded 4.52e+01 0 1 3.92e+03 1.98e+01 0.00e+00 0.00e+00 2.13e+00 3.03e+00 1 0 0 0 0 0 0 0 Rejected
747	U-27 Solve_Succeeded 2.10e+00 0 1 5.24e+01 2.29e+00 0.00e+00 0.00e+00 4.58e-01 6.12e+00 1 0 0 0 0 0 0 0 Rejected
748	Results for Test#18 (OCP#2 - Context U):
749	Solutions: 24
750	Candidates: 0
751	Rejected: 24
752	Solutions Improperly Accepted as Candidate (IC): 0
753	Solutions Improperly Rejected (IR): 0
754	Effective Classifications: 24
755	Wrong Classifications: 0
756	Effectiveness in Context= 100.00%
757	
758	Summary for PC0#2 - Effectiveness: 72.78%
759	
760	Sign Test Result: The statistical data corroborate the Null Hypothesis of median estimative for Effectiveness >= 70.00%
761	Results for Test#19 (OCP#3 - Context A):
762	x_NRMSE_max=1.000e+00%
763	u_NRMSE_max=1.000e+00%
764	Sim# return_status objective JOK con u_Energy u_RMS x_SAT_IND(%) u_SAT_IND(%) x_NRMSE(%) u_NRMSE(%) x_IR x_IC u_IR u_IC IRI IC solution
765	A-1 Solve_Succeeded 8.08e-07 1 1 1.06e+00 1.03e+00 0.00e+00 0.00e+00 2.1e-01 -1.00e+00 0 0 0 0 0 0 0 0 Candidate
766	A-2 Solve_Succeeded 3.32e-08 1 1 1.07e+00 1.03e+00 0.00e+00 0.00e+00 4.61e-02 -1.00e+00 0 0 0 0 0 0 0 0 Candidate
767	A-3 Solve_Succeeded 1.34e-01 0 1 2.44e-01 4.94e-01 0.00e+00 0.00e+00 2.56e+01 -1.00e+00 0 0 1 0 0 0 0 0 Rejected

1004	Wrong Classifications: 6																		
1005	Effectiveness in Context= 78.00%																		
1006																			
1007	Results for Test#25 (OCP#3 - Context S):																		
1008	x_NRMSE_max=1.000e+00%																		
1009	u_NRMSE_max=1.000e+00%																		
1010	Sim#	return_status	objective	JOK	con	u_Energy	u_RMS	x_SAT_IND(%)	u_SAT_IND(%)	x_NRMSE(%)	u_NRMSE(%)	x_IR	x_IC	u_IR	u_IC	IR	IC	solution	
1011	S-1	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1012	S-2	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1013	S-3	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1014	S-4	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1015	S-5	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1016	S-6	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1017	S-7	Solve_Succeeded	1.34e-01	0	1	2.43e-01	4.93e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1018	S-8	Solve_Succeeded	1.34e-01	0	1	2.43e-01	4.93e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1019	S-9	Solve_Succeeded	1.34e-01	0	1	2.43e-01	4.93e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1020	S-10	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1021	S-11	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1022	S-12	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1023	S-13	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1024	S-14	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1025	S-15	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1026	S-16	Solve_Succeeded	1.34e-01	0	1	2.43e-01	4.93e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1027	S-17	Solve_Succeeded	1.34e-01	0	1	2.43e-01	4.93e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1028	S-18	Solve_Succeeded	1.34e-01	0	1	2.43e-01	4.93e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1029	S-19	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1030	S-20	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1031	S-21	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1032	S-22	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1033	S-23	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1034	S-24	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1035	S-25	Solve_Succeeded	1.34e-01	0	1	2.43e-01	4.93e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1036	S-26	Solve_Succeeded	1.34e-01	0	1	2.43e-01	4.93e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1037	S-27	Solve_Succeeded	1.34e-01	0	1	2.43e-01	4.93e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1038	Results for Test#25 (OCP#3 - Context S):																		
1039	Solutions: 27																		
1040	Candidates: 0																		
1041	Rejected: 27																		
1042	Solutions Improperly Accepted as Candidate (IC): 0																		
1043	Solutions Improperly Rejected (IR): 0																		
1044	Effective Classifications: 27																		
1045	Wrong Classifications: 0																		
1046	Effectiveness in Context= 100.00%																		
1047																			
1048	Results for Test#26 (OCP#3 - Context T):																		
1049	x_NRMSE_max=2.000e+00%																		
1050	u_NRMSE_max=2.000e+00%																		
1051	Sim#	return_status	objective	JOK	con	u_Energy	u_RMS	x_SAT_IND(%)	u_SAT_IND(%)	x_NRMSE(%)	u_NRMSE(%)	x_IR	x_IC	u_IR	u_IC	IR	IC	solution	
1052	T-1	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1053	T-2	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1054	T-3	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1055	T-4	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1056	T-5	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1057	T-6	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1058	T-7	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1059	T-8	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1060	T-9	Solve_Succeeded	1.34e-01	0	1	2.43e-01	4.93e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1061	T-10	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	
1062	T-11	Solve_Succeeded	1.34e-01	0	1	2.44e-01	4.94e-01	0.00e+00	0.00e+00	2.56e+01	-1.00e+00	0	0	1	0	0	0	Rejected	

1299	u_NRMSE_max=2.000e+00%																	
1300	Sim#	return_status	objective	JOK	con	u_Energy	u_RMS	x_SAT_IND(%)	u_SAT_IND(%)	x_NRMSE(%)	u_NRMSE(%)	x_IR	x_IC	u_IR	u_IC	IR	IC	solution
1301	K-1	Solve_Succeeded	1.68e-02	1	0	8.51e-01	9.22e-01	0.00e+00	8.50e+01	9.05e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1302	K-2	Solve_Succeeded	1.73e-02	1	0	8.04e-01	8.97e-01	0.00e+00	8.00e+01	9.12e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1303	K-3	Restoration_Fail	7.43e-02	-1	-1	0.00e+00	0.00e+00	0.00e+00	0.00e+00	-1.00e+00	-1.00e+00	0	0	1	0	0	0	Not Solv
1304	K-4	Solve_Succeeded	1.42e-02	1	0	8.53e-01	9.24e-01	0.00e+00	8.50e+01	8.44e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1305	K-5	Solve_Succeeded	1.69e-02	1	0	8.03e-01	8.96e-01	0.00e+00	8.00e+01	8.82e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1306	K-6	Solve_Succeeded	1.76e-02	1	0	8.05e-01	8.97e-01	0.00e+00	8.00e+01	8.91e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1307	K-7	Solve_Succeeded	1.67e-02	1	0	8.04e-01	8.97e-01	0.00e+00	8.00e+01	8.89e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1308	K-8	Solve_Succeeded	1.67e-02	1	0	8.05e-01	8.97e-01	0.00e+00	7.00e+01	8.81e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1309	K-9	Solve_Succeeded	1.76e-02	1	1	8.04e-01	8.97e-01	0.00e+00	0.00e+00	8.92e+00	-1.00e+00	0	1	0	0	0	0	1 Candidate
1310	K-10	Solve_Succeeded	1.68e-02	1	0	8.51e-01	9.22e-01	0.00e+00	8.50e+01	9.05e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1311	K-11	Solve_Succeeded	1.47e-02	1	0	9.04e-01	9.51e-01	0.00e+00	9.00e+01	8.56e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1312	K-12	Solve_Succeeded	1.43e-02	1	0	8.53e-01	9.24e-01	0.00e+00	8.50e+01	8.34e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1313	K-13	Solve_Succeeded	1.67e-02	1	0	8.52e-01	9.23e-01	0.00e+00	8.50e+01	9.11e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1314	K-14	Solve_Succeeded	1.72e-02	1	0	8.02e-01	8.96e-01	0.00e+00	8.00e+01	8.97e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1315	K-15	Solve_Succeeded	1.70e-02	1	0	8.05e-01	8.97e-01	0.00e+00	8.00e+01	8.90e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1316	K-16	Solve_Succeeded	1.67e-02	1	0	8.04e-01	8.97e-01	0.00e+00	8.00e+01	8.89e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1317	K-17	Solve_Succeeded	1.67e-02	1	0	8.05e-01	8.97e-01	0.00e+00	7.00e+01	8.81e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1318	K-18	Solve_Succeeded	1.76e-02	1	1	8.04e-01	8.97e-01	0.00e+00	0.00e+00	8.92e+00	-1.00e+00	0	1	0	0	0	0	1 Candidate
1319	K-19	Solve_Succeeded	1.50e-02	1	0	9.04e-01	9.51e-01	0.00e+00	9.00e+01	8.65e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1320	K-20	Solve_Succeeded	1.67e-02	1	0	8.04e-01	8.97e-01	0.00e+00	8.00e+01	8.86e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1321	K-21	Solve_Succeeded	1.76e-02	1	0	8.05e-01	8.97e-01	0.00e+00	8.00e+01	8.91e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1322	K-22	Solve_Succeeded	1.68e-02	1	0	8.51e-01	9.22e-01	0.00e+00	8.50e+01	9.05e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1323	K-23	Solve_Succeeded	1.67e-02	1	0	8.04e-01	8.97e-01	0.00e+00	8.00e+01	8.86e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1324	K-24	Solve_Succeeded	6.87e-02	0	0	9.18e-01	9.58e-01	0.00e+00	9.00e+01	1.52e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1325	K-25	Solve_Succeeded	1.67e-02	1	0	8.04e-01	8.97e-01	0.00e+00	8.00e+01	8.89e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1326	K-26	Solve_Succeeded	1.67e-02	1	0	8.05e-01	8.97e-01	0.00e+00	7.00e+01	8.81e+00	-1.00e+00	0	0	1	0	0	0	Rejected
1327	K-27	Solve_Succeeded	1.76e-02	1	1	8.04e-01	8.97e-01	0.00e+00	0.00e+00	8.92e+00	-1.00e+00	0	1	0	0	0	0	1 Candidate
1328	Results for Test#32 (OCP#4 - Context K):																	
1329	Solutions: 26																	
1330	Candidates: 3																	
1331	Rejected: 23																	
1332	Solutions Improperly Accepted as Candidate (IC): 3																	
1333	Solutions Improperly Rejected (IR): 0																	
1334	Effective Classifications: 23																	
1335	Wrong Classifications: 3																	
1336	Effectiveness in Context= 88.00%																	
1337																		
1338	Results for Test#33 (OCP#4 - Context L):																	
1339	x_NRMSE_max=3.000e+00%																	
1340	u_NRMSE_max=3.000e+00%																	
1341	Sim#	return_status	objective	JOK	con	u_Energy	u_RMS	x_SAT_IND(%)	u_SAT_IND(%)	x_NRMSE(%)	u_NRMSE(%)	x_IR	x_IC	u_IR	u_IC	IR	IC	solution
1342	L-1	Solve_Succeeded	3.22e-02	1	0	7.11e-01	8.43e-01	0.00e+00	7.00e+01	1.18e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1343	L-2	Solve_Succeeded	2.32e-02	1	0	8.02e-01	8.95e-01	0.00e+00	8.00e+01	1.05e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1344	L-3	Solve_Succeeded	2.58e-02	1	0	8.04e-01	8.97e-01	0.00e+00	8.00e+01	1.12e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1345	L-4	Solve_Succeeded	3.26e-02	1	0	7.16e-01	8.46e-01	0.00e+00	7.00e+01	1.20e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1346	L-5	Solve_Succeeded	2.29e-02	1	0	8.02e-01	8.96e-01	0.00e+00	8.00e+01	1.06e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1347	L-6	Solve_Succeeded	2.45e-02	1	0	8.00e-01	8.95e-01	0.00e+00	8.00e+01	1.08e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1348	L-7	Solve_Succeeded	3.26e-02	1	0	7.16e-01	8.46e-01	0.00e+00	7.00e+01	1.20e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1349	L-8	Solve_Succeeded	3.47e-02	1	0	7.14e-01	8.45e-01	0.00e+00	6.00e+01	1.21e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1350	L-9	Solve_Succeeded	3.01e-02	1	0	7.09e-01	8.42e-01	0.00e+00	3.00e+01	1.17e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1351	L-10	Solve_Succeeded	5.13e-02	0	0	9.00e-01	9.49e-01	0.00e+00	9.00e+01	1.52e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1352	L-11	Maximum_CPU_Time	0.00e+00	-1	-1	0.00e+00	0.00e+00	0.00e+00	0.00e+00	-1.00e+00	-1.00e+00	0	0	1	0	0	0	Not Solv
1353	L-12	Solve_Succeeded	3.27e-02	1	0	7.08e-01	8.41e-01	0.00e+00	7.00e+01	1.16e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1354	L-13	Solve_Succeeded	2.58e-02	1	0	8.04e-01	8.97e-01	0.00e+00	8.00e+01	1.13e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1355	L-14	Solve_Succeeded	2.32e-02	1	0	8.02e-01	8.95e-01	0.00e+00	8.00e+01	1.13e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1356	L-15	Solve_Succeeded	2.32e-02	1	0	8.02e-01	8.95e-01	0.00e+00	8.00e+01	1.05e+01	-1.00e+00	0	0	1	0	0	0	Rejected
1357	L-16	Solve_Succeeded	3.26e-02	1	0	7.16e-01	8.46e-01	0.00e+00	7.00e+01	1.20e+01	-1.00e+00	0	0	1	0	0	0	Rejected

1417	Wrong Classifications: 6
1418	Effectiveness in Context= 75.00%
1419	
1420	Results for Test#35 (OCP#4 - Context T):
1421	x_NRMSE_max=2.000e+00%
1422	u_NRMSE_max=2.000e+00%
1423	Sim# return_status objective JOK con u_Energy u_RMS x_SAT_IND(%) u_SAT_IND(%) x_NRMSE(%) u_NRMSE(%) x_IR x_IC u_IR u_IC IR IC solution
1424	T-1 Solve_Succeeded 2.27e-02 1 0 1.85e+00 1.36e+00 0.00e+00 0.00e+00 1.06e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1425	T-2 Solve_Succeeded 3.09e-02 1 0 1.88e+00 1.37e+00 0.00e+00 0.00e+00 1.24e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1426	T-3 Solve_Succeeded 2.61e-02 1 0 1.87e+00 1.37e+00 0.00e+00 0.00e+00 1.14e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1427	T-4 Solve_Succeeded 3.12e-02 1 0 1.85e+00 1.36e+00 0.00e+00 0.00e+00 1.24e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1428	T-5 Solve_Succeeded 3.09e-02 1 0 1.88e+00 1.37e+00 0.00e+00 0.00e+00 1.24e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1429	T-6 Solve_Succeeded 2.57e-02 0 0 1.51e+00 1.23e+00 0.00e+00 0.00e+00 1.12e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1430	T-7 Solve_Succeeded 1.12e-01 0 0 1.76e+00 1.33e+00 0.00e+00 0.00e+00 2.39e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1431	T-8 Solve_Succeeded 1.25e-01 0 0 1.82e+00 1.35e+00 0.00e+00 0.00e+00 2.33e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1432	T-9 Solve_Succeeded 1.12e-01 0 0 1.76e+00 1.33e+00 0.00e+00 0.00e+00 2.19e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1433	T-10 Solve_Succeeded 2.35e-02 1 0 1.83e+00 1.35e+00 0.00e+00 0.00e+00 1.08e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1434	T-11 Solve_Succeeded 3.71e-02 0 0 1.86e+00 1.36e+00 0.00e+00 0.00e+00 1.35e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1435	T-12 Solve_Succeeded 3.09e-02 1 0 1.88e+00 1.37e+00 0.00e+00 0.00e+00 1.24e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1436	T-13 Solve_Succeeded 2.27e-02 1 0 1.85e+00 1.36e+00 0.00e+00 0.00e+00 1.06e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1437	T-14 Solve_Succeeded 3.71e-02 0 0 1.89e+00 1.37e+00 0.00e+00 0.00e+00 1.34e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1438	T-15 Solve_Succeeded 3.71e-02 0 0 1.89e+00 1.37e+00 0.00e+00 0.00e+00 1.34e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1439	T-16 Solve_Succeeded 1.12e-01 0 0 1.76e+00 1.33e+00 0.00e+00 0.00e+00 2.33e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1440	T-17 Solve_Succeeded 1.25e-01 0 0 1.82e+00 1.35e+00 0.00e+00 0.00e+00 2.33e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1441	T-18 Solve_Succeeded 1.12e-01 0 0 1.76e+00 1.33e+00 0.00e+00 0.00e+00 2.19e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1442	T-19 Solve_Succeeded 2.60e-02 1 0 1.49e+00 1.22e+00 0.00e+00 0.00e+00 1.13e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1443	T-20 Solve_Succeeded 3.71e-02 0 0 1.89e+00 1.37e+00 0.00e+00 0.00e+00 1.34e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1444	T-21 Solve_Succeeded 3.09e-02 1 0 1.88e+00 1.37e+00 0.00e+00 0.00e+00 1.24e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1445	T-22 Solve_Succeeded 2.25e-02 1 0 1.49e+00 1.22e+00 0.00e+00 0.00e+00 1.66e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1446	T-23 Solve_Succeeded 1.12e-01 0 0 1.76e+00 1.33e+00 0.00e+00 0.00e+00 2.19e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1447	T-24 Solve_Succeeded 3.71e-02 0 0 1.89e+00 1.37e+00 0.00e+00 0.00e+00 1.34e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1448	T-25 Solve_Succeeded 1.12e-01 0 0 1.76e+00 1.33e+00 0.00e+00 0.00e+00 2.19e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1449	T-26 Solve_Succeeded 1.25e-01 0 0 1.82e+00 1.35e+00 0.00e+00 0.00e+00 2.33e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1450	T-27 Solve_Succeeded 1.12e-01 0 0 1.76e+00 1.33e+00 0.00e+00 0.00e+00 2.19e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1451	Results for Test#35 (OCP#4 - Context T):
1452	Solutions: 27
1453	Candidates: 0
1454	Rejected: 27
1455	Solutions Improperly Accepted as Candidate (IC): 0
1456	Solutions Improperly Rejected (IR): 0
1457	Effective Classifications: 27
1458	Wrong Classifications: 0
1459	Effectiveness in Context= 100.00%
1460	
1461	Results for Test#36 (OCP#4 - Context U):
1462	x_NRMSE_max=3.000e+00%
1463	u_NRMSE_max=3.000e+00%
1464	Sim# return_status objective JOK con u_Energy u_RMS x_SAT_IND(%) u_SAT_IND(%) x_NRMSE(%) u_NRMSE(%) x_IR x_IC u_IR u_IC IR IC solution
1465	U-1 Solve_Succeeded 4.02e-02 1 0 1.79e+00 1.34e+00 0.00e+00 0.00e+00 1.33e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1466	U-2 Solve_Succeeded 4.98e-02 1 0 1.77e+00 1.33e+00 0.00e+00 0.00e+00 1.54e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1467	U-3 Solve_Succeeded 4.98e-02 1 0 1.77e+00 1.33e+00 0.00e+00 0.00e+00 1.54e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1468	U-4 Solve_Succeeded 4.02e-02 1 0 1.79e+00 1.34e+00 0.00e+00 0.00e+00 1.33e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1469	U-5 Solve_Succeeded 4.98e-02 1 0 1.77e+00 1.33e+00 0.00e+00 0.00e+00 1.33e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1470	U-6 Solve_Succeeded 4.02e-02 1 0 1.79e+00 1.34e+00 0.00e+00 0.00e+00 1.33e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1471	U-7 Solve_Succeeded 1.13e-01 0 0 2.01e+00 1.42e+00 0.00e+00 0.00e+00 2.23e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1472	U-8 Solve_Succeeded 1.36e-01 0 0 2.15e+00 1.47e+00 0.00e+00 0.00e+00 2.45e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1473	U-9 Solve_Succeeded 8.67e-02 0 0 1.88e+00 1.37e+00 0.00e+00 0.00e+00 1.95e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1474	U-10 Solve_Succeeded 4.02e-02 1 0 1.79e+00 1.34e+00 0.00e+00 0.00e+00 1.33e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected
1475	U-11 Solve_Succeeded 4.98e-02 1 0 1.77e+00 1.33e+00 0.00e+00 0.00e+00 1.54e+01 -1.00e+00 0 0 1 0 0 0 0 Rejected


```

1535 Test #26: Effectiveness=100.00%
1536 Test #27: Effectiveness=100.00%
1537 Test #28: Effectiveness=100.00%
1538 Test #29: Effectiveness=100.00%
1539 Test #30: Effectiveness= 63.00%
1540 Test #31: Effectiveness= 67.00%
1541 Test #32: Effectiveness= 88.00%
1542 Test #33: Effectiveness= 92.00%
1543 Test #34: Effectiveness= 75.00%
1544 Test #35: Effectiveness=100.00%
1545 Test #36: Effectiveness=100.00%
1546 Summary for PCO#1
1547 Example pag 319 - Figure 13.2 pag 320 Ref[Triola,1999]
1548 Sign Test Result: The statistical data corroborate the Null Hypothesis of median estimative for Effectiveness >= 70.00%
1549 Summary for PCO#2
1550 Example pag 319 - Figure 13.2 pag 320 Ref[Triola,1999]
1551 Sign Test Result: The statistical data corroborate the Null Hypothesis of median estimative for Effectiveness >= 70.00%
1552 Summary for PCO#3
1553 Example pag 319 - Figure 13.2 pag 320 Ref[Triola,1999]
1554 Sign Test Result: The statistical data corroborate the Null Hypothesis of median estimative for Effectiveness >= 70.00%
1555 Summary for PCO#4
1556 Example pag 319 - Figure 13.2 pag 320 Ref[Triola,1999]
1557 Sign Test Result: The statistical data corroborate the Null Hypothesis of median estimative for Effectiveness >= 70.00%
1558 Kruskal Wallis H-test test:
1559 [78.0, 100.0, 100.0, 67.0, 100.0, 0.0, 68.0, 68.0, 93.0]
1560 [100.0, 63.0, 45.0, 100.0, 0.0, 64.0, 100.0, 83.0, 100.0]
1561 [100.0, 100.0, 93.0, 100.0, 89.0, 78.0, 100.0, 100.0, 100.0]
1562 [100.0, 100.0, 63.0, 67.0, 88.0, 92.0, 75.0, 100.0, 100.0]
1563 H-statistic: 3.97956246402
1564 P-Value: 0.26367948262
1565 pval = 0.264 > significance (alpha) = 0.05000
1566 Accept NULL hypothesis - No significant difference between groups.
1567 The significance (alpha) is 0.05000 (confidence=95.00000%)
1568 *****
1569 References
1570 Sign Test[1]
1571 [1] Gibbons, J. D., and S. Chakraborti. Nonparametric Statistical Inference, 5th Ed. Boca Raton, FL: Chapman & Hall/CRC Press,
1572 Taylor & Francis Group, 2011.
1573 Kruskal-Wallis Test [2]
1574 [2] Daniel, Wayne W. (1990). 'Kruskal-Wallis one-way analysis of variance by ranks'. Applied Nonparametric Statistics (2nd ed.).
1575 Boston: PWS-Kent. pp. 226-234. ISBN 0-534-91976-6.
1576 *****
1577

```


APPENDIX C

THE OUTPUT OF THE KRUSKAL-WALLIS TEST

GET

```
FILE='C:\Users\User\Documents\SPSS_OCP\JM2.2_OCP.sav'.
DATASET NAME DataSet1 WINDOW=FRONT.
FREQUENCIES VARIABLES=Effectiveness OCP
/ORDER=ANALYSIS.
```

Frequencies

Notes		
Output Created		09-JUL-2018 18:32:58
Comments		
Input	Data	C:\Users\User\Documents\SPSS_OCP\JM2.2_OCP.sav
	Active Dataset	DataSet1
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	36
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics are based on all cases with valid data.
Syntax		FREQUENCIES VARIABLES=Effectiveness OCP /ORDER=ANALYSIS.
Resources	Processor Time	00:00:00,02
	Elapsed Time	00:00:00,02

[DataSet1] C:\Users\User\Documents\SPSS_OCP\JM2.2_OCP.sav

Statistics

		Effectiveness	OCP
N	Valid	36	36
	Missing	0	0

Frequency Table

Effectiveness

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	2	5,6	5,6	5,6
	45	1	2,8	2,8	8,3
	63	2	5,6	5,6	13,9
	64	1	2,8	2,8	16,7
	67	2	5,6	5,6	22,2
	68	2	5,6	5,6	27,8
	75	1	2,8	2,8	30,6
	78	2	5,6	5,6	36,1
	83	1	2,8	2,8	38,9
	88	1	2,8	2,8	41,7
	89	1	2,8	2,8	44,4
	92	1	2,8	2,8	47,2
	93	2	5,6	5,6	52,8
	100	17	47,2	47,2	100,0
	Total	36	100,0	100,0	

OCP

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	9	25,0	25,0	25,0
	2	9	25,0	25,0	50,0
	3	9	25,0	25,0	75,0
	4	9	25,0	25,0	100,0
	Total	36	100,0	100,0	

*Nonparametric Tests: Independent Samples.

NPTESTS

/INDEPENDENT TEST (Effectiveness) GROUP (OCP) KRUSKAL_WALLIS COMPARE=PAIRWISE)
 E) MEDIAN(TESTVALUE=SAMPLE COMPARE=PAIRWISE)
 /MISSING SCOPE=ANALYSIS USERMISSING=EXCLUDE
 /CRITERIA ALPHA=0.05 CILEVEL=95.

Nonparametric Tests

Notes

Output Created		09-JUL-2018 18:33:34
Comments		
Input	Data	C:\Users\User\Documents\S PSS_OCP\JM2.2_OCP. sav
	Active Dataset	DataSet1
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	36
Syntax		NPTESTS /INDEPENDENT TEST (Effectiveness) GROUP (OCP) KRUSKAL_WALLIS (COMPARE=PAIRWISE) MEDIAN (TESTVALUE=SAMPLE COMPARE=PAIRWISE) /MISSING SCOPE=ANALYSIS USERMISSING=EXCLUD E /CRITERIA ALPHA=0.05 ...
Resources	Processor Time	00:00:00,27
	Elapsed Time	00:00:00,22

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The medians of Effectiveness are the same across categories of OCP.	Independent-Samples Median Test	,548	Retain the null hypothesis.
2	The distribution of Effectiveness is the same across categories of OCP.	Independent-Samples Kruskal-Wallis Test	,264	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is ,05.

NPAR TESTS

```

/K-W=Effectiveness BY OCP(1 4)
/MEDIAN=Effectiveness BY OCP(1 4)
/MISSING ANALYSIS.

```

NPar Tests

Notes		
Output Created		09-JUL-2018 18:34:04
Comments		
Input	Data	C:\Users\User\Documents\S PSS_OCP\JM2.2_OCP. sav
	Active Dataset	DataSet1
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	36
Missing Value Handling	Definition of Missing	User-defined missing values are treated as missing.
	Cases Used	Statistics for each test are based on all cases with valid data for the variable(s) used in that test.
Syntax		NPAR TESTS /K-W=Effectiveness BY OCP(1 4) /MEDIAN=Effectiveness BY OCP(1 4) /MISSING ANALYSIS.
Resources	Processor Time	00:00:00,00
	Elapsed Time	00:00:00,00
	Number of Cases Allowed ^a	449389

a. Based on availability of workspace memory.

Kruskal-Wallis Test

Ranks

	OCP	N	Mean Rank
Effectiveness	1	9	15,89
	2	9	15,67
	3	9	23,89
	4	9	18,56
	Total	36	

Test Statistics^{a,b}

Effectiveness	
Kruskal-Wallis H	3,980
df	3
Asymp. Sig.	,264

a. Kruskal Wallis Test

b. Grouping Variable: OCP

Median Test**Frequencies**

		OCP			
		1	2	3	4
Effectiveness	> Median	3	4	6	4
	<= Median	6	5	3	5

Test Statistics^a

Effectiveness	
N	36
Median	93,00
Chi-Square	2,118 ^b
df	3
Asymp. Sig.	,548

a. Grouping Variable: OCP

b. 8 cells (100,0%) have expected frequencies less than 5. The minimum expected cell frequency is 4,3.

INDEX

- Bolza, 95
- CONCLUSION, 91
- CONTRIBUTIONS, 41
 - Algorithm to Treat Nonconvex OCPs, 41
 - ASAF - Automated Simulation and Analysis Framework, 52
 - Description of the Process of Algorithm Application, 48
 - Sequence of the Algorithm Application, 44
 - The Integrated and Automated Environment for Modeling, Simulation and Optimization, 48
- Definitions, 46
 - Context, 46
 - Type-I Parameters Group, 46
 - Type-II Parameters Group, 46
 - Type-III Parameters Group, 46
- DISCUSSION OF THE RESULTS, 87
- Environment for Modeling, Simulation and Optimization
 - HSL Routines, 51
 - IPOPT, 50
 - JModelica.org, 49
 - Python and other tools, 51
- Graphic Description of the Contribution of the Thesis, 27
- Hypothesis
 - Analysis of the Optimizations Results for OCP#1, 68
 - First OCP Used in the Hypothesis Verification, 59
 - Fourth OCP Used in the Hypothesis Verification, 64
 - Optimizations Results, 65
 - Optimizations Results for OCP#1, 67
 - Second OCP Used in the Hypothesis Verification, 62
 - The *MATLAB*[®] Code for the Hypothesis Verification, 84
 - The Hypothesis Verification by Statistical Tests, 84
 - Third OCP Used in the Hypothesis Verification, 64
 - Verification, 59
 - Verification by *Kruskal-Wallis Test*, 85
 - Verification by *Signal Test*, 84

Verification by the application IBM[®]

SPSS[®] *Statistics*, 85

INTRODUCTION, 25

Lagrange, 95

integrand, 95

Lagrangian, 95

Mayer, 95

METHODOLOGY, 31

Bibliographical Review, 31

Most Cited Articles on the Topic, 34

Searches in Databases, 33

Statistical Analysis, 38

The Hypothesis, 35

The Refutation Method, 36

The Scientific Problem, 35

Tools to Verify the Hypothesis, 38

Results, 65

STATISTICAL ANALYSIS OF THE SIMULATIONS,

101

SUMMARY OF THE *DYNAMIC OPTIMIZATION*

BY DIRECT COLLOCATION, 93

Collocation Polynomials, 96

NLP, 97

Nonlinear Program, 97

The Transformation of the OCP, 97

THE HYPOTHESIS VERIFICATION, 59

THE OUTPUT OF THE KRUSKAL-WALLIS

TEST, 129